Accelerator physics

A brief introduction

Z.Martí

ALBA

Outline

- Introduction
- Accelerator technology
- Beam dynamics

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Why would someone make so much efforts to break something?...

Why accelerators

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To see what there is inside!

Why accelerators

- The more energy we put in the particles the bigger the detail!

$\updownarrow$

$$\lambda = \frac{h}{p}$$

- As de Broglie (1923) would say: the more momentum (p), the smaller the associated *wavelength* (λ): the deeper in the laws of nature we can reach!

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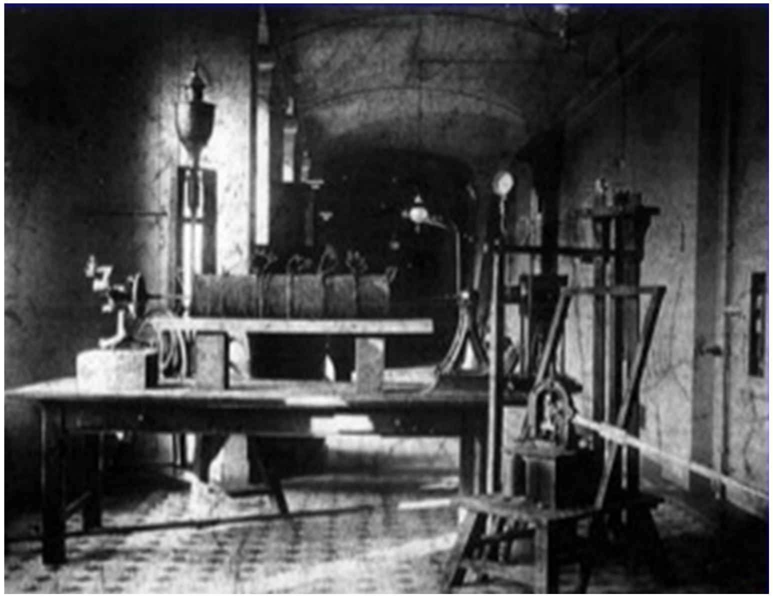
Roentgen's Lab (1895):



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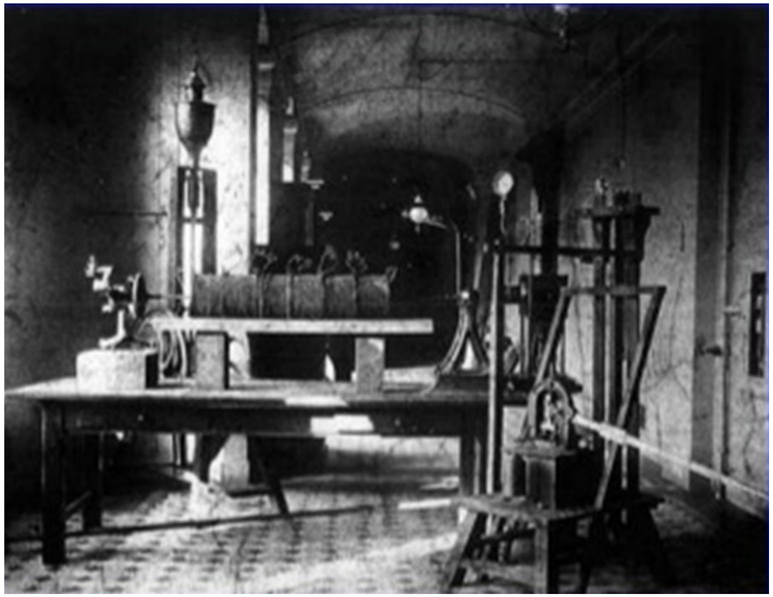
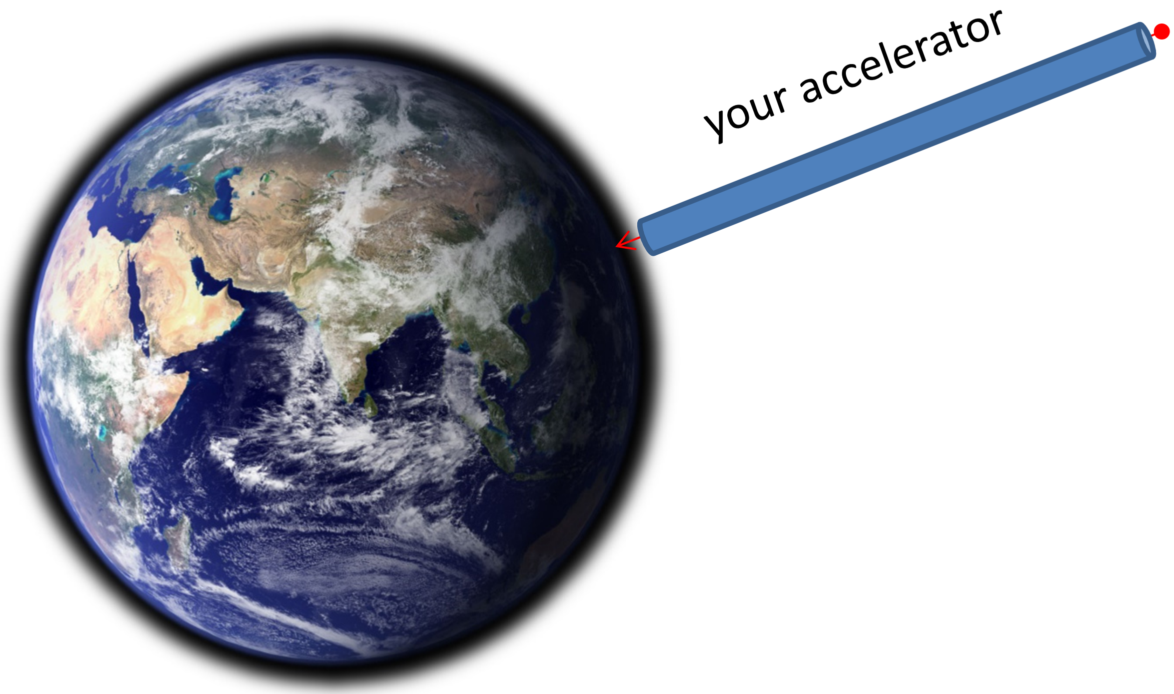


The first x-ray picture: Roentgen's wife hand!

How to build your accelerator

We could use gravitational fields!



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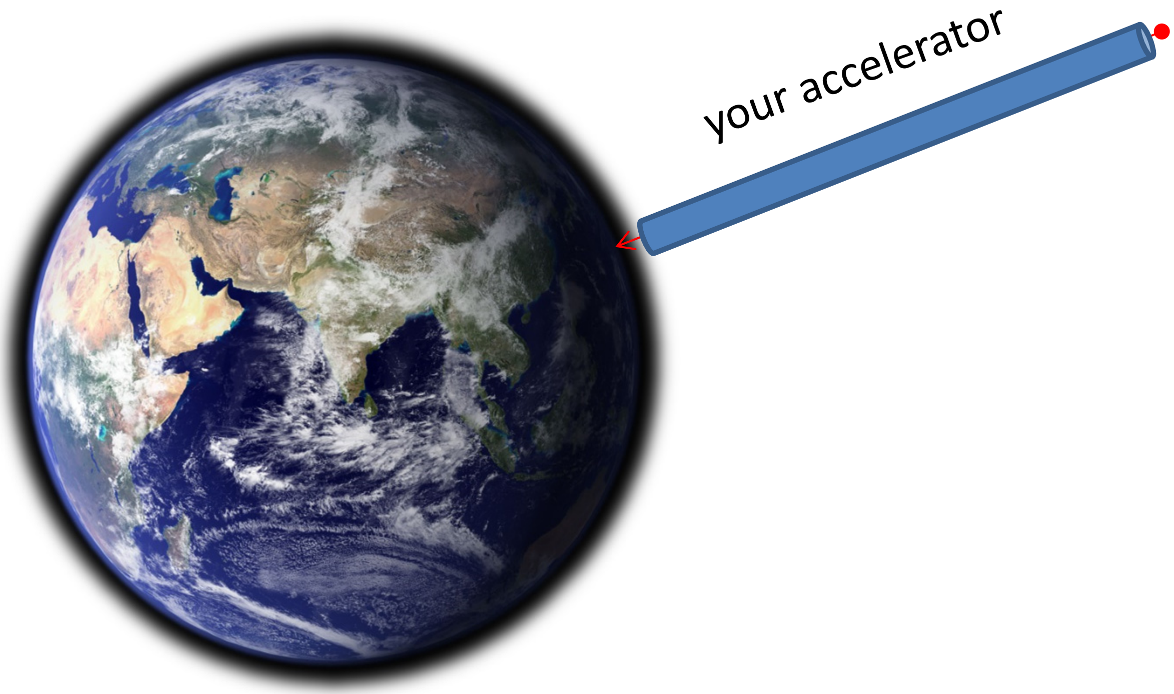
Using the Earth: 11,2 km/s << c=300000 km/s



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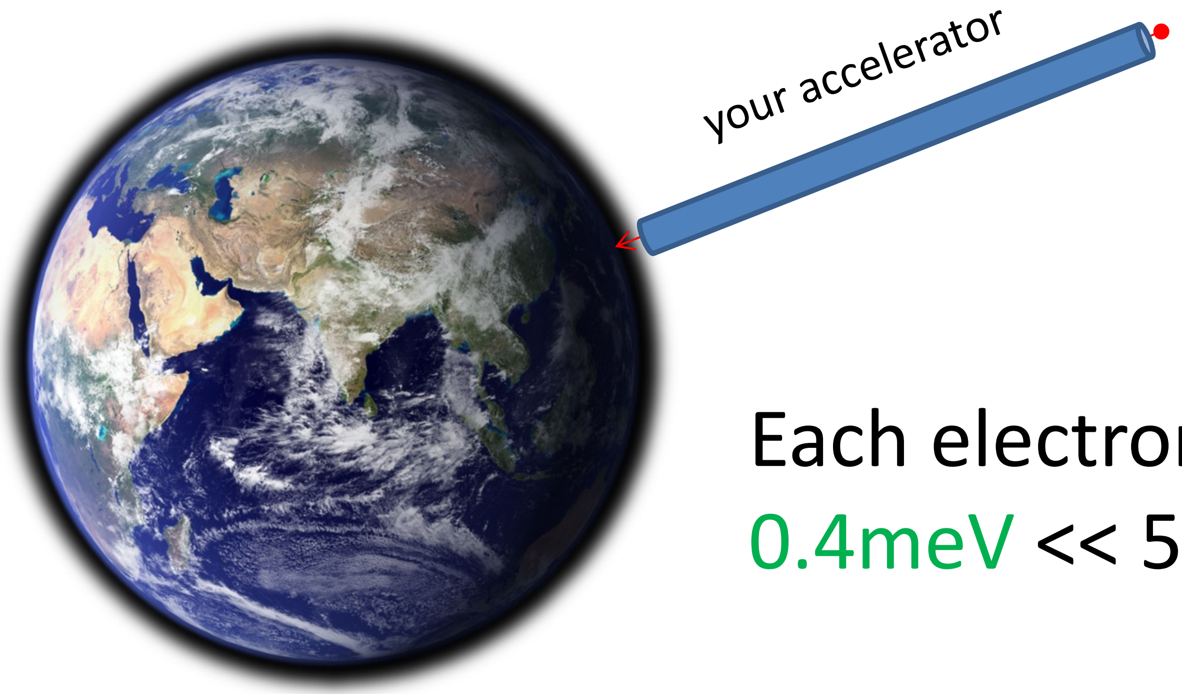

Each electron:

0.4meV << 511keV!!

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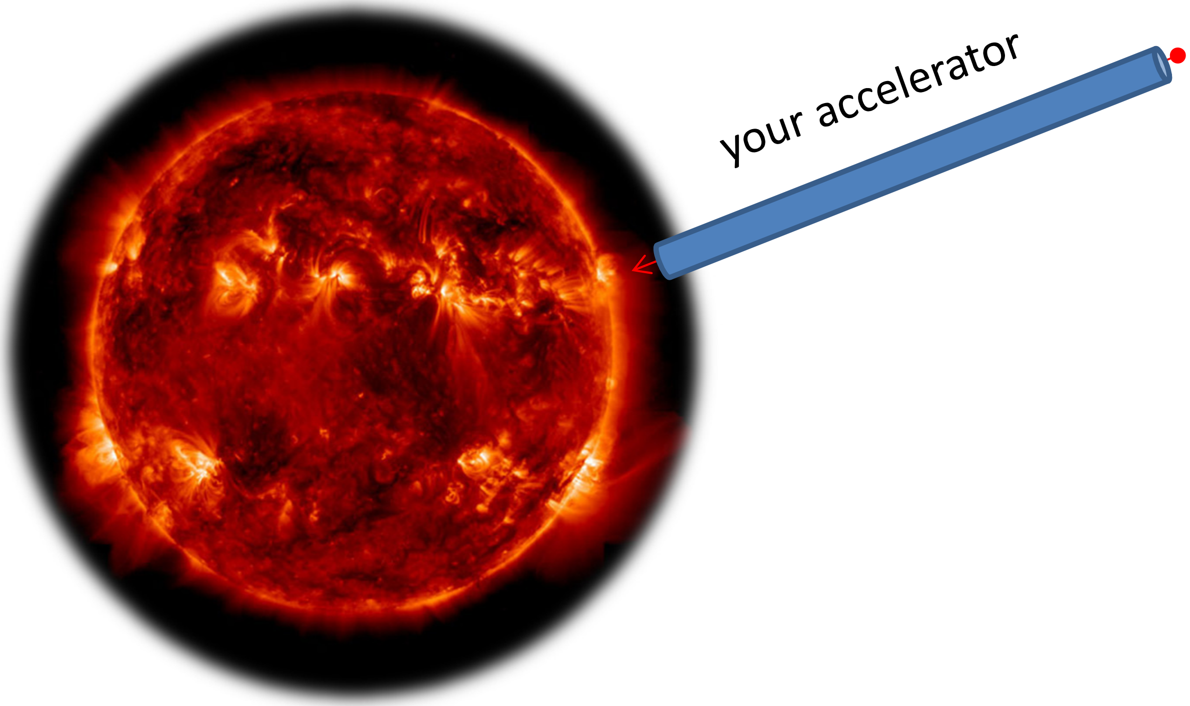

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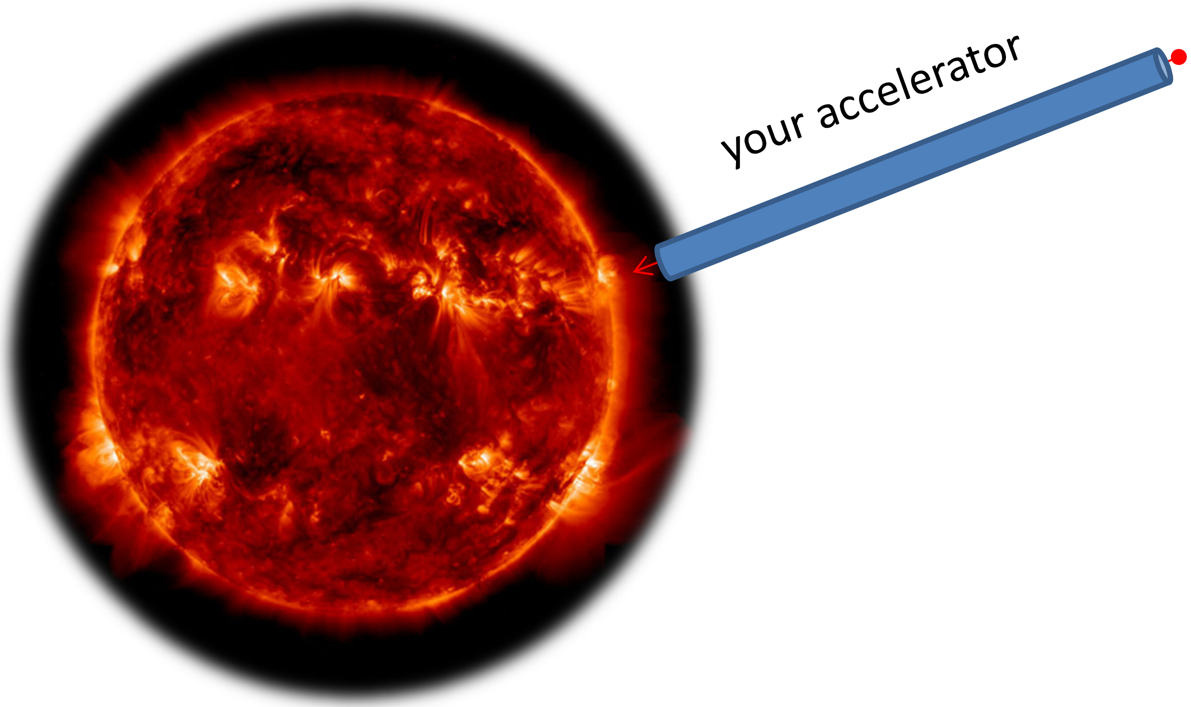

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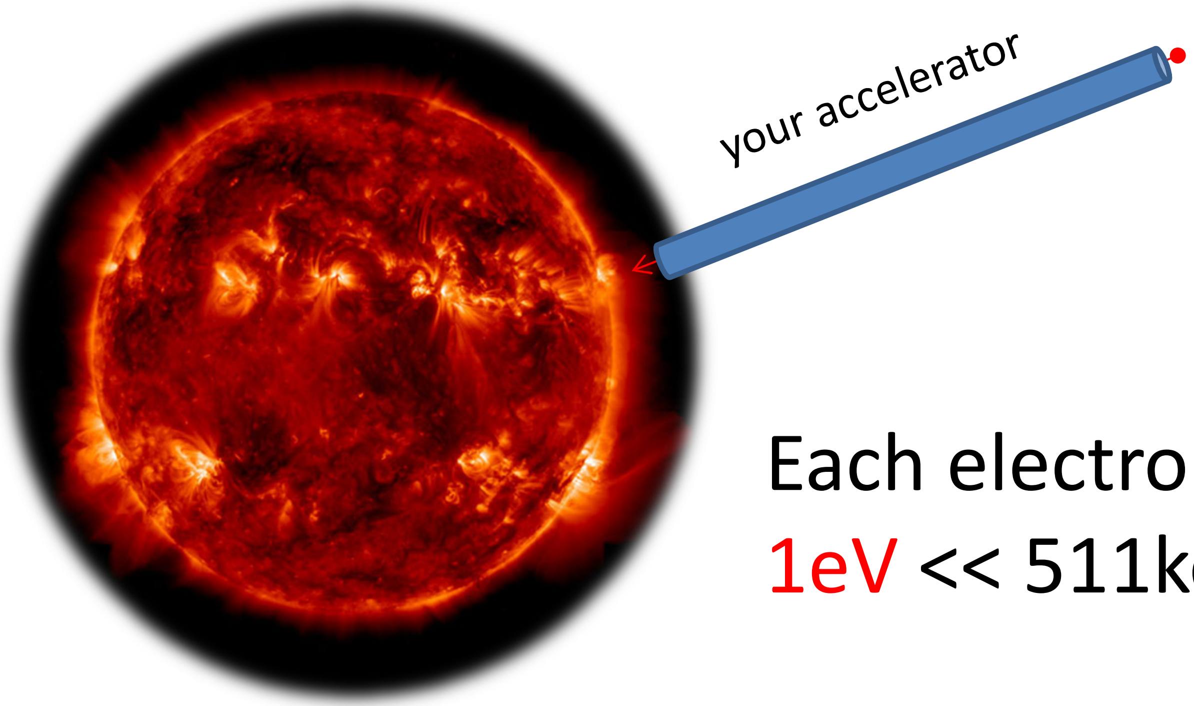

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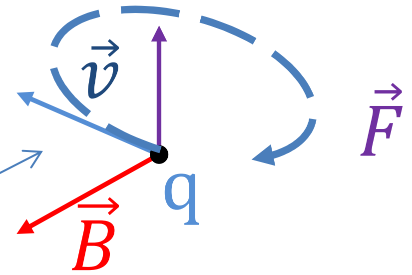

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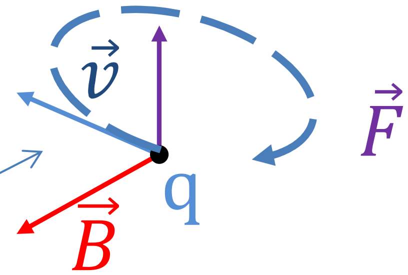

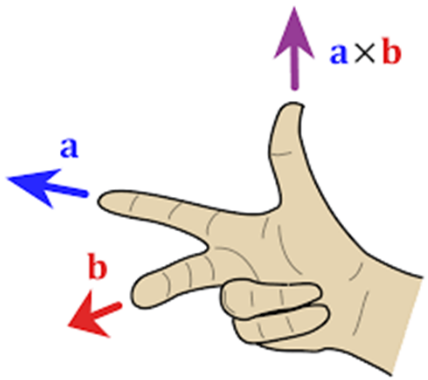

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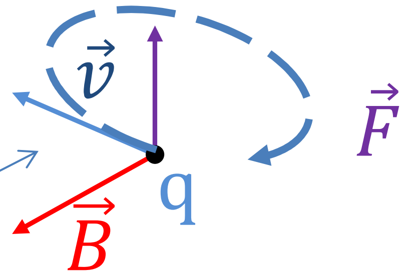

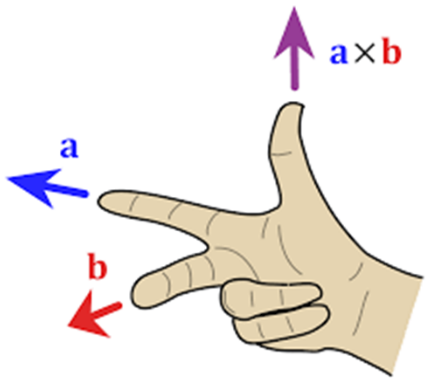

Only acts upon charged particles!

It is not very useful to accelerate since it does not increase the particle energy:

$$\int \vec{F}\overrightarrow{ds} = q \int \left(\frac{\overrightarrow{ds}}{dt} \times \vec{B} \right) \overrightarrow{ds} = 0$$

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It does a good job, but has a some limitations…

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A **static** electric field accelerates only in a certain region, which is not good for multi pass acc. (circular acc.)…

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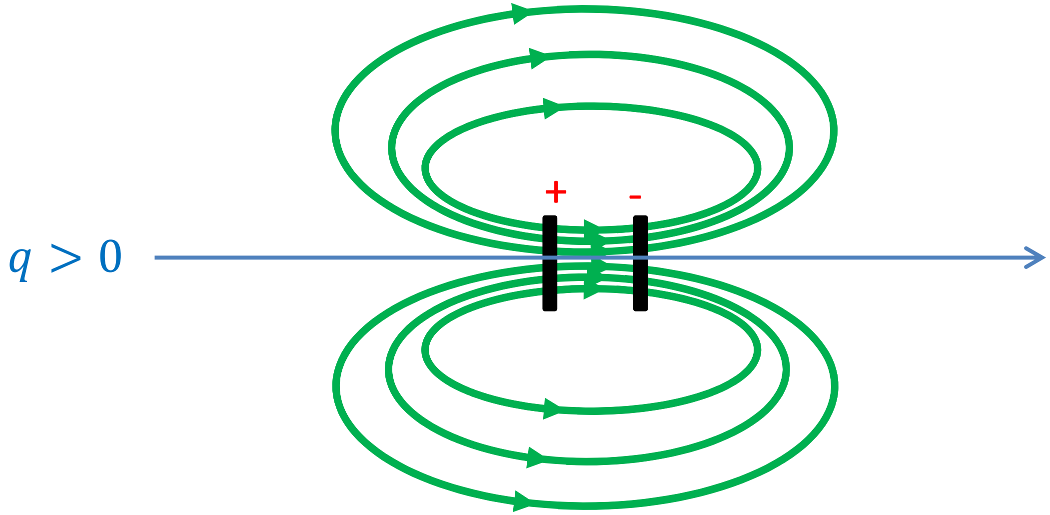

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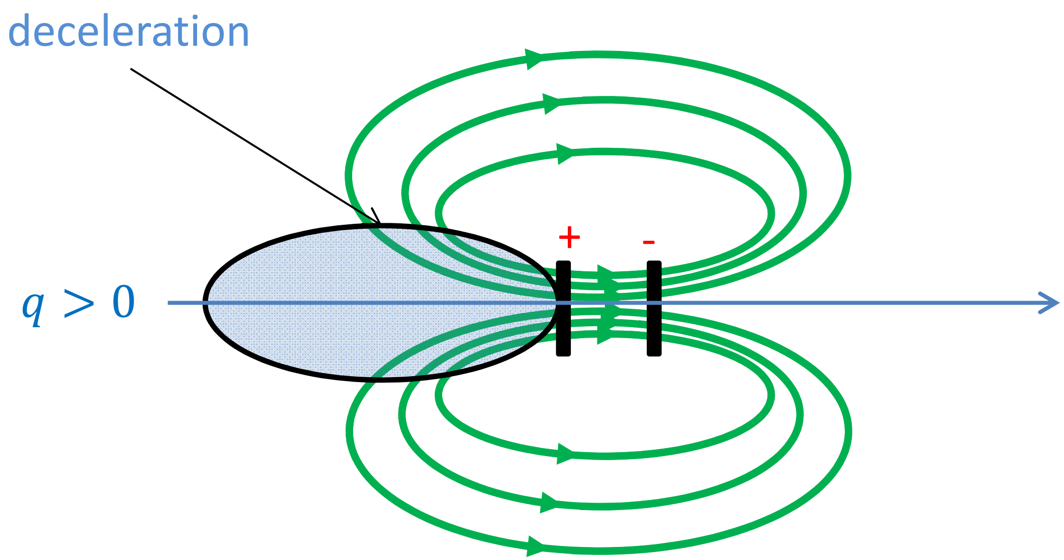

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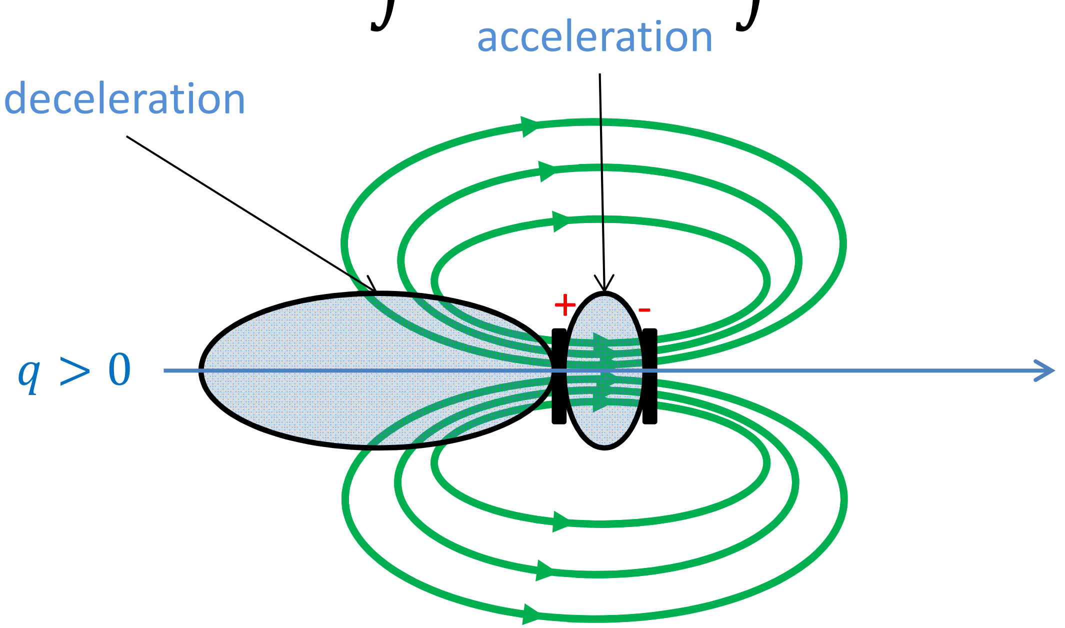

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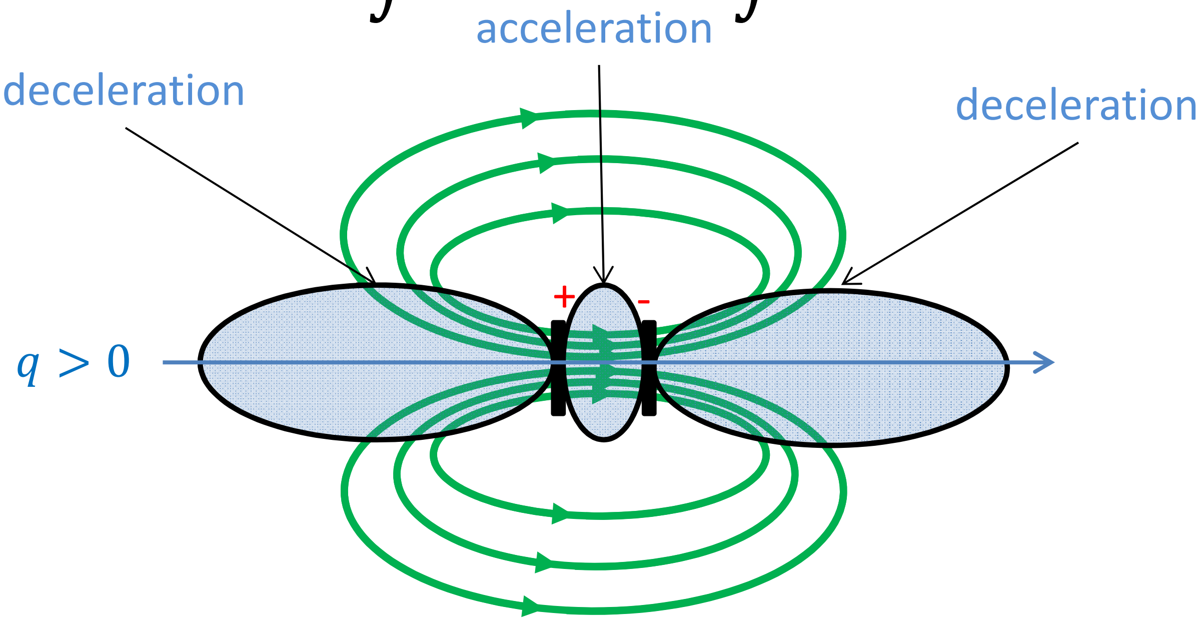

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Which is equivalent to **Faraday's induction law**.

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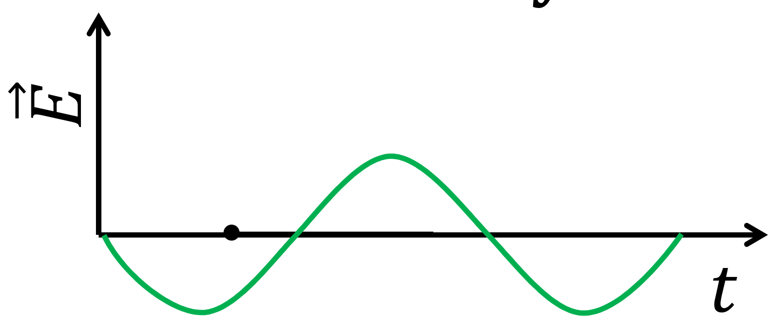

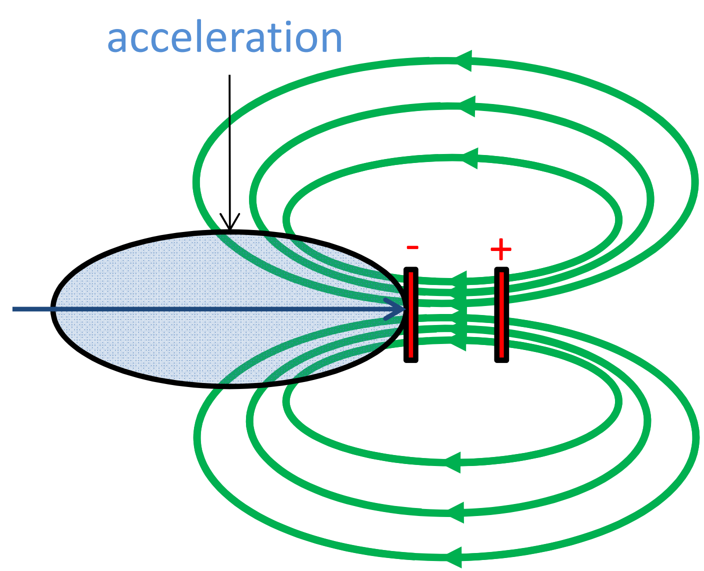

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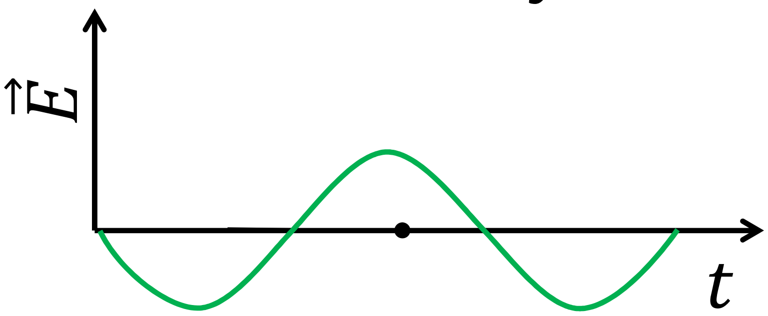

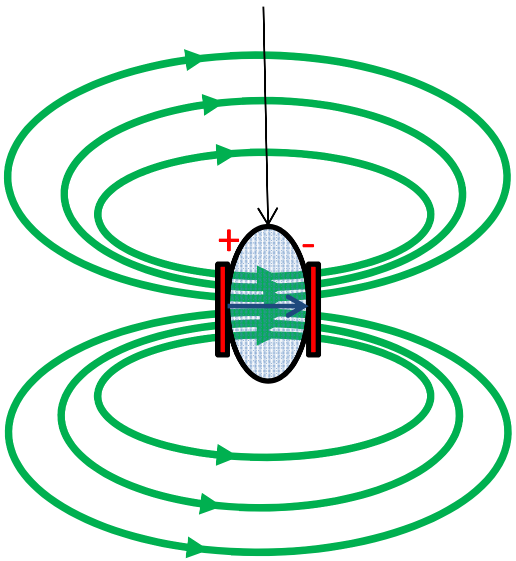

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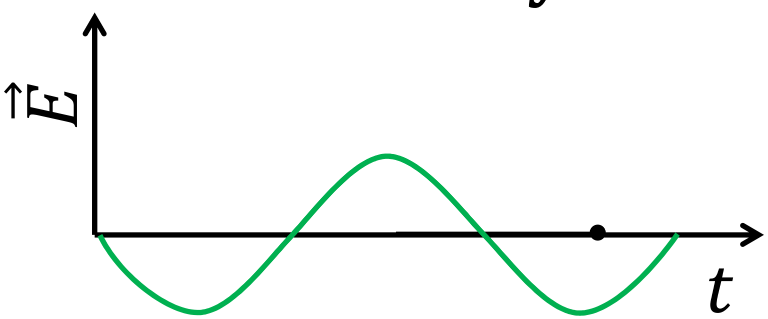

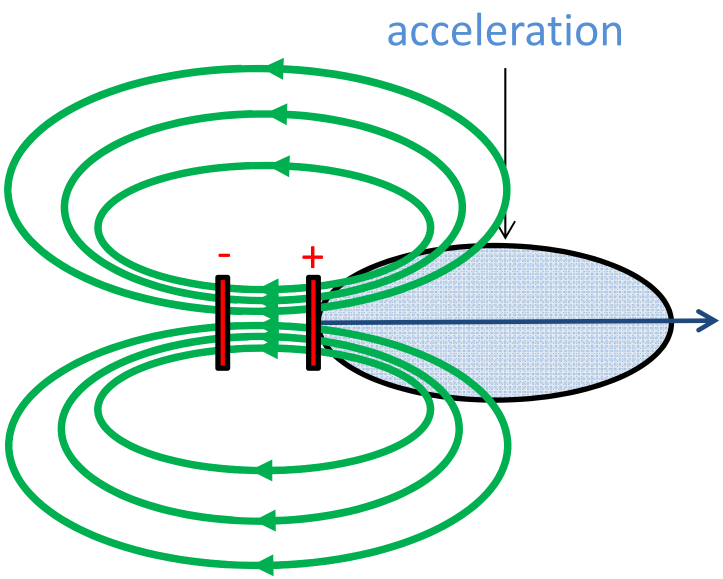

Caution: temporal structure in the beam!

How to build your accelerator

In summary, mother nature offers us:

1. Gravitational field: is too weak.
2. Magnetic field: is strong but it is only good for bending, it does not increase the particle energy.
3. Electric field: can produce higher energy gains if it varies with time.

How to build your accelerator

So, basically everything we do with accelerators is explained by Maxwell laws:

Coulomb's (Gauss') law:

$$q = -\oiint \epsilon \vec{E}\,\overrightarrow{da}$$

Gauss' law:

$$\oiint \vec{B}\,\overrightarrow{da} = 0$$

Faraday's law:

$$\oint \vec{E}\,\overrightarrow{ds} = -\iint \frac{\partial \vec{B}}{\partial t}\,\overrightarrow{da}$$

Ampere's law:

$$\oint \frac{\vec{B}}{\mu}\,\overrightarrow{ds} = \iint \frac{\partial \epsilon \vec{E}}{\partial t}\,\overrightarrow{da} + \iint \vec{J}\,\overrightarrow{da}$$

A bit of relativity

Fact: the speed of light is irrelative!

- $= mc^2.$
- The measured elapsed times and the distances are different in different inertial reference systems (time dilatation and length contraction).

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- Particles have a maximum propagation velocity: the speed of light (c).
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Fact: the speed of light is irrelative!

As Albert Einstein showed to the world in 1905:

- Particles have a maximum propagation velocity: the speed of light (c).
- Particles have a minimum energy ("rest energy"): $E\ 0\ 0\ 0\ = mc^2$.
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E 0 =*mm c* 2 *cc c* 2 *2 c 2 .*

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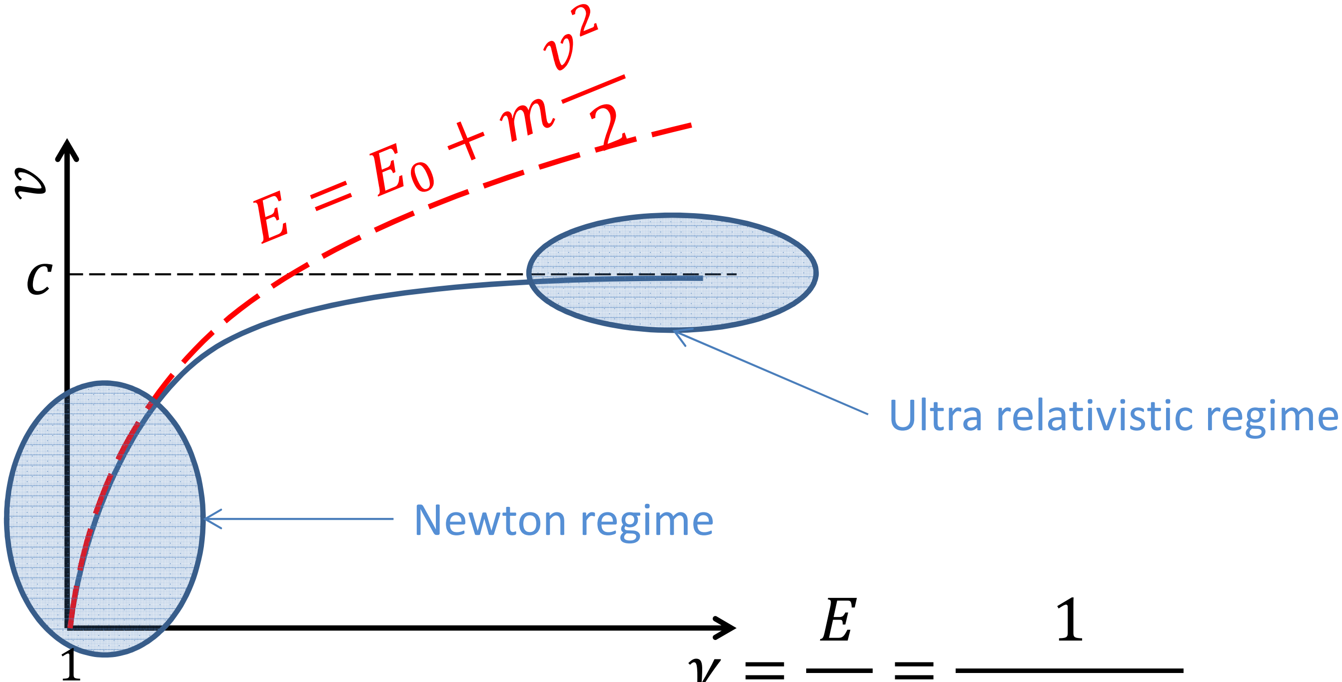
The particle's momentum p and the kinematic energy E are expressed as follows:

$$E = \gamma m c^2 \; \left(\frac{1}{2} m v^2\right)$$

$$p = \gamma m v \quad (mv)$$

A bit of relativity

If the truth be told, nowadays, many accelerators do almost no acceleration:



$$\gamma = \frac{E}{E_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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The muon experiment

(forget about acc. for 1 sec.):

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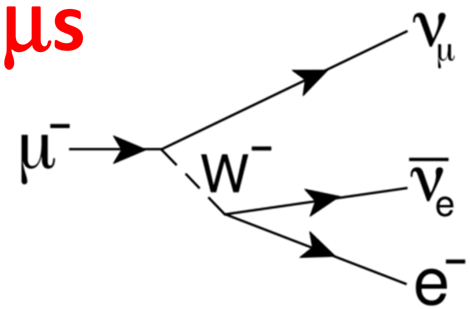
T_0=2.2μs



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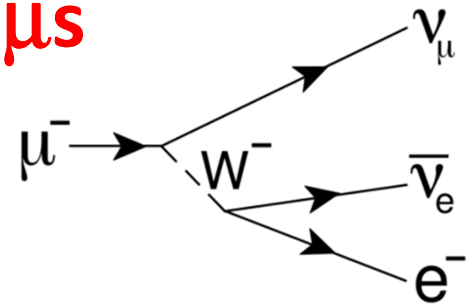
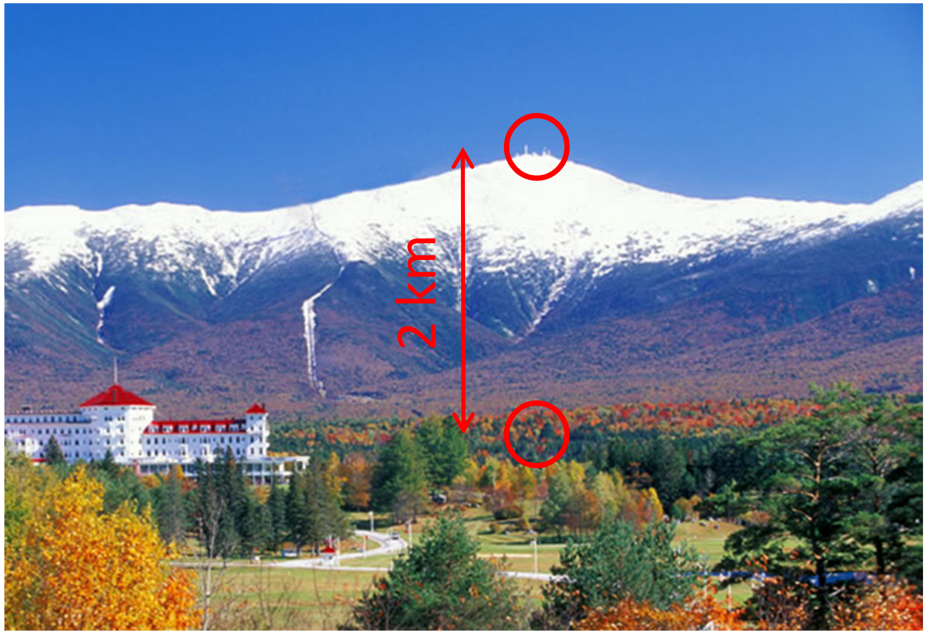


Rossi, B. and Hall (1941) in Mt.Washington in New Hampshire



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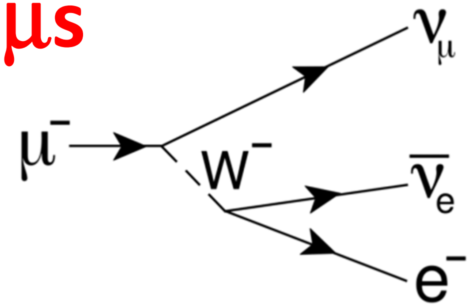

Given such lifetime, measuring muon's energy and flux at the two labs, whithout time dilatation should be 22, but it is 1.4 in agreement with relativity.

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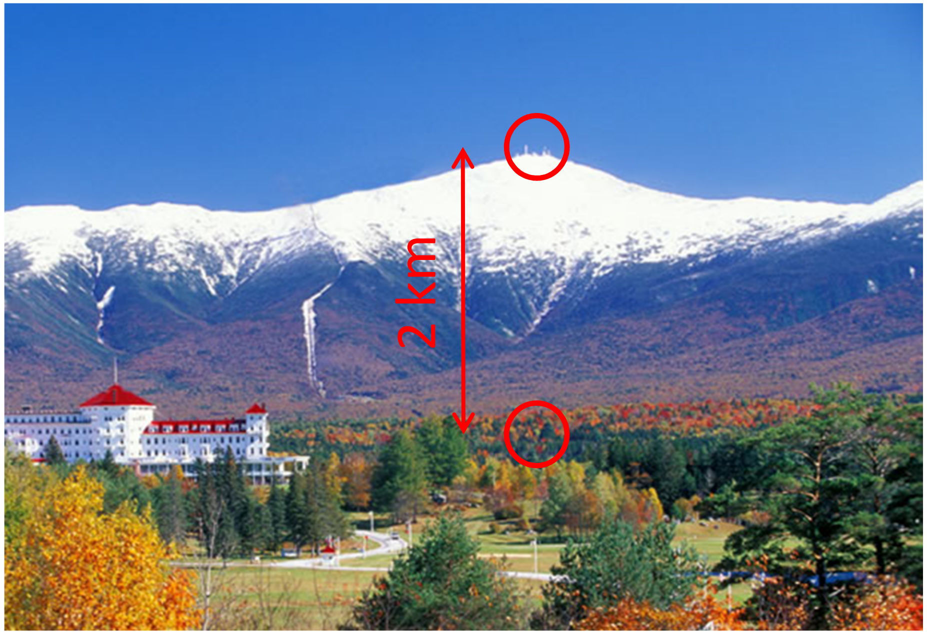

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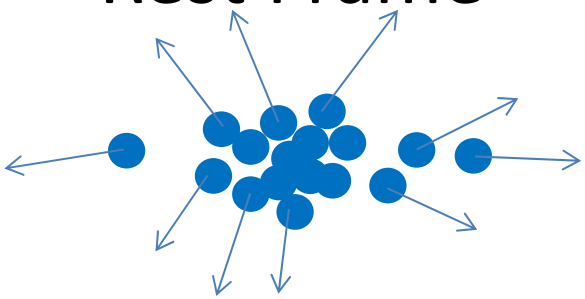
Rest Frame



Δt

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Δt $\gamma\Delta t$

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Rest Frame

Lab Frame

Δt $\qquad$ $\gamma \Delta t$

They don't hold together, but in the lab frame, due to time dilatation they separate slowly! (same as muons lifetime seems longer)

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1. After some acceleration, particles do not get faster any more, which is good, because we can transfer more energy without changing the accelerator geometry!
2. Yes, we can accelerate a bunch of particles with the same charge if we accelerate them rapidly!

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However, the maximum electric field achievable is limited and a series of accelerating structures are disposed one after the other…

This leads to the concept of linear accelerator (Linac) in contrast to circular accelerator.

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Linear acceleration $E_0 \gg E \; (v \ll c)$:

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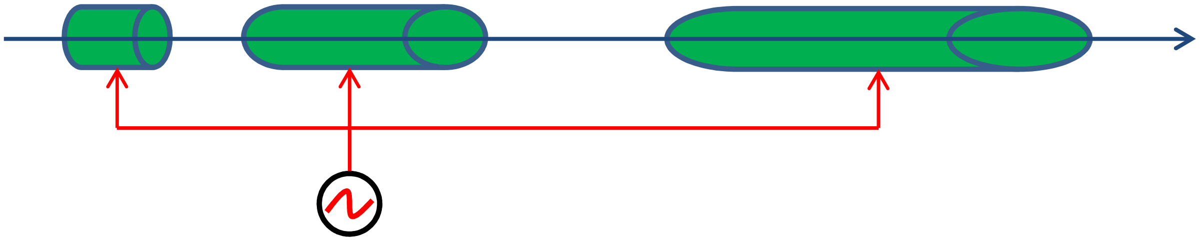

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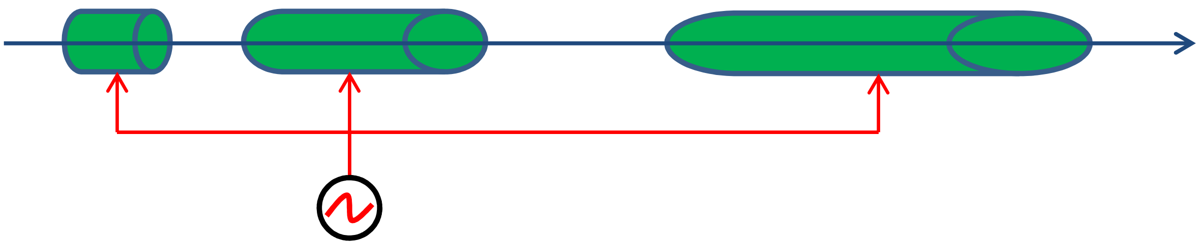

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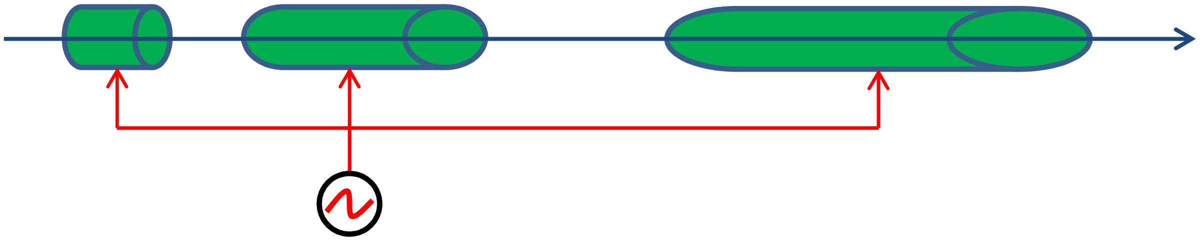
Accelerators geometry

Linear acceleration $E_0 \gg E \ (v \ll c)$:



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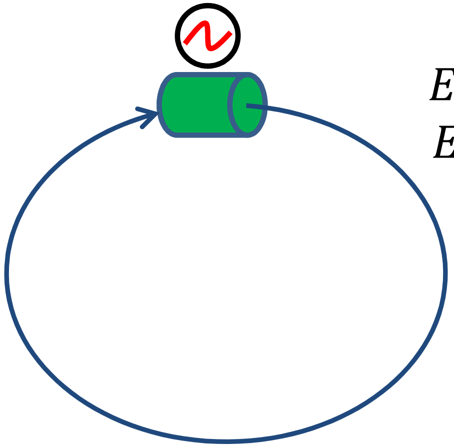
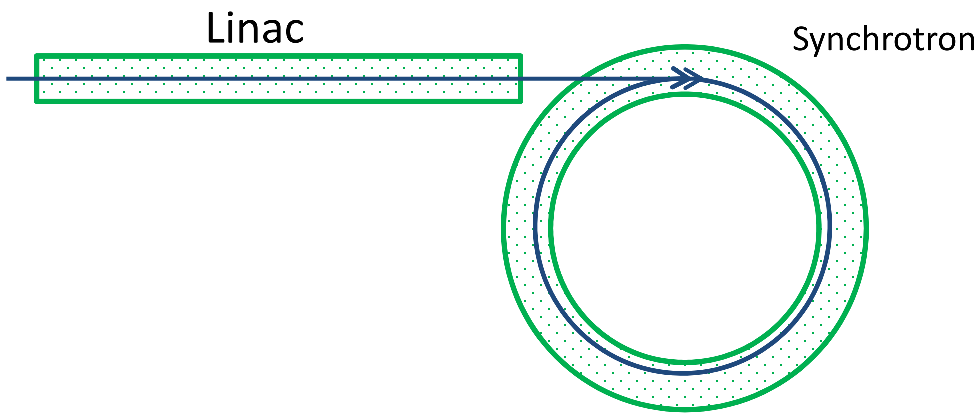


$E_0 \ll E \ (v \approx c) \rightarrow \ freq = ct$

$E_0 \sim E \ (v \lesssim c) \rightarrow \ freq \neq ct$

Accelerators geometry

Typically the two schemes are combined :

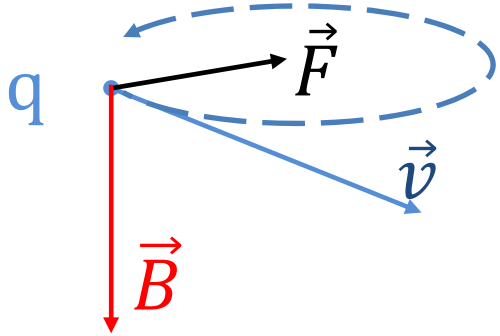


There are around 90 synchrotron radiation facilities in the world using this configuration.

Circular accelerators

First of all, how do we close the circle? Magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B}$$



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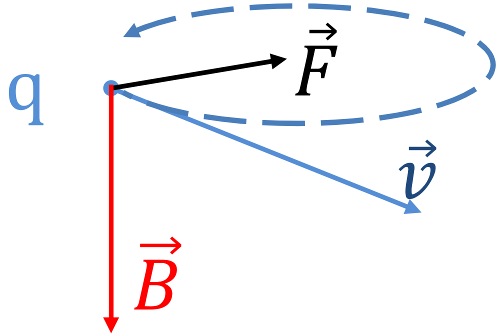

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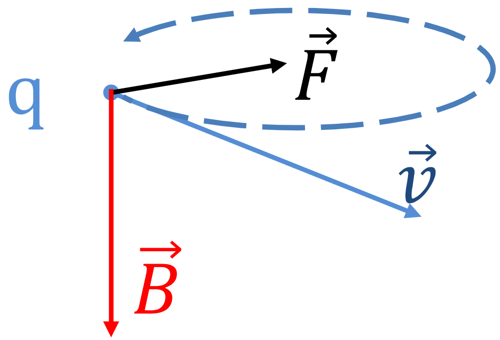

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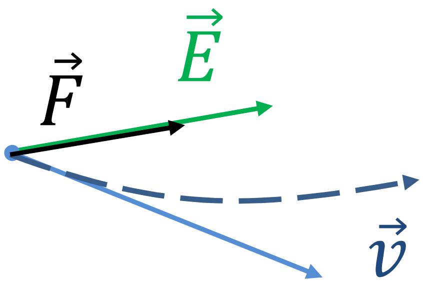

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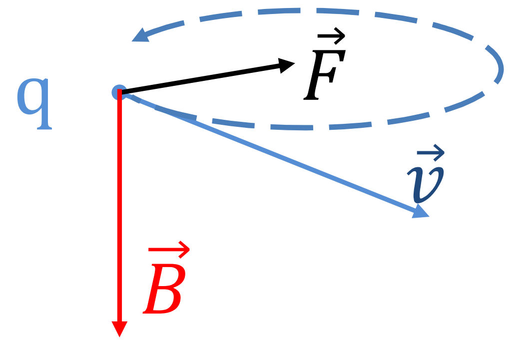

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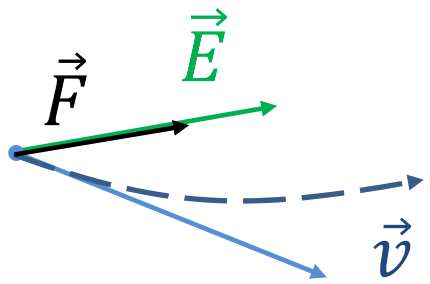

Yes, but…

Circular accelerators

If, $v \approx c$ (3x10^{8}m/s) then, a magnetic field of 1T (modest field) equals in **strength** to a 3MV/cm electric field (high tech field)!

$\ll c$, electric fields will always be more effective.

In circular accelerators most guiding fields are magnetic while in the first Linac stages guiding field is electric.

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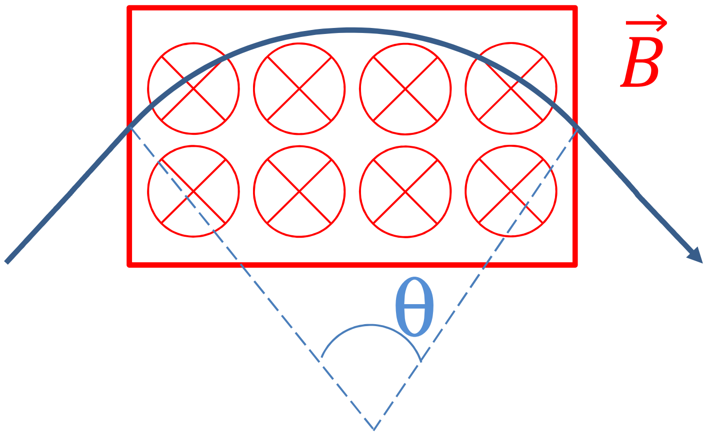

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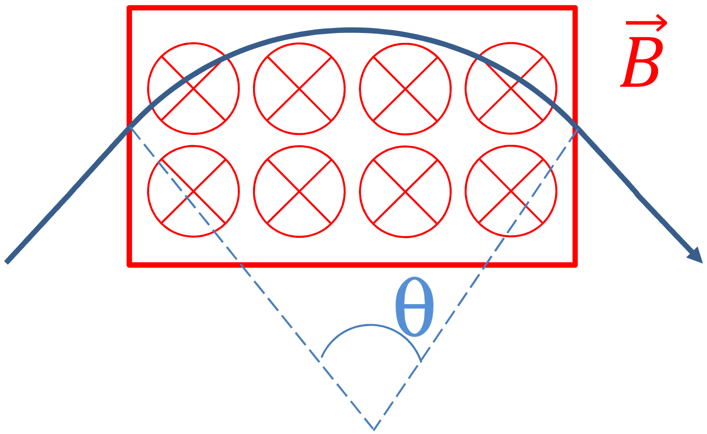

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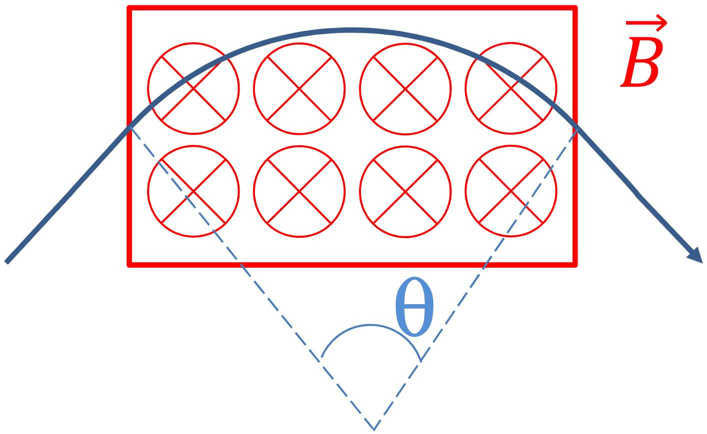

	ALBA	LHC
Num. Mag.	32	1232
θ[°]	11.25	0.3

Circular accelerators

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Most circular accelerator work in the ultra relativistic regime (p $= \gamma m v \approx \gamma m c \approx E/c$):

$$\rho = \frac{E}{qcB}$$

Synchrotron accelerators

Compared with other circular accelerators, the synchrotron design (1945) opened the route to large scale facilities and a rapid increase of the particles energy…

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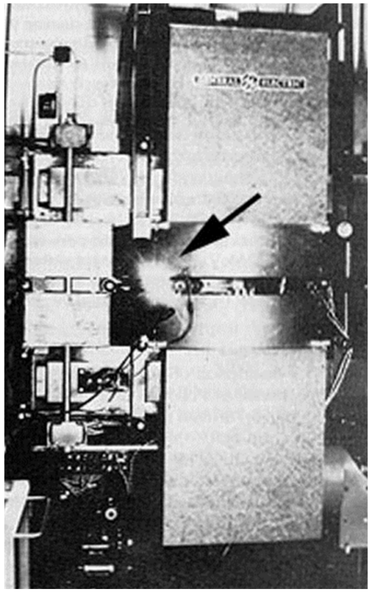
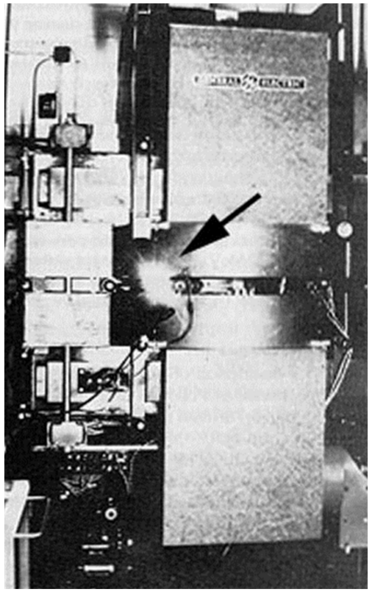


General Electric synchrotron accelerator

Synchrotron accelerators

Compared with other circular accelerators, the synchrotron design (1945) opened the route to large scale facilities and a rapid increase of the particles energy…

At higher energies, in 1946 they found out some light was coming out when particles passed through the guiding magnetic field.



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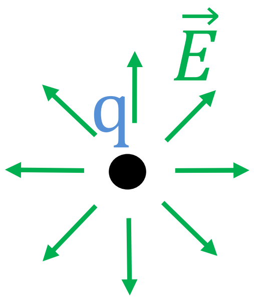

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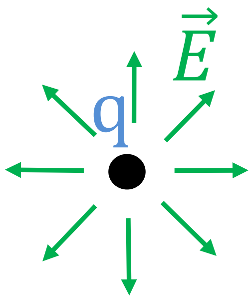
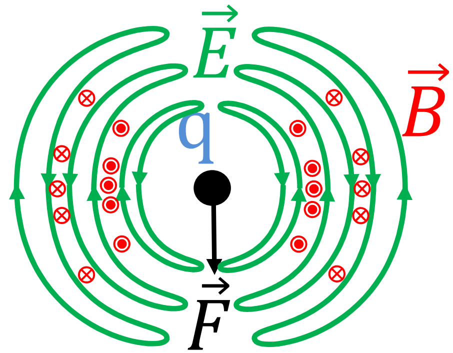


Non Inertial rest Frame



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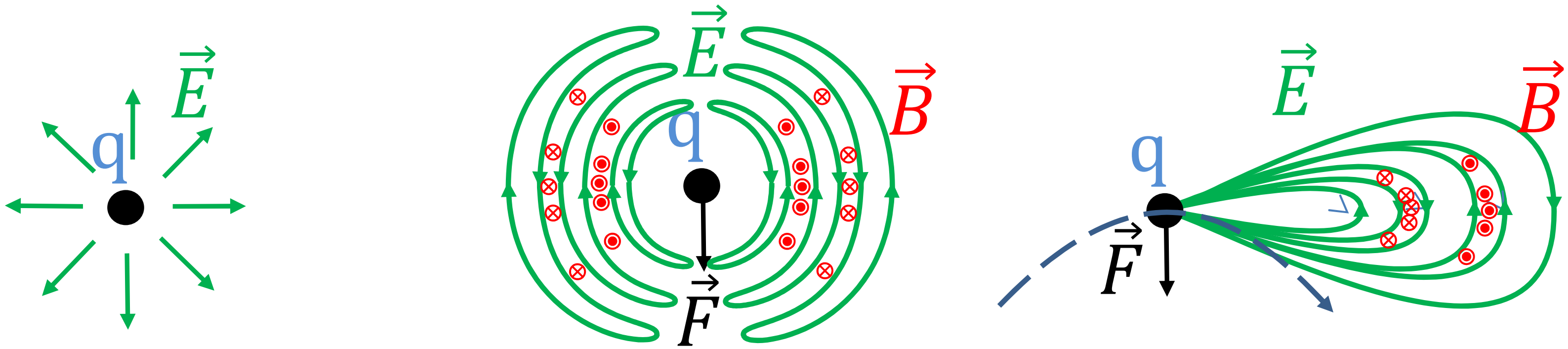

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Synchrotron accelerators

Synchrotron light is not different to the antenna radiation. However:

1. Particles move with a velocity respect to the observer, which adds a relativistic Doppler effect: a factor γ.
2. At relativistic speeds, time dilatation also adds another factor γ.

As particles travel in the accelerator they find magnetic elements at a frequency around GHz, then, X rays are produced!

Focusing the beam

Both in case of colliders or synchrotron radiation facilities, the smaller the beam size, the better!

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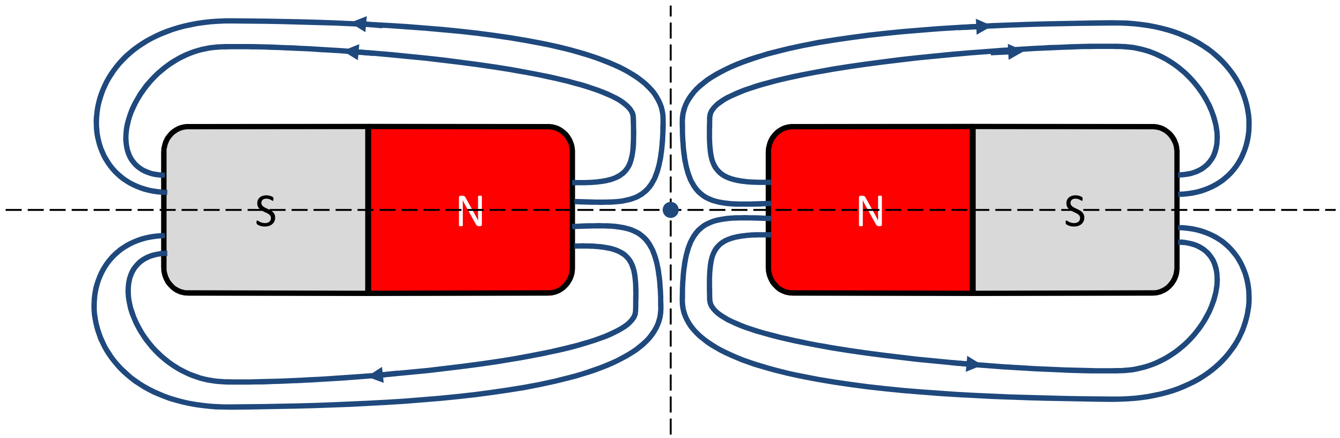
We need to cancel the field at the beam center!

Focusing the beam

A simple: way opposing two magnets!



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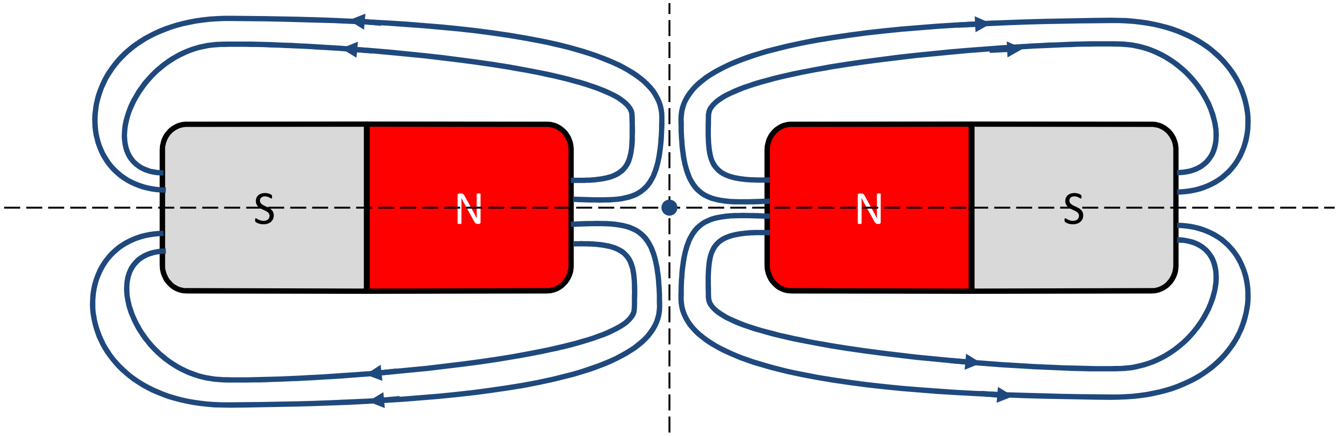
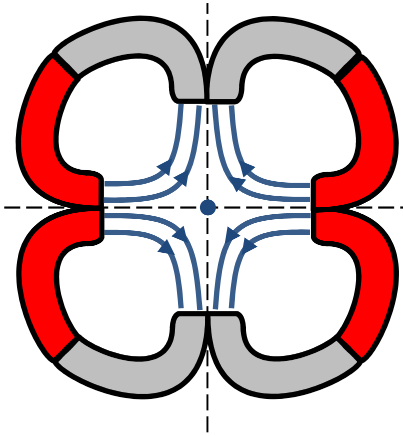


Or more efficient (4 poles working):



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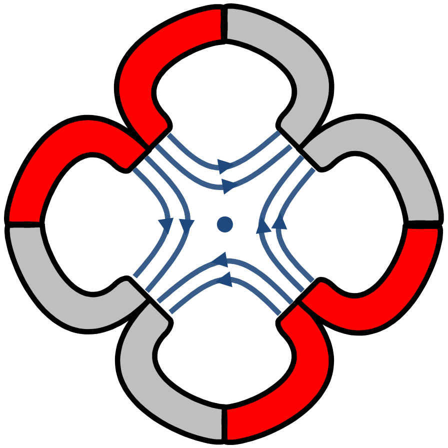
For a proper focusing we need to place the poles 45 deg (called quadrupole):



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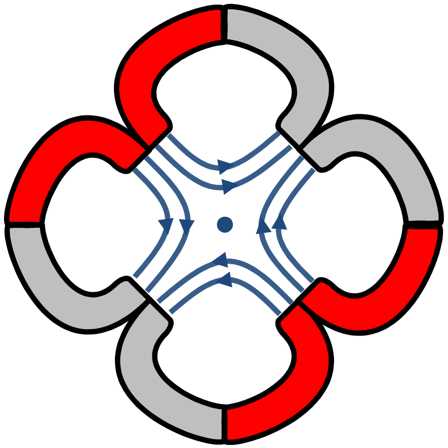

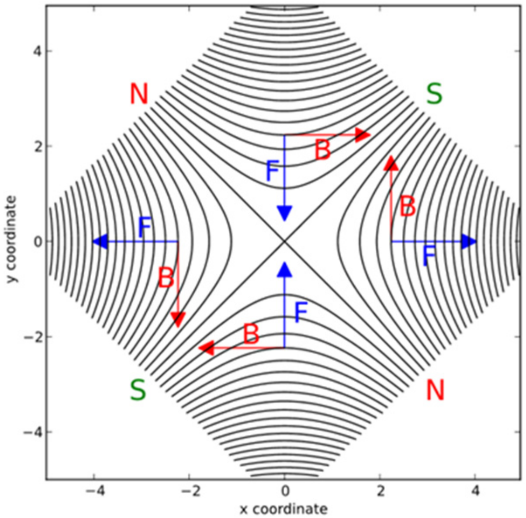

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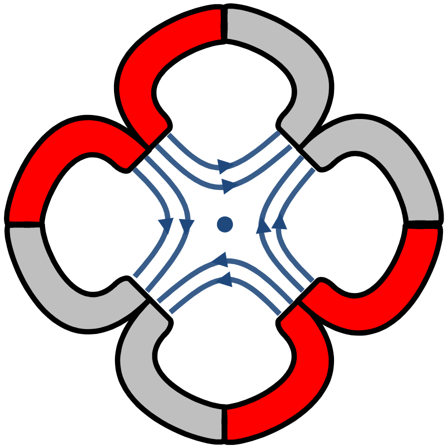

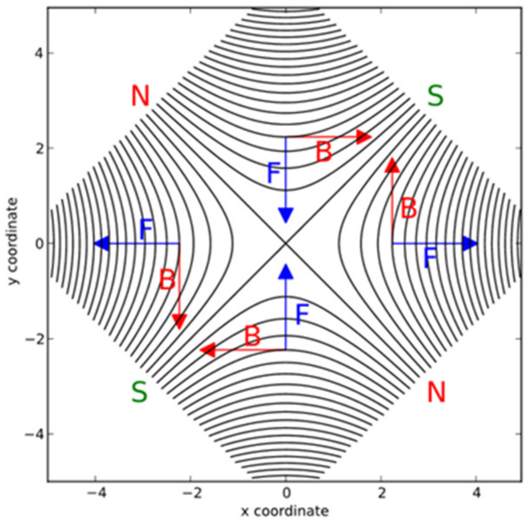

Unfortunately we focus in one plane and defocus in the other:

$$F_x = k\Delta x \qquad F_y = -k\Delta y$$

Outline

- Introduction
- Accelerator technology
- Beam dynamics

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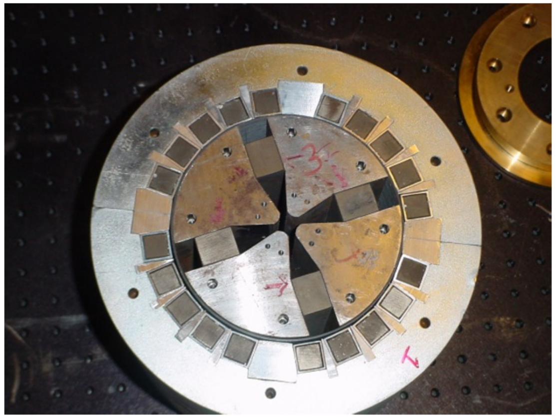
- Vacuum systems.

Magnet technology

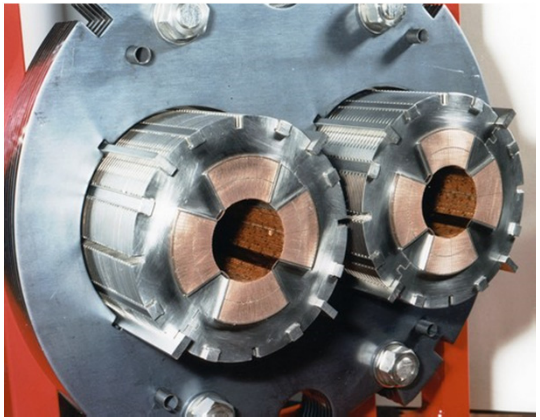
Regarding magnetic field usage (bend and focus), up to three different technologies are used:

Permanent magnets:



Normal conducting electromagnets:

Superconducting electromagnets:



Magnet technology

Ampere's law (intra medium)

$$\oint \frac{\vec{B}}{\mu} \overrightarrow{ds} = \iint \frac{\partial \epsilon \vec{E}}{\partial t} \overrightarrow{da} + \iint \vec{J} \overrightarrow{da}$$

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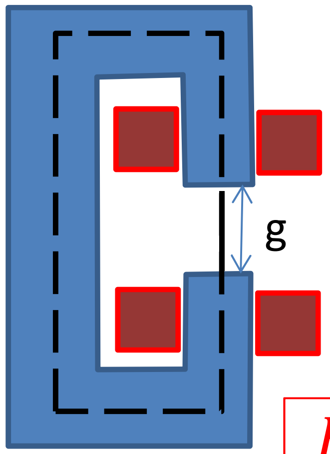

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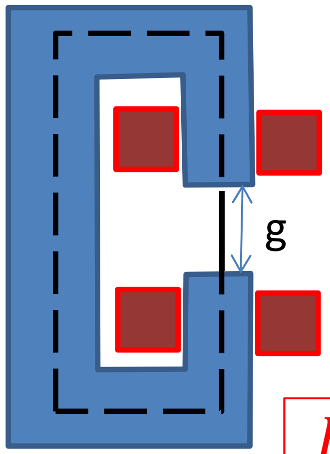

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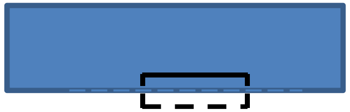

For a very thin loop in the boundary:

$$\oint \frac{\vec{B}}{\mu}\overrightarrow{ds} = 0$$

For an small enough loop:

$$\frac{B_{\parallel}}{\mu} = \frac{B_{0,\parallel}}{\mu_0}$$

Magnet technology

High permeability ($\mu \gg \mu_0$) materials allow to:

Substance	Permeability
Air, dry	Slightly more than 1
Aluminum	Slightly more than 1
Bismuth	Slightly less than 1
Cobalt	60 to 70
Ferrite	100 to 3000
Free space (vacuum)	1 (exactly)
Iron	60 to 8000
Nickel	50 to 60
Permalloy	3000 to 30,000
Silver	Slightly less than 1
Steel	300 to 600
Specialized alloys	100,000 to 1,000,000

1. Concentrate flux lines in the gap:

$$B_0 \simeq \frac{I\mu_0}{g}$$

2. Generate equipotential surfaces:

$$B_{0,\parallel} \simeq 0$$

Magnet technology

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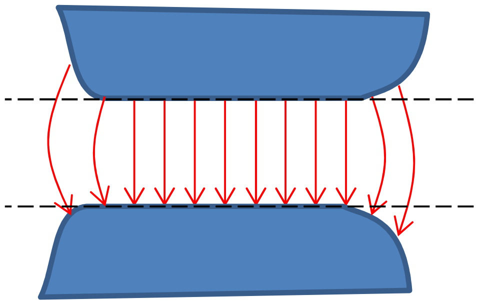

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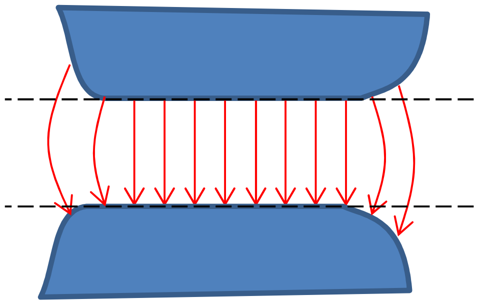
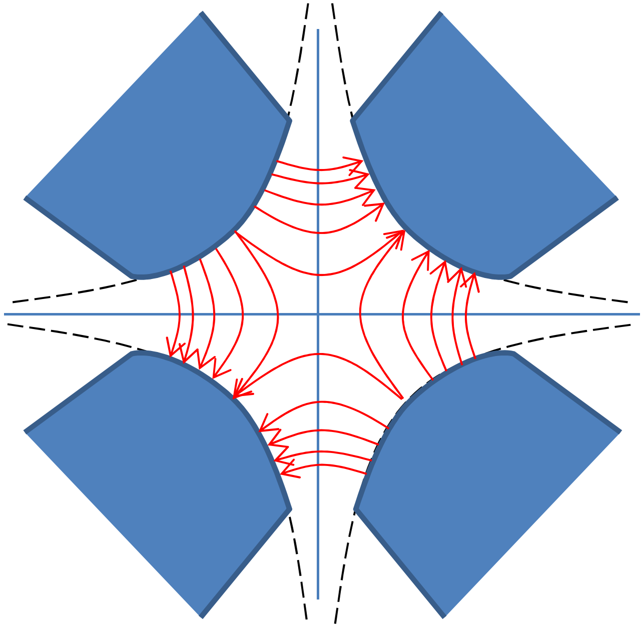
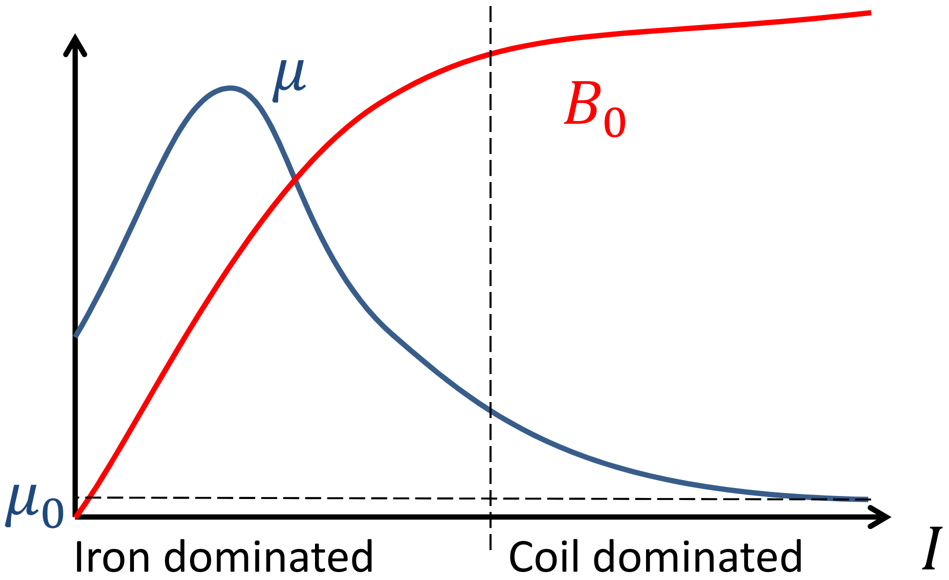


1. Dipoles: constant magnetic field, equipotential lines are straight lines.
2. Quadrupoles: linear magnetic field, equipotential lines are hyperbolas.



Magnet technology

Both permanent magnets and electromagnets are used in the iron dominated regime:



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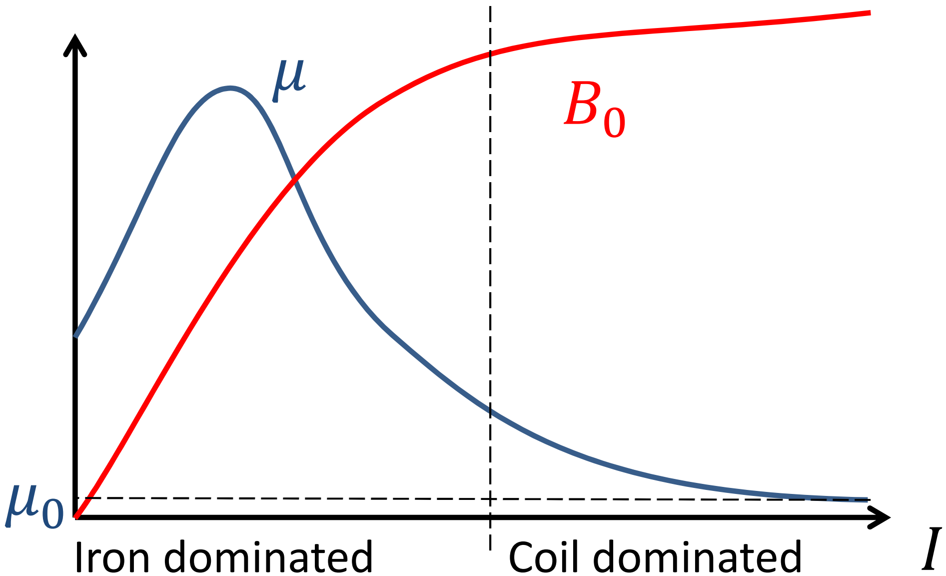

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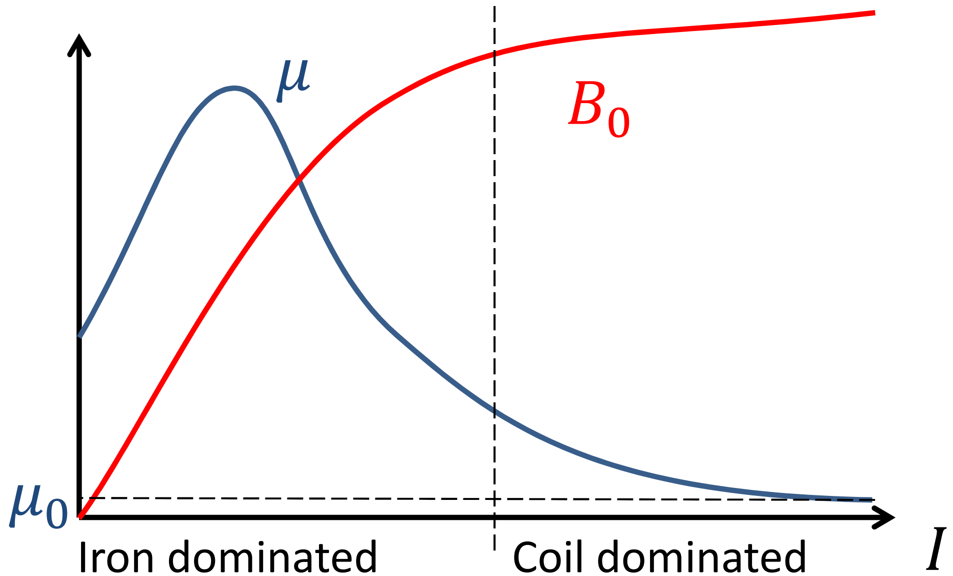

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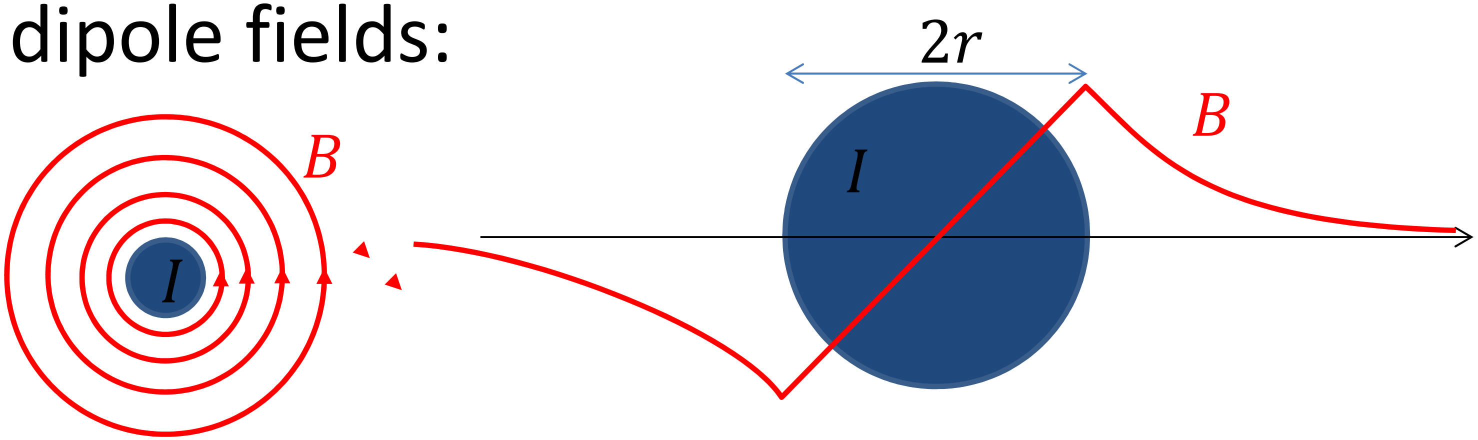
Superconducting electromagnets

Magnet technology

Coil dominated magnets can produce also pure dipole fields:



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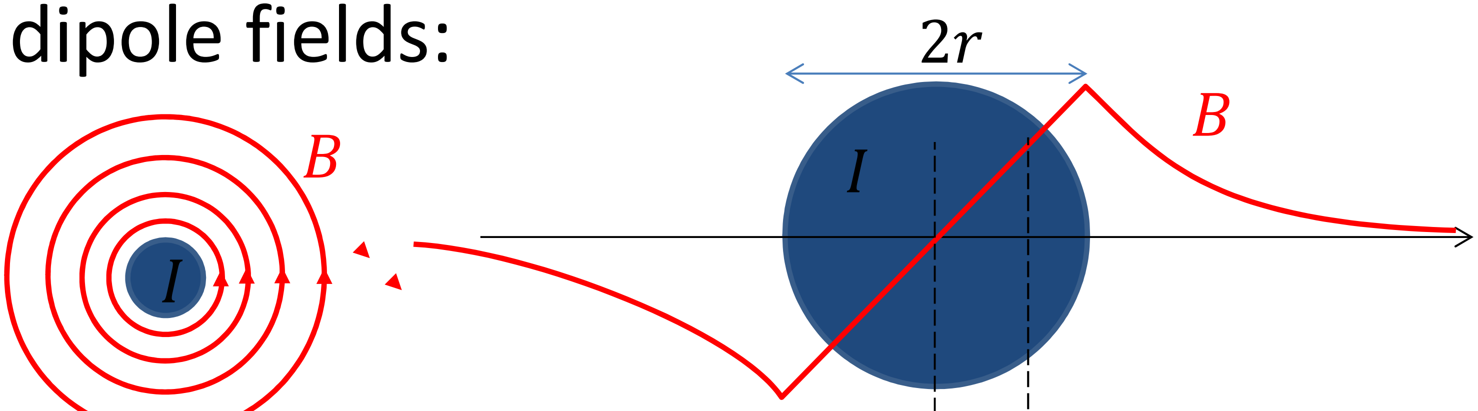
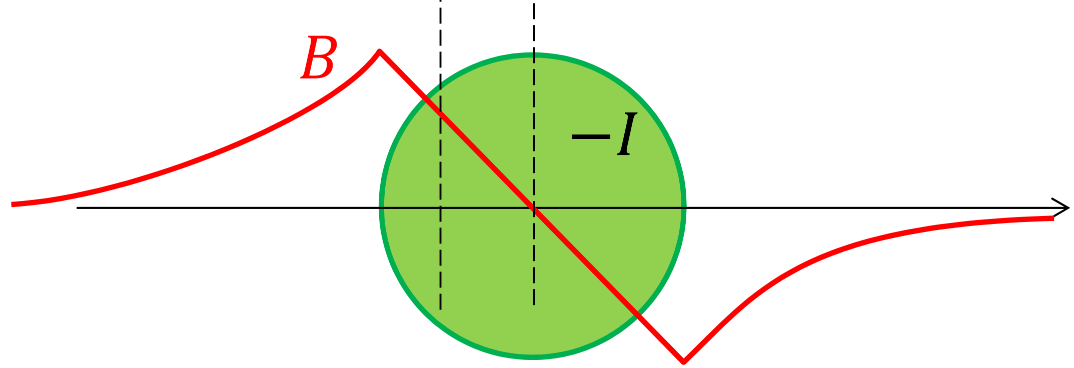


And adding a displaced cable with opposite current…



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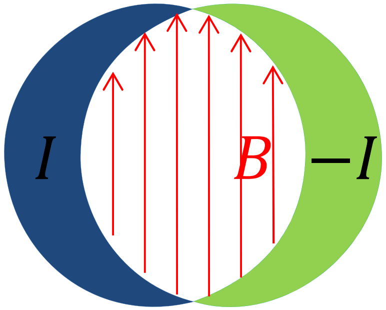

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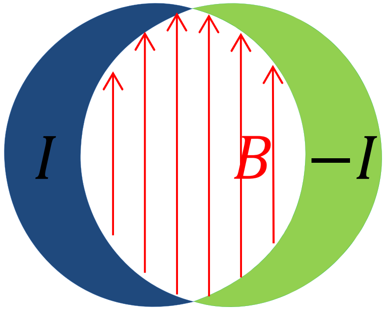
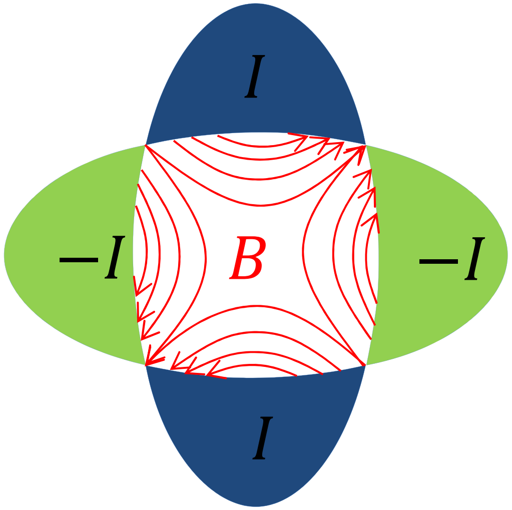


Similarly, overlapping concentric ellipses gives a pure quadrupolar field:



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Coil dominated (SC):

1. Finite malleability.
2. Finite size.
3. Finite T_c/B_c.
4. Quenching.

Magnet technology

Now you can probably better recognize the three different technologies of these quadrupoles:

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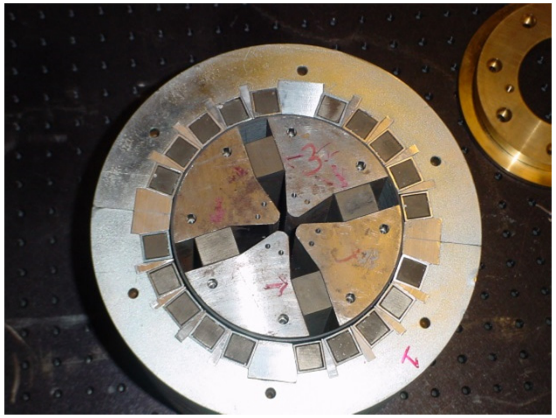

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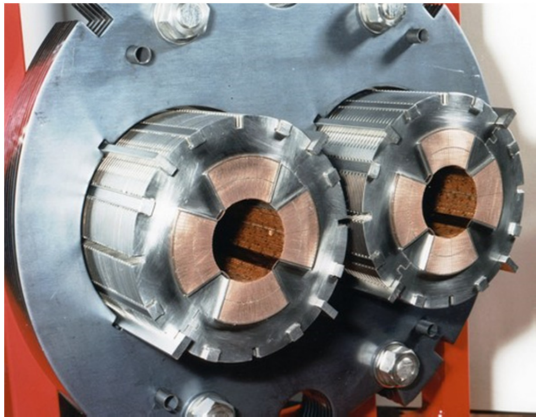

Coil dominated

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4. **Accumulate**: RF (resonant) cavities, which can be normal or super conducting.

RF technology

Major breakthroughs in the development of RF accelerating happened at the end of World War II, when RF power sources in MW ranges at frequencies of MHz were developed…

All because they wanted more powerful Radars…

RF technology

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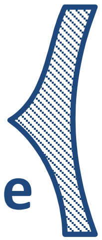

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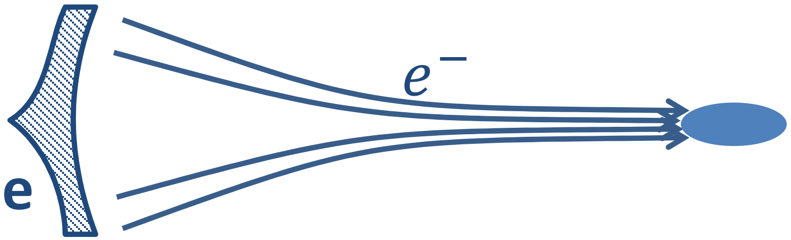

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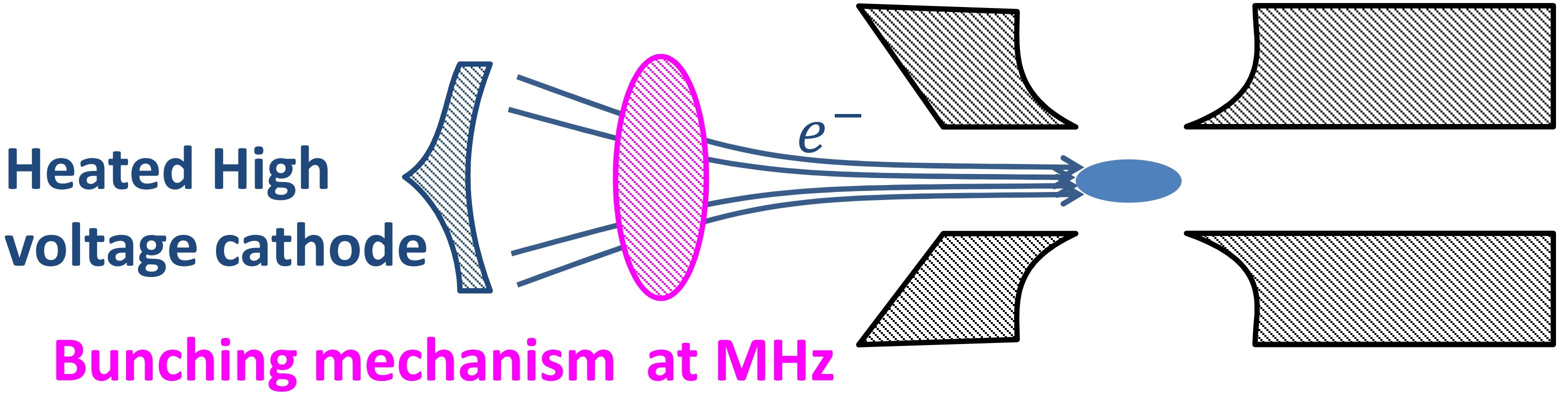

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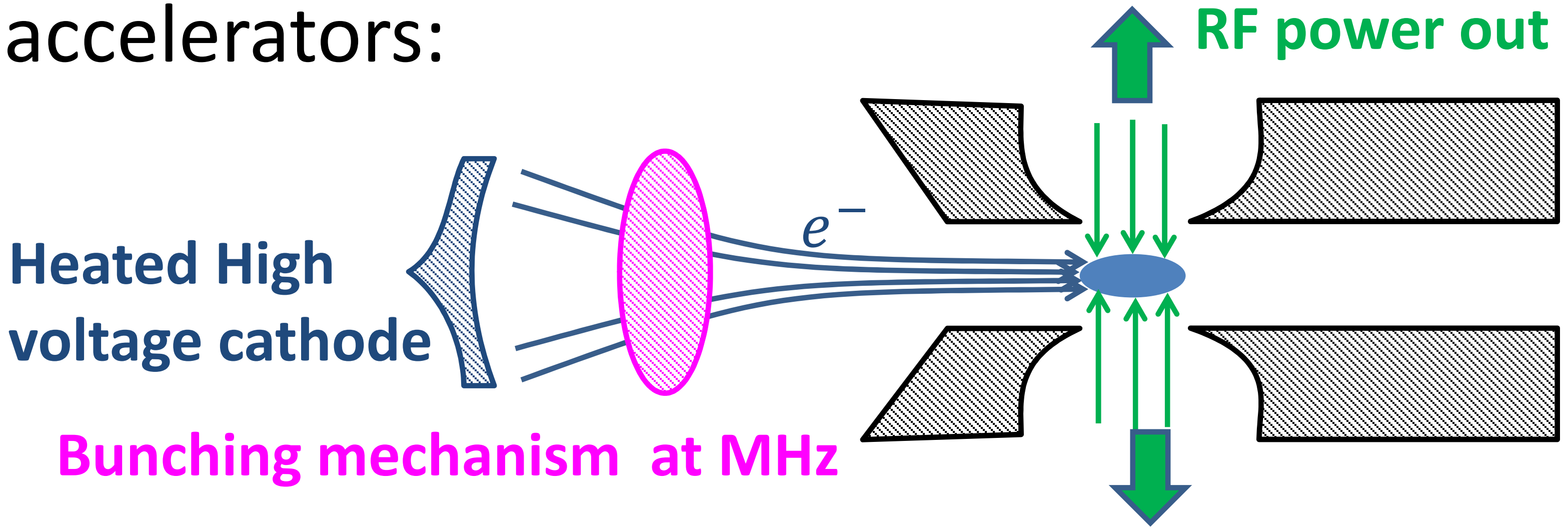
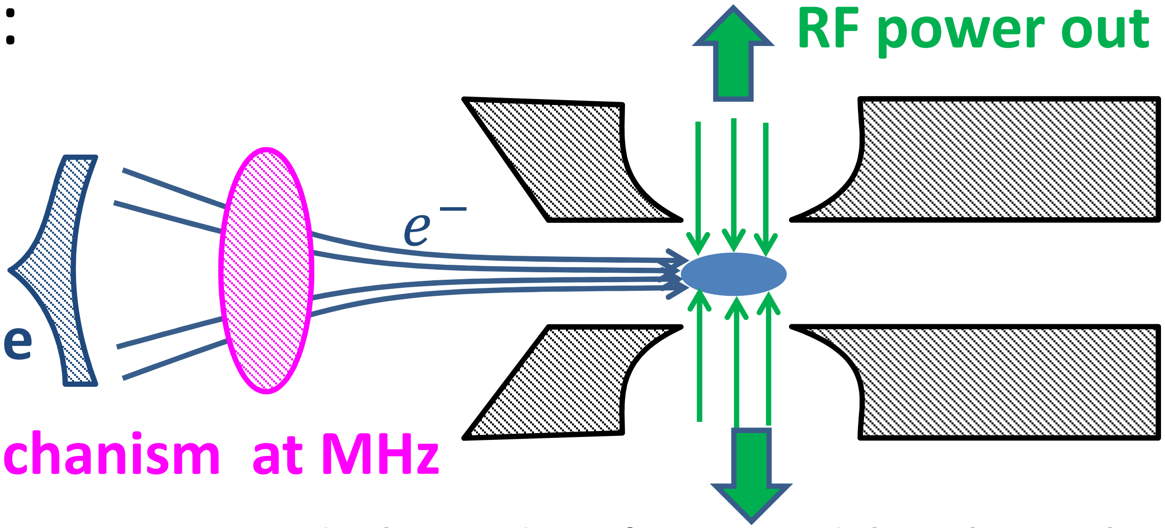


RF technology

Vacuum tubes (Klystrons, Magnetrons, Tetrodes, IOTs) are still the most used RF power source in accelerators:



However, SSA are modular, don't need high voltage and are more stable and endurable… Just wait for the market to grow!

RF technology

At **ALBA** (500MHz), an IOT (like a klystron but bunches with a grid) broke and it has been used for demonstration purposes:

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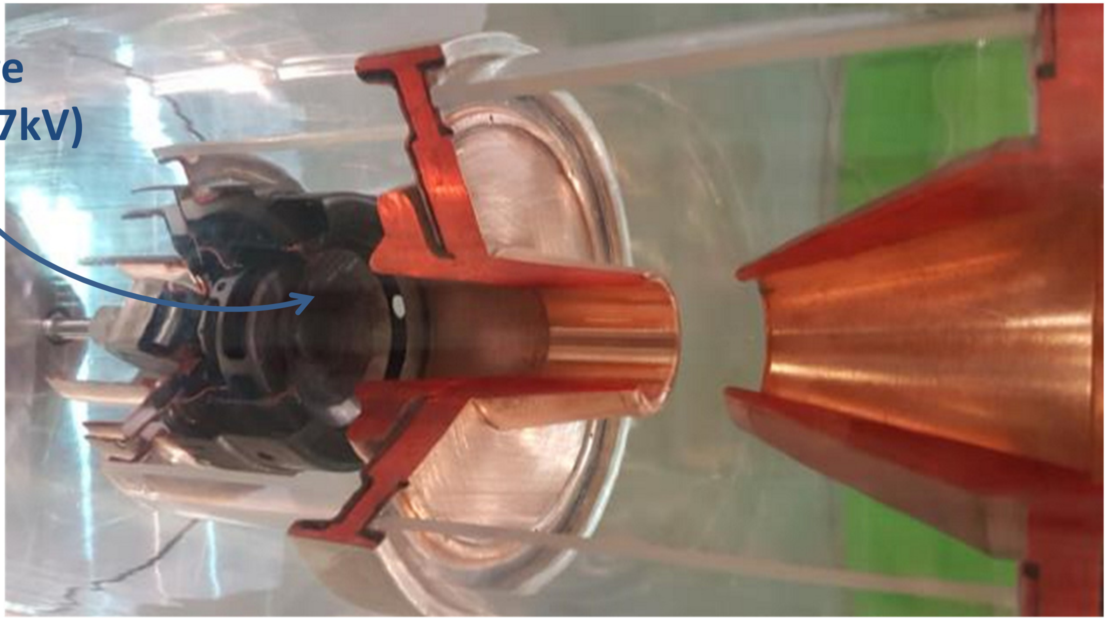

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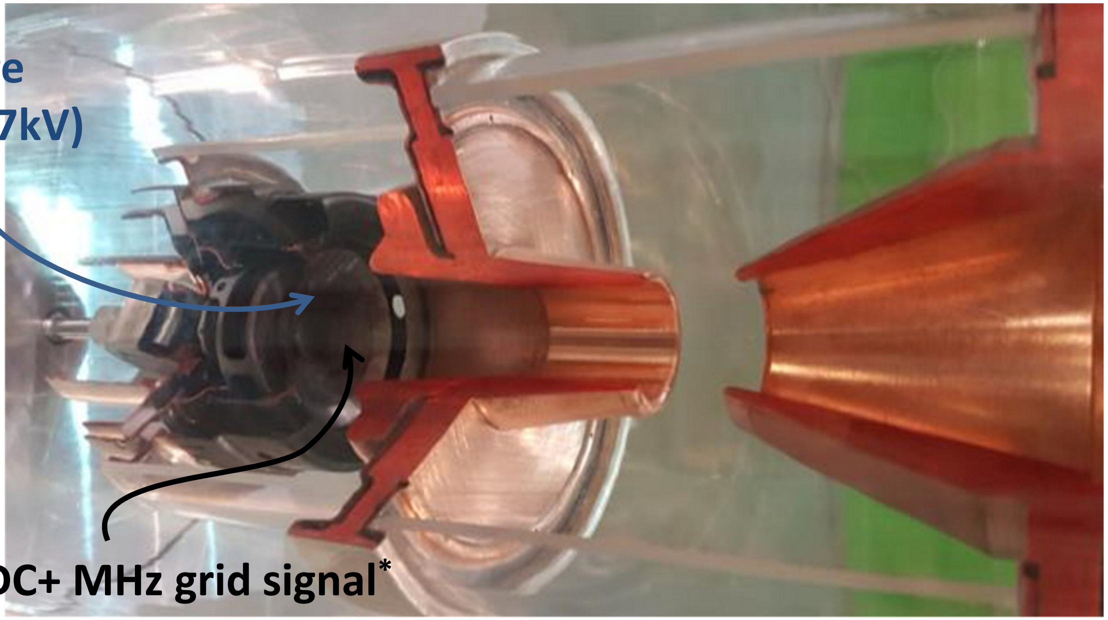

*The **grid** must be very close to cathode, (1/4 mm) otherwise electrons do not have time to cross it!

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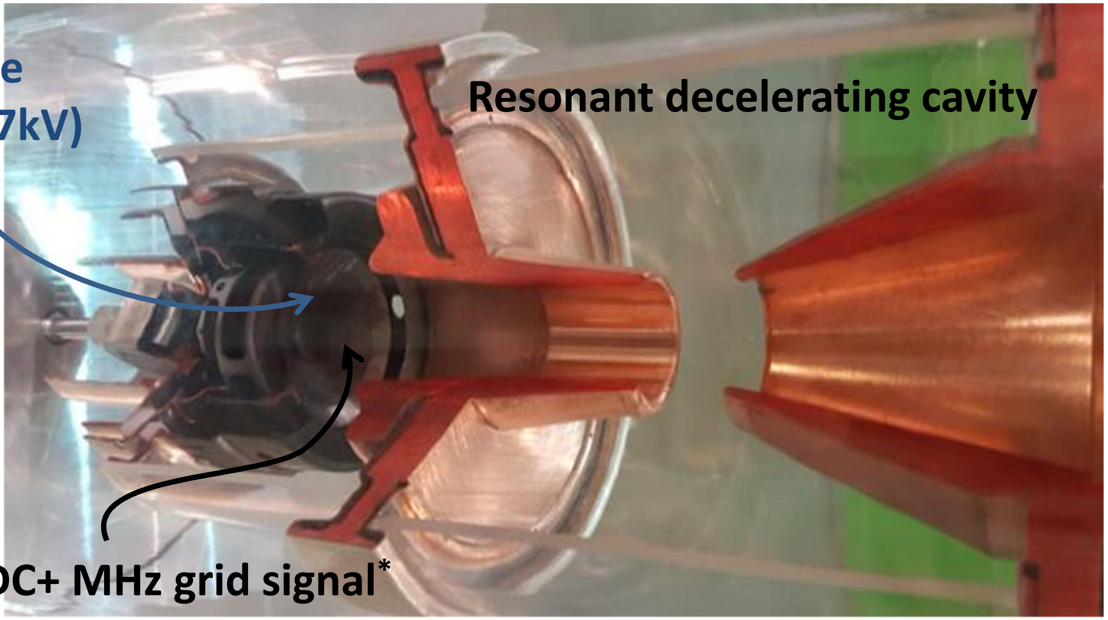
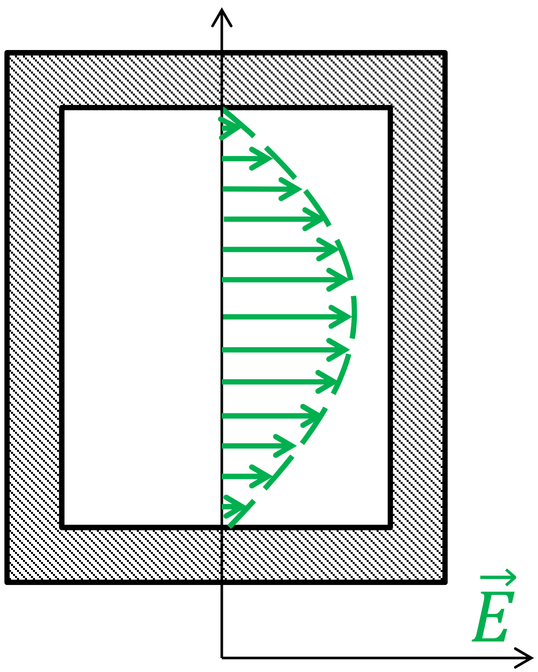


*The **grid** must be very close to cathode, (1/4 mm) otherwise electrons do not have time to cross it!

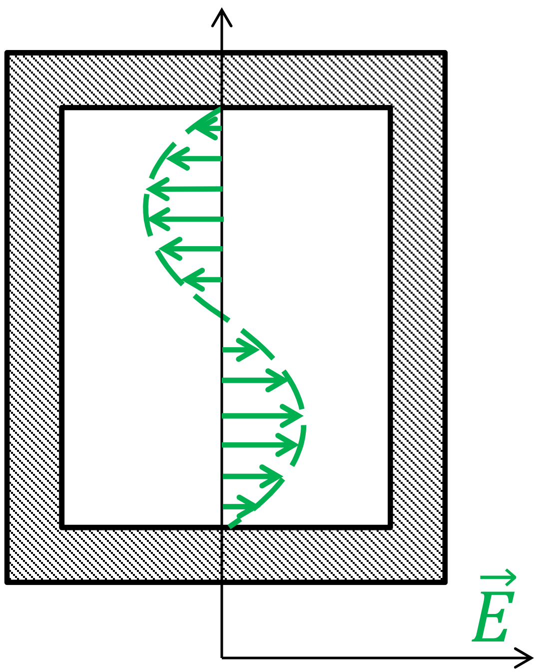
RF technology

RF Cavities are **conductor geometries** designed so that specific frequencies (modes) can be accumulated without losses (resonate).

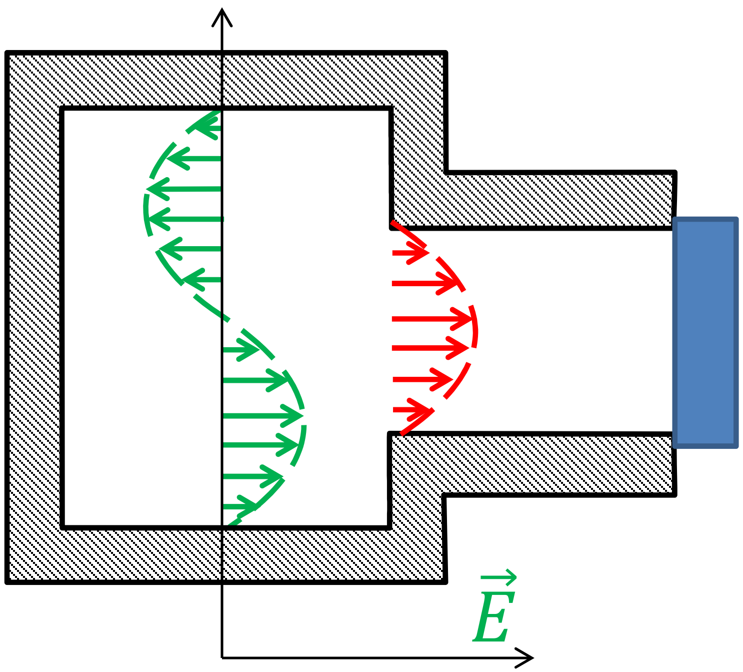


RF technology

Unavoidably, high order modes (HOM) can resonate as well. This can be avoided by feeding only the main mode and damping the other modes.



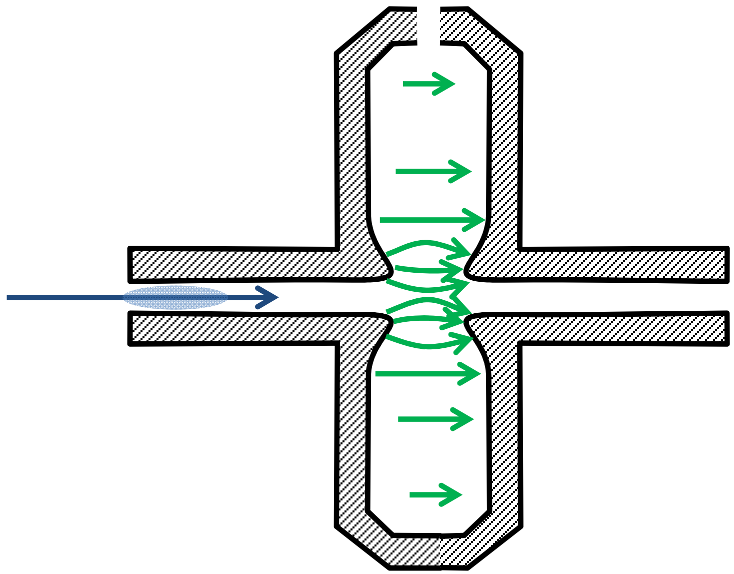
Damping HOMs can be achieved by enabling those modes to **escape the cavity** and being absorbed (for example with ferrites*).



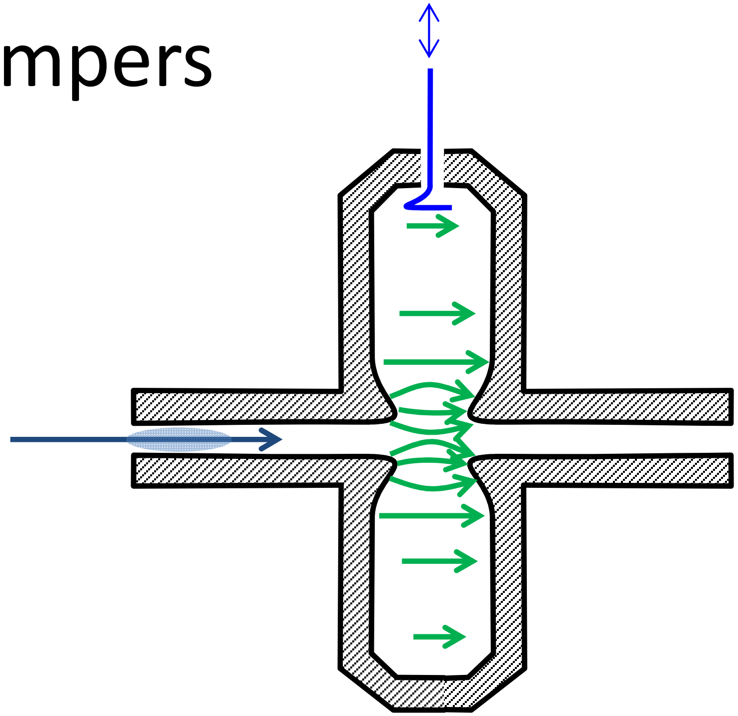
*ferrites: high permeability and high resistivity.

RF technology

The Cavity can be normal conducting (Cu) or superconducting. Normal conducting have limited current densities, hence, the geometry has to enhance the field, also getting closer to the beam:



The frequency adjustment can be done thought a hole in the cavity, modifying the cavity geometry. Since the beam pipe is small, specific HOM dampers are needed:

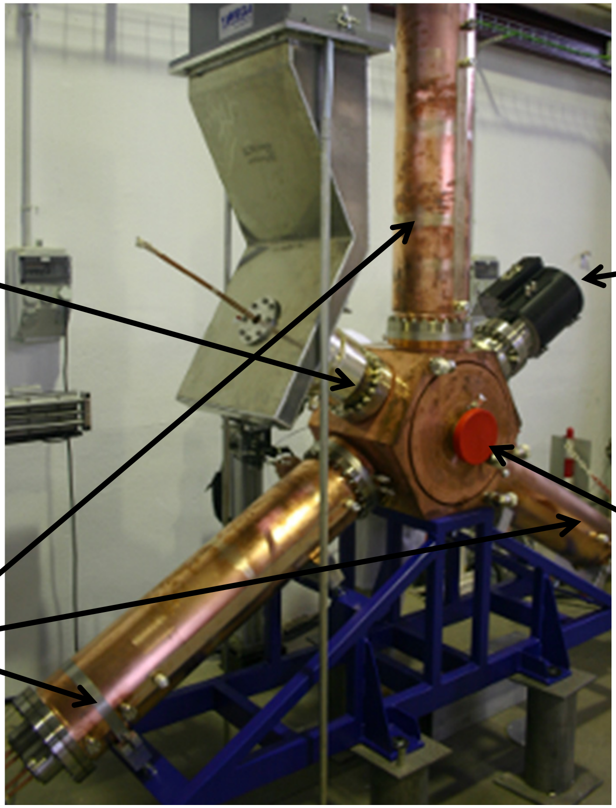


RF technology

ALBA (500MHz) RF cavity:

RF power input

Frequency tuner

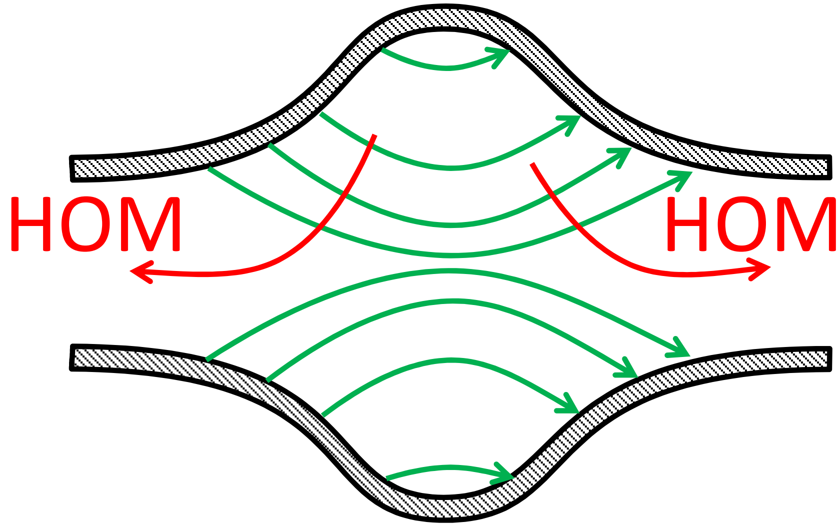


Beam pipe

HOM dampers

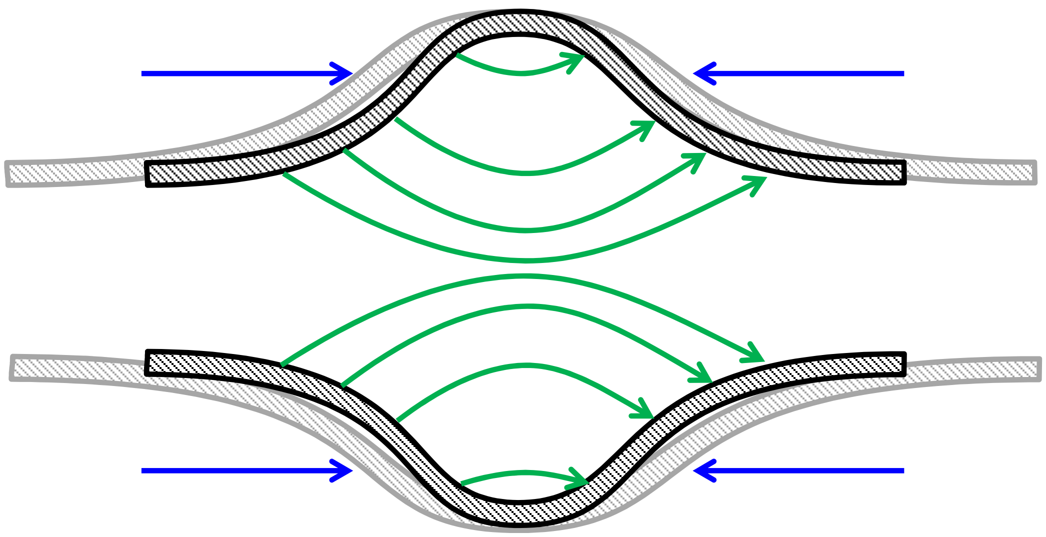
RF technology

In superconducting cavities, electric field can be much higher. However surface should be smooth to avoid quenches (multipacting). The separation to the beam can be bigger allowing HOM to escape through the vacuum chamber.



RF technology

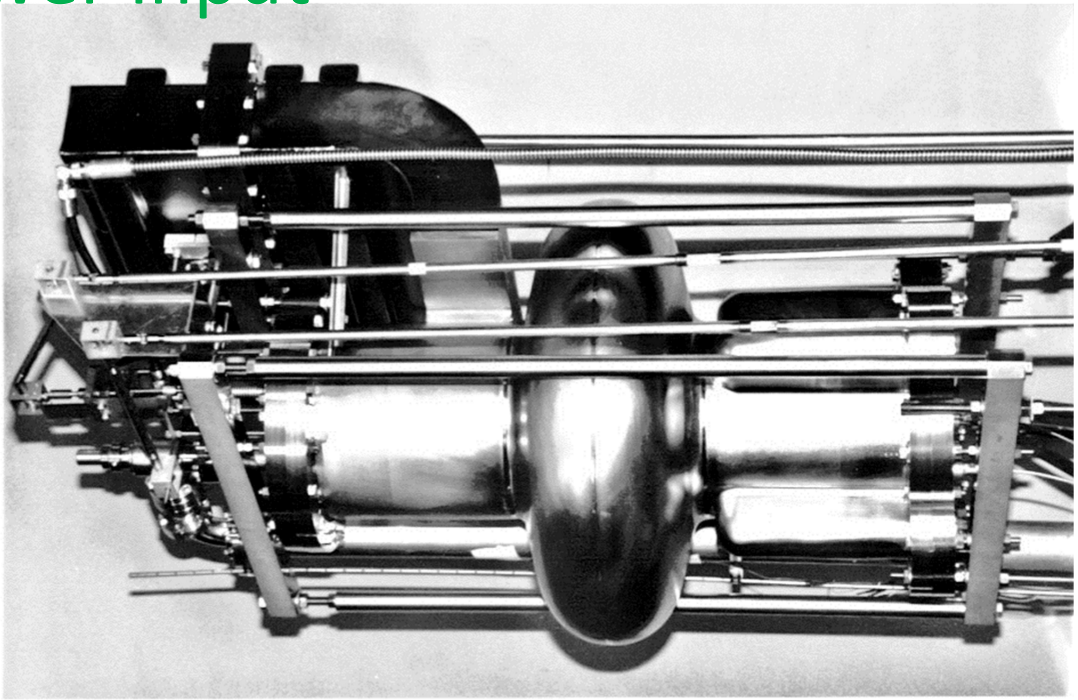
In this case, the frequency adjustment can be done by mechanically deformation of the cavity. The resaon is again avoiding sharp shapes to prevent quenching.



RF technology

CESR (500MHz) supeconducting RF cavity:

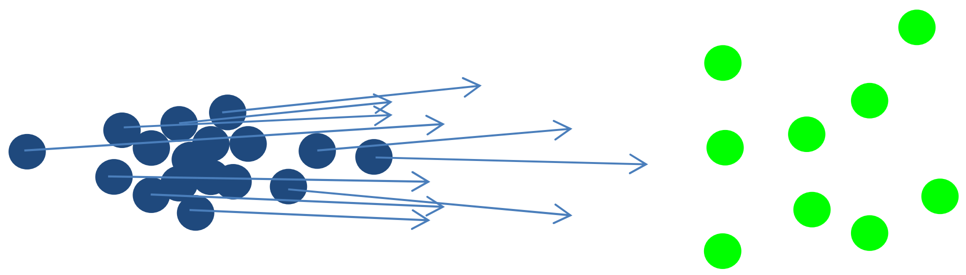
RF power input



Beam pipe

Vacuum system

Ok, we can accelerate, bend and focus charged particle beams. But, charged particles interact **very strongly** with air (or any gas) since air is full of charged particles!



Vacuum system

Electrons/Protons at GeV energies stop after few/several km of air (lifetime is **microseconds**).

bar) (UHV).

At UHV the beam lifetime is around **hours**.

Vacuum system

$0\ 10\ -12\ -12\ 10\ -12$ bar) (UHV).

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Typically Vacuum systems in modern accelerators guarantee vacuum levels around pbar ($10\ -$
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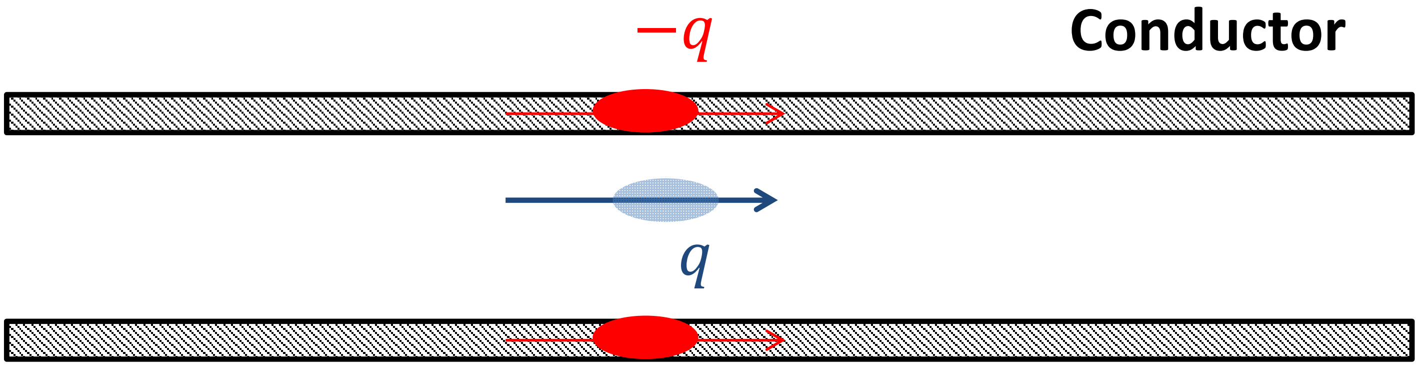
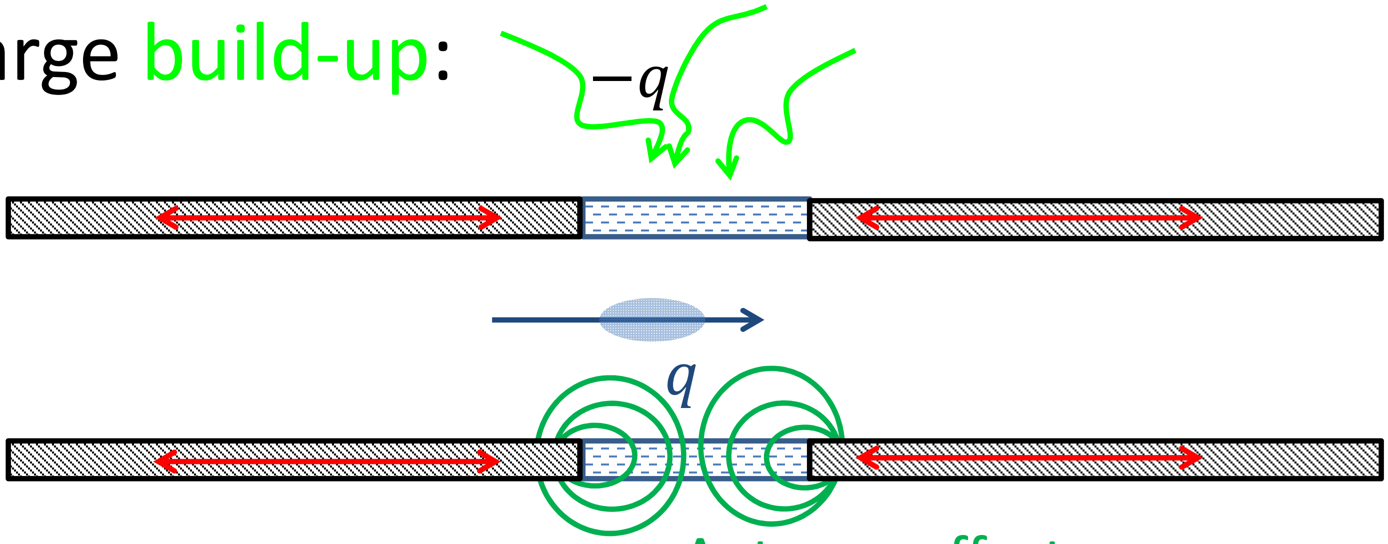
Vacuum system

To keep UHV the vacuum chamber has to ensure, apart from good sealing, resistance to radiation, conductivity…



To keep UHV the vacuum chamber has to ensure, apart from good sealing, resistance to radiation, conductivity… which is needed to ensure the image beam continuity and avoid charge build-up:



Vacuum system

Vacuum chambers are then metallic (steel, Al or Cu). For some particular applications chambers can also be ceramic.

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However, the beam pipe image current continuity is always guaranteed.

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However, the beam pipe image current continuity is always guaranteed.

Vacuum systems is a large topic (as the others), but we will stop here…

The end of the first part…

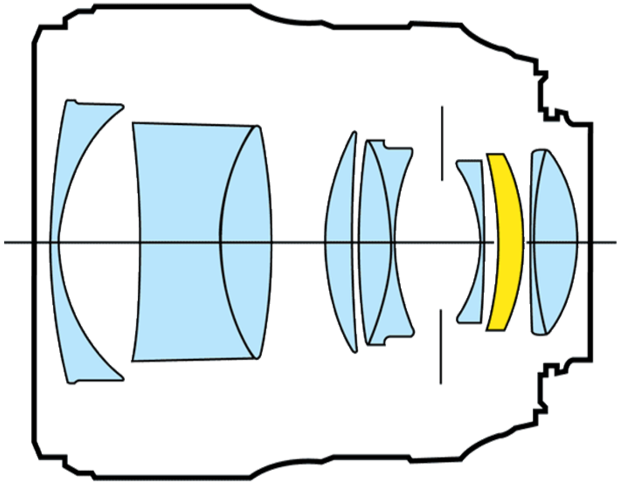
Thanks! Questions?

Outline

- Introduction
- Accelerator technology
- Beam dynamics

Beam optics

Quadrupoles act like lenses for the charged particle beams: accelerators layouts (also called lattice) are similar to a camera focusing systems:

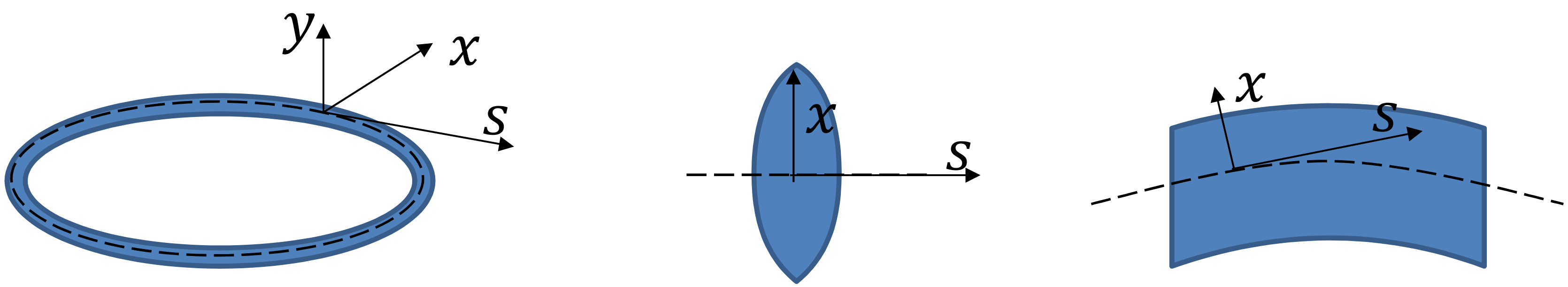


The question is, how do we choose the lattice?

To know how the beam is focused, we need to know the individual particle trajectories.

First, the proper **reference system** should be used:

Accelerator **reference system**: Quadrupole **reference system**: Dipole **reference system**:



Beam optics

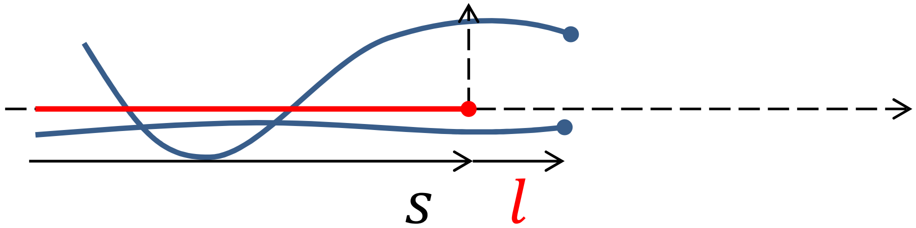
We will restrict ourselves to the paraxial approximation:

$$x \ll \rho \text{ and } y \ll \rho$$

This is generally the case since the beam movement is of the order of mm, and the accelerator bending radius around tens of m:

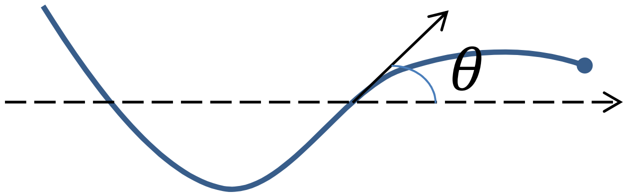
	ALBA	LHC
$\rho\ [m]$	7	2812
$\theta[°]$	11.25	0.3

The longitudinal position s with respect to the reference particle is chosen as independent variable (not the time t).



Notice that the trajectory angle is now the derivate respect to the independent variable:

$$\frac{dx}{ds} \equiv x' \approx \theta$$

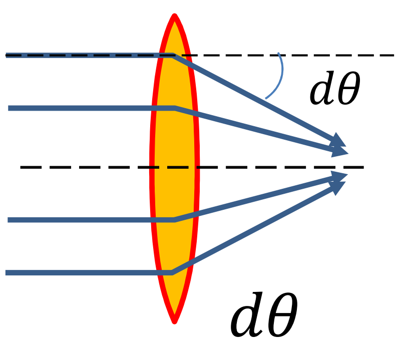


Beam optics

Well aligned Quadrupole magnets have a linear magnetic field:

$$B_x = -gy \qquad B_y = gx$$

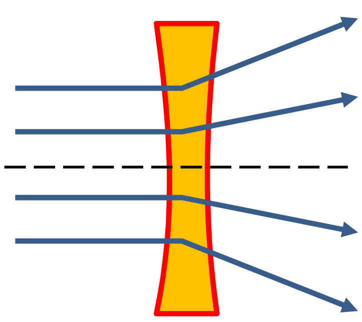
If it focuses in one plane it defocuses in the other plane $k = {}^{g}/_{B\rho}$:



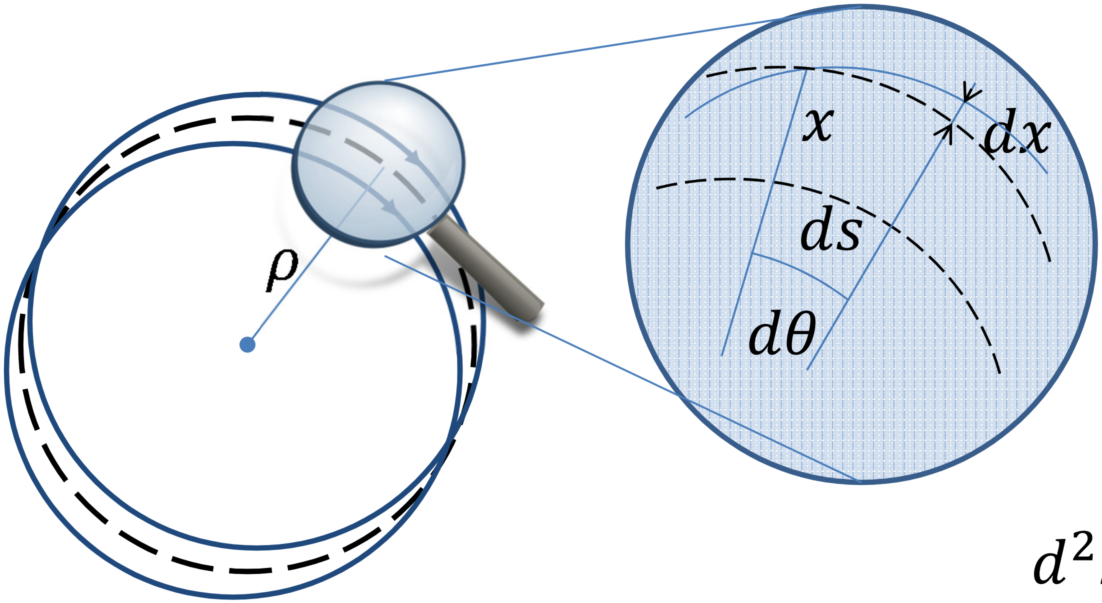
$$\frac{d\theta}{ds} = -kx$$

$$\frac{d^2x}{ds^2} = -kx$$

$$\frac{d^2y}{ds^2} = ky$$



In this reference system, bending magnets are similar to simple drift sections except they add some extra focusing in the **horizontal plane**:



Circular orbits accomplish:

$$\frac{d^2x}{ds^2} = \frac{1}{\rho^2}\frac{d^2x}{d\theta^2} = -\frac{1}{\rho^2}x$$

Equations of motion

Then the equations of motion are:

$$\frac{d^2x}{ds^2} + \left(k(s) + \frac{1}{\rho^2(s)}\right)x = 0$$

$$\frac{d^2y}{ds^2} - k(s)y = 0$$

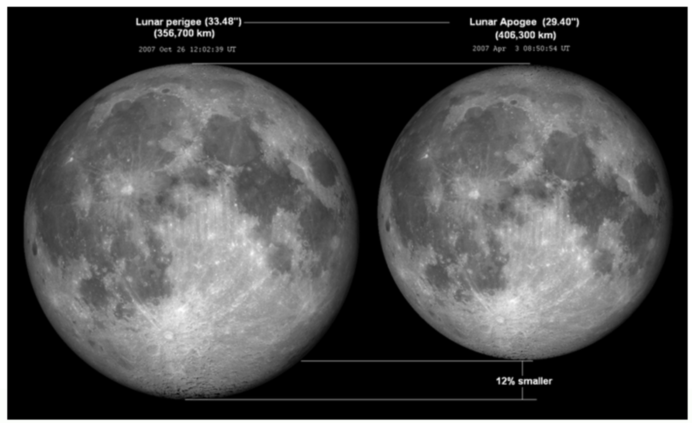
In circular accelerators (of circumference L) these quantities are periodic:

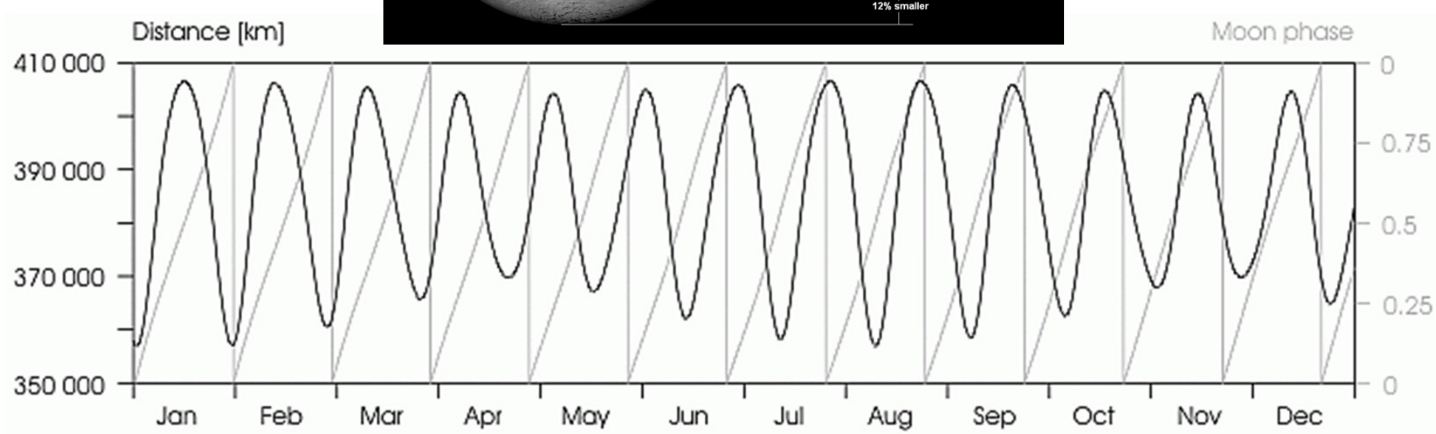
$$k(s+L) = k(s)$$

$$\rho(s+L) = \rho(s)$$

Equations of motion

Such type of equation had been solved in 1886 by **G.W.Hill** when studying the motion of the moons perigee.

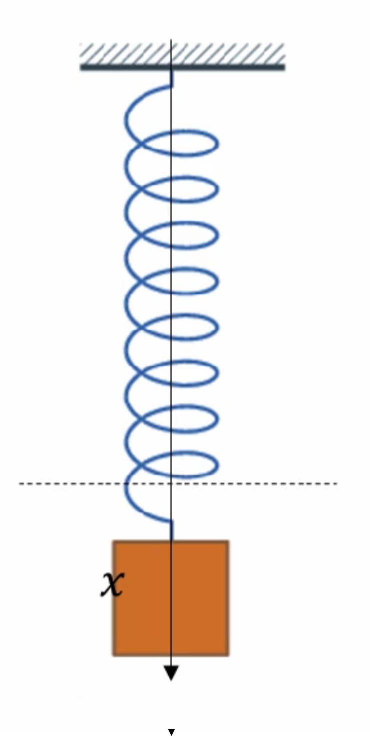




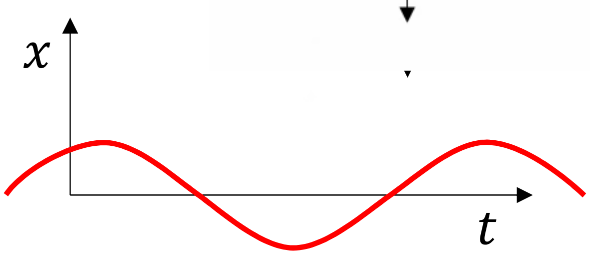
Equations of motion

Hill's equations are also called pseudo-harmonic oscillation, indeed they resemble the harmonic oscillator:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$



Solution: $x(t) = A cos(\omega t - \mu_0)$



Amplitude (constant)

Phase (grows linearly)

Particle trajectories

The solution are the *betatron* oscillations:

$$x(s) = \sqrt{J\beta(s)}\cos(\mu(s) - \boldsymbol{\mu_0})$$

Amplitude-like   Phase-like

- J: action [m]: depends on **initial conditions.**
- μ_0: Initial phase [rad]:depends on **initial conditions.**
- $\beta(s)$: beta function [m]: it is periodic: $\beta(s+L) = \beta(s)$
- μ: betatron phase [rad]: does NOT grow linearly:

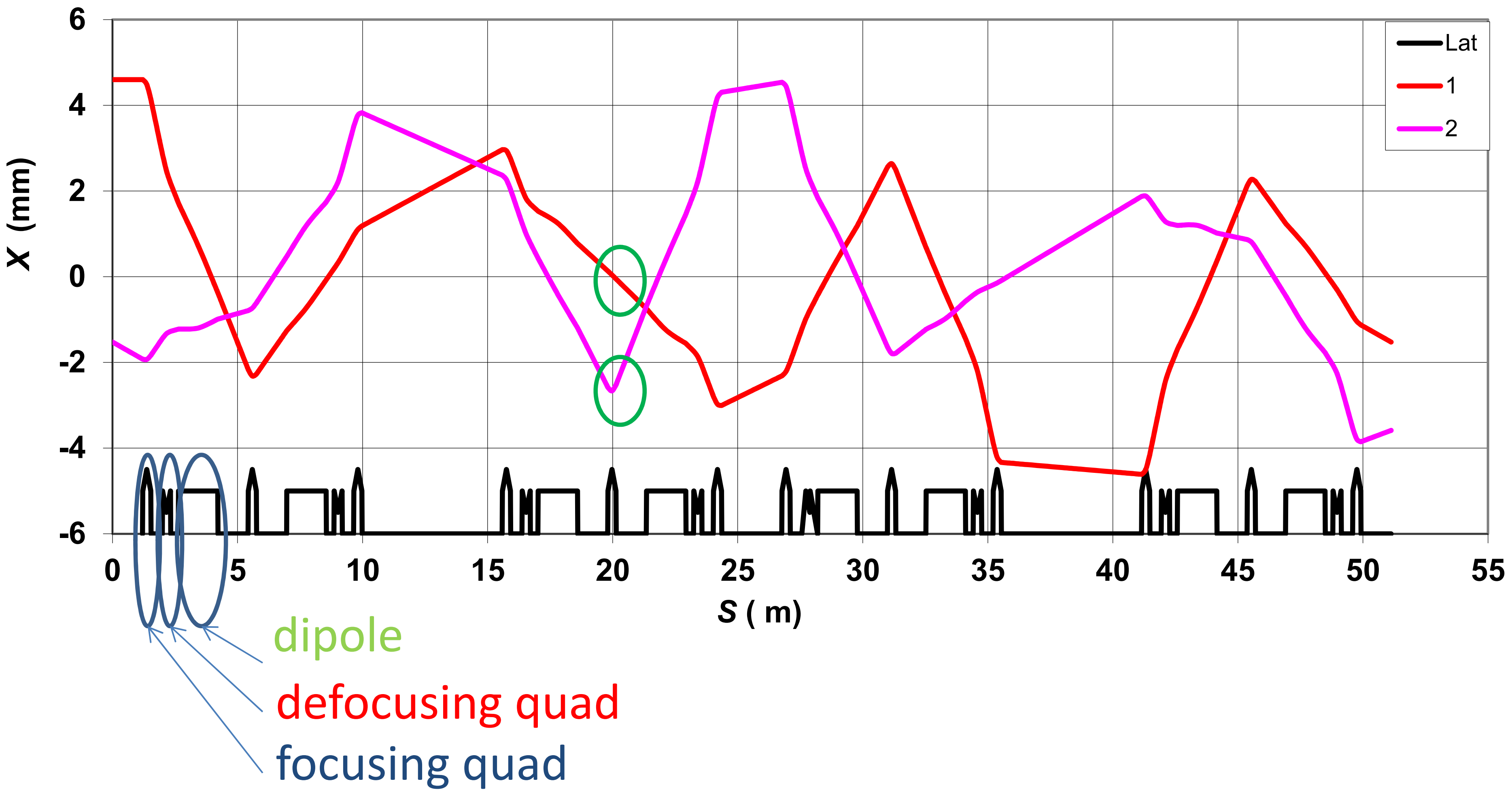
$$\mu(s) = \int_0^s \frac{ds}{\beta(s)}$$

Particle trajectories

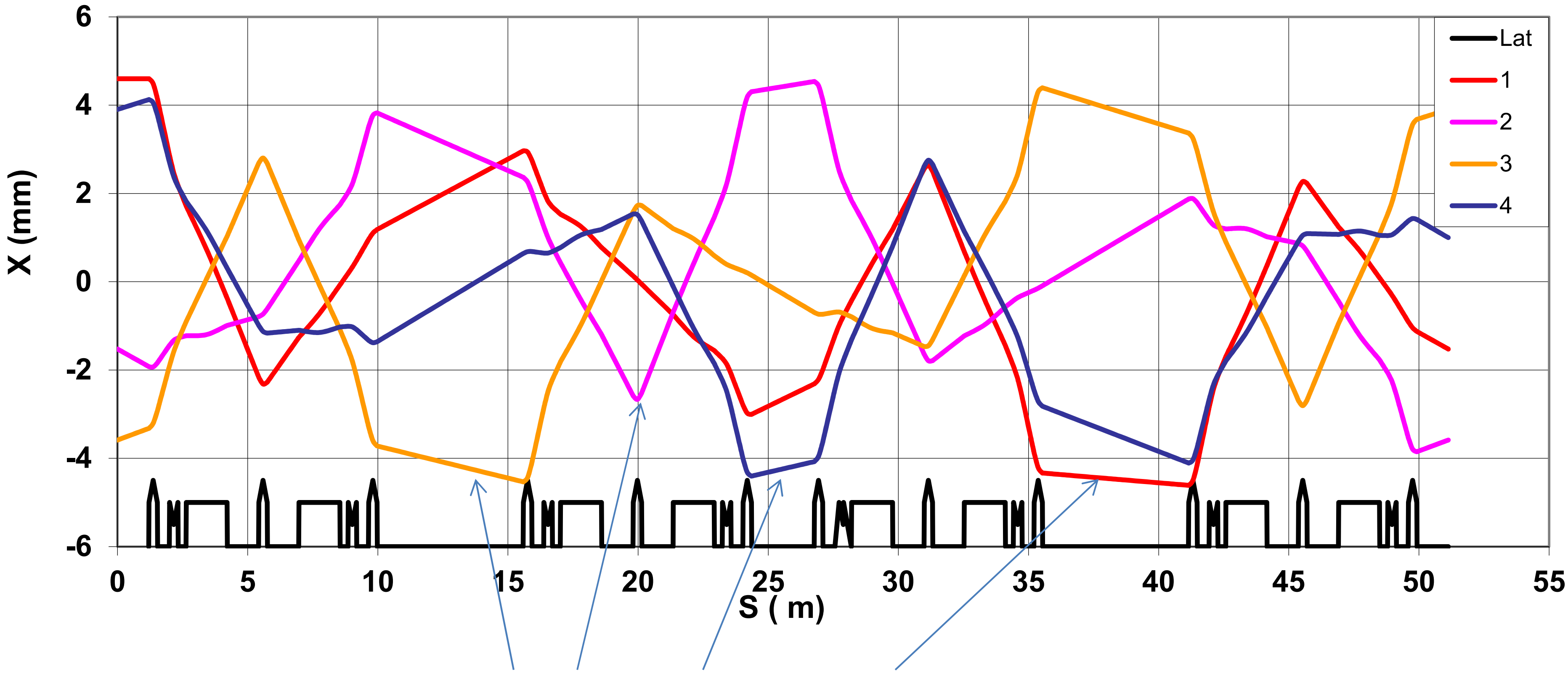
The beta function depends on the lattice focusing elements distribution (lattice).

Trajectories of different particles with different **initial conditions** have the same envelope given by the beta function.

Particle trajectories



Particle trajectories



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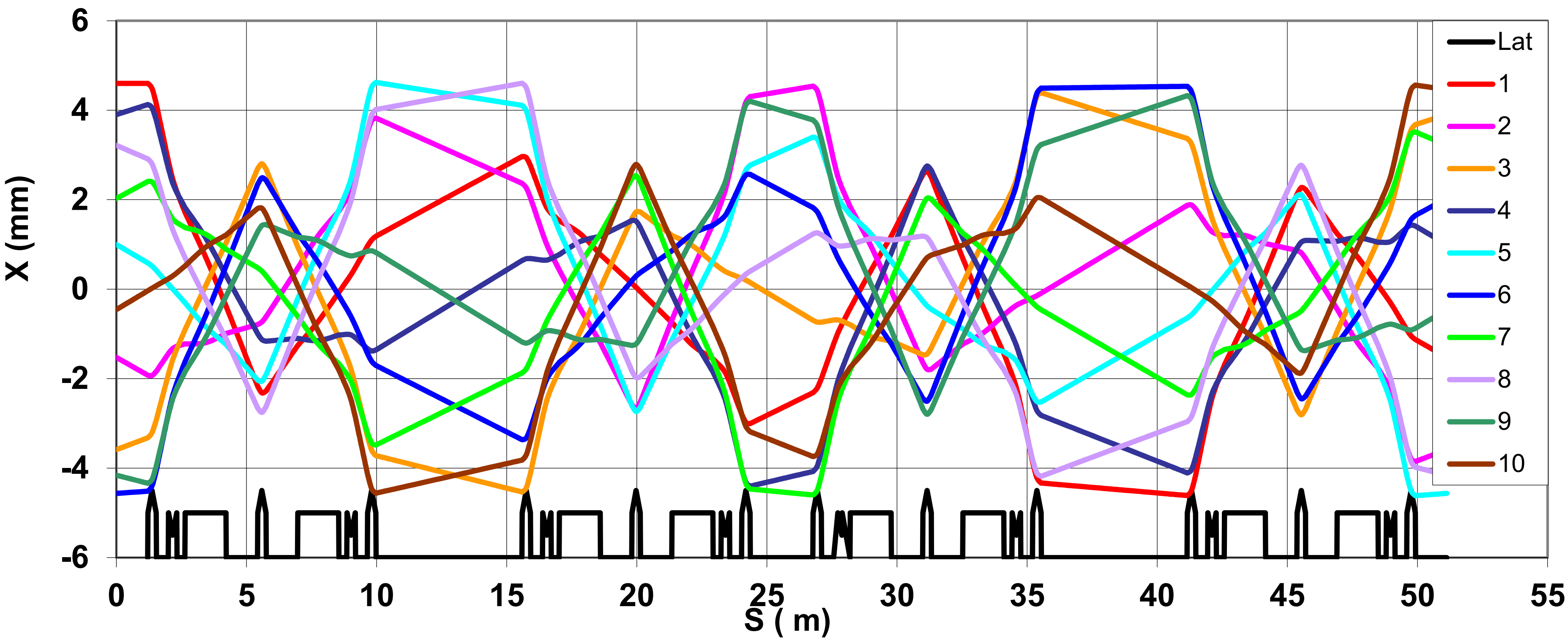
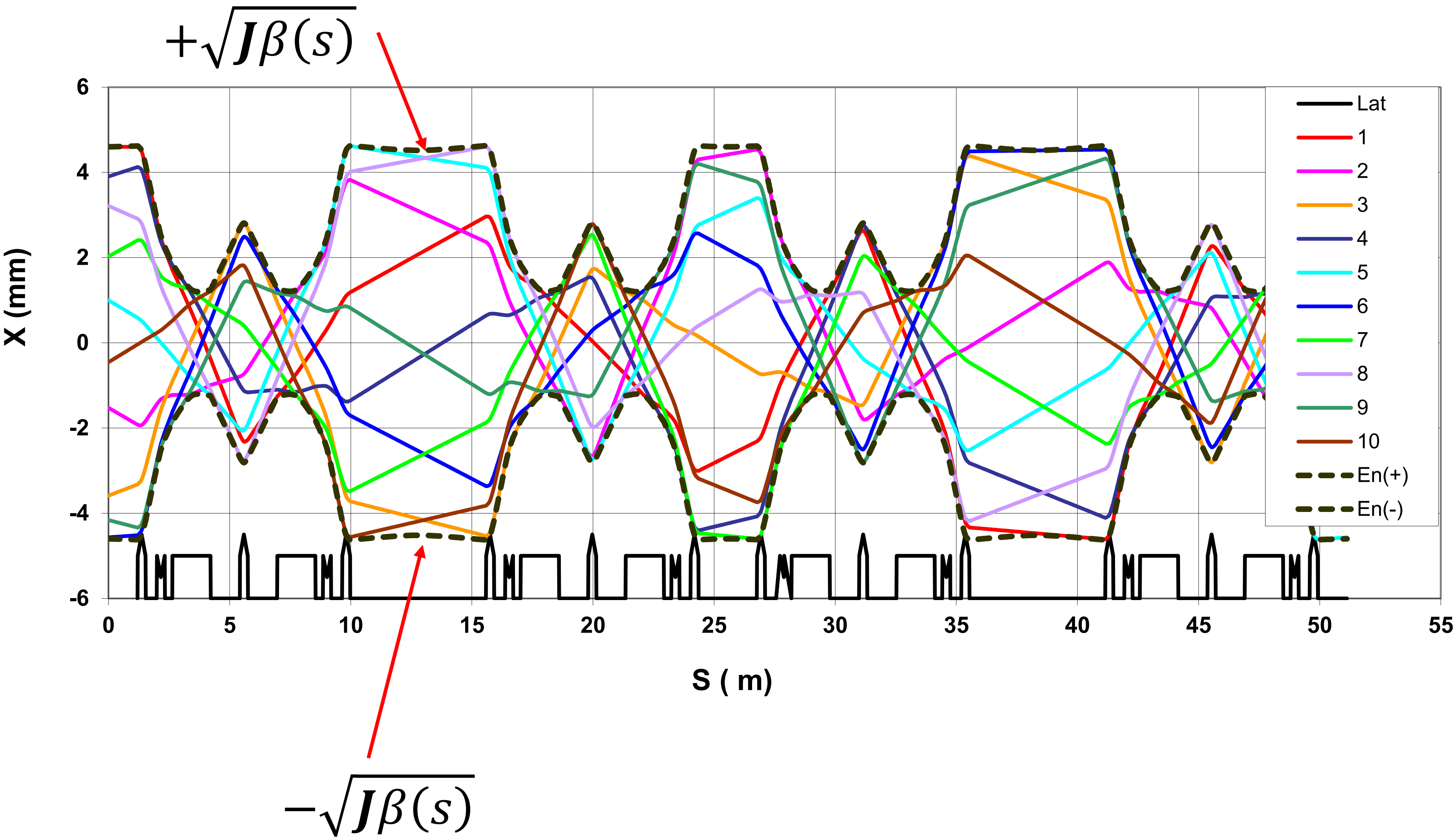
Particle trajectories



The tracks correspond to different turns of the same particle, but could also be different particles starting with an initial phases changed by $\mu(L)$

Particle trajectories



Particle trajectories

Ok, we know how a single particle varies its position.

However, circular accelerators are characterized by the emittance which is a measure of the beam focalization and collimation.

Emittance lives in the position-angle space.

Positions and angles

Every particle trajectory can be parameterized by $(\boldsymbol{J},\boldsymbol{\mu_0})$. Equivalently we can use initial position and angle $(\boldsymbol{x_0},\boldsymbol{x'_0})$. The angle can be obtained by simple derivation:

$$x'(s) = \sqrt{\frac{\boldsymbol{J}}{\beta(s)}} sin(\mu(s) - \boldsymbol{\mu_0}) - \alpha(s)\sqrt{\boldsymbol{J}\beta(s)} cos(\mu(s) - \boldsymbol{\mu_0})$$

Where:

$$\alpha(s) = -\frac{\beta'(s)}{2}$$

Positions and angles

So, position and angle:

$$x(s) = \sqrt{J\beta(s)}cos(\mu(s) - \boldsymbol{\mu_0})$$

$$x'(s) = \sqrt{\frac{J}{\beta(s)}}sin(\mu(s) - \boldsymbol{\mu_0}) - \alpha(s)\sqrt{J\beta(s)}cos(\mu(s) - \boldsymbol{\mu_0})$$

We can isolate the phase dependence:

$$\frac{x^2(s)}{J\beta(s)} = cos^2(\mu(s) - \boldsymbol{\mu_0})$$

$$\gamma(s) \equiv \frac{1 + \alpha^2(s)}{\beta(s)}$$

Substituting:

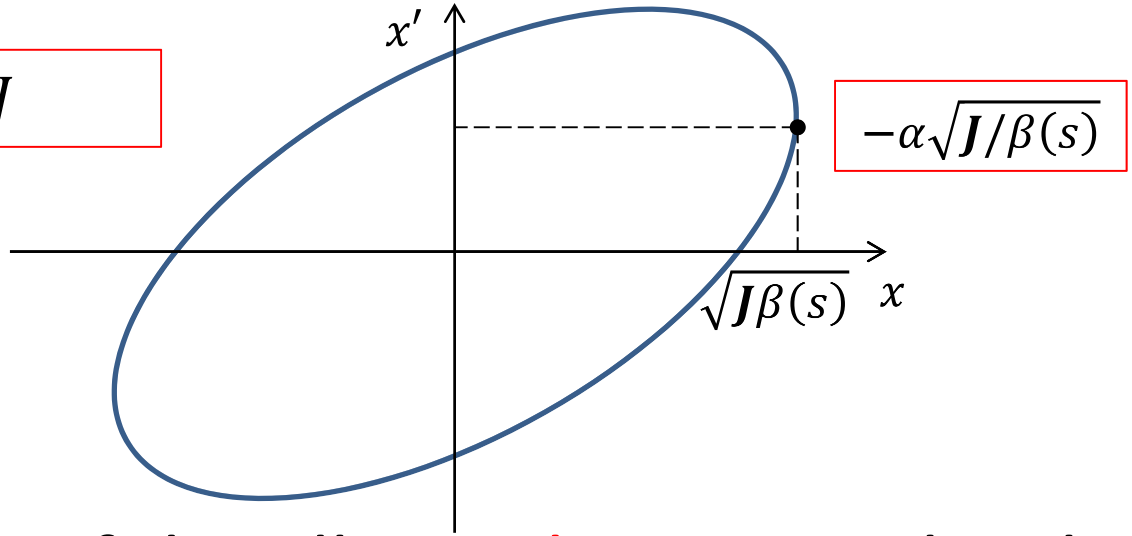
$$\boldsymbol{J} = \beta(s)x'^2(s) + 2\alpha(s)x(s)x'(s) + \gamma(s)x^2(s)$$

Positions and angles

position and angle describe an ellipse:

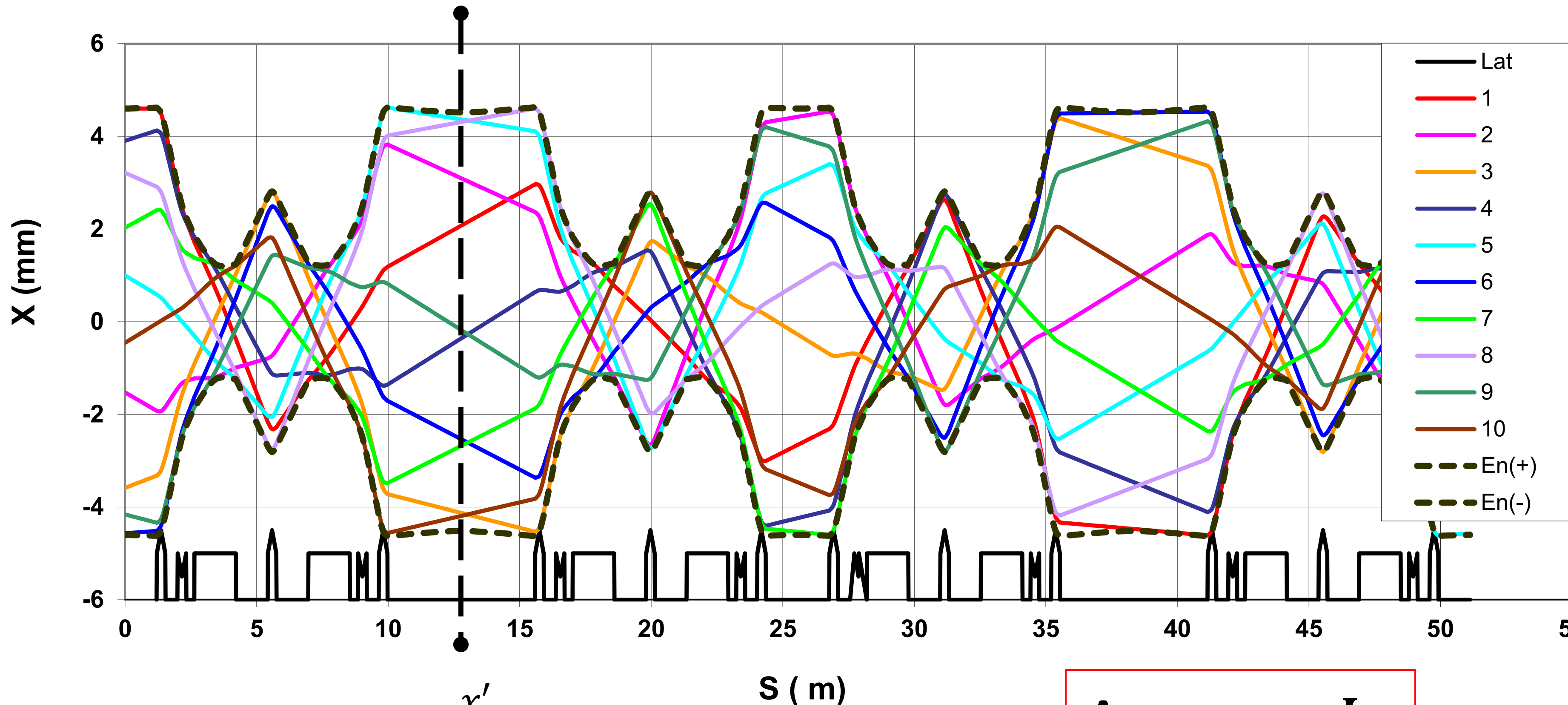
$$J = \beta(s)x'^2(s) + 2\alpha(s)x(s)x'(s) + \gamma(s)x^2(s)$$

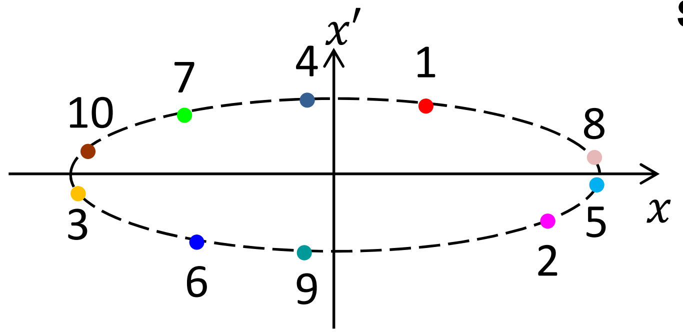
Area = πJ



The shape of the ellipse changes with s, but the area is constant: J is **invariant**.

Positions and angles

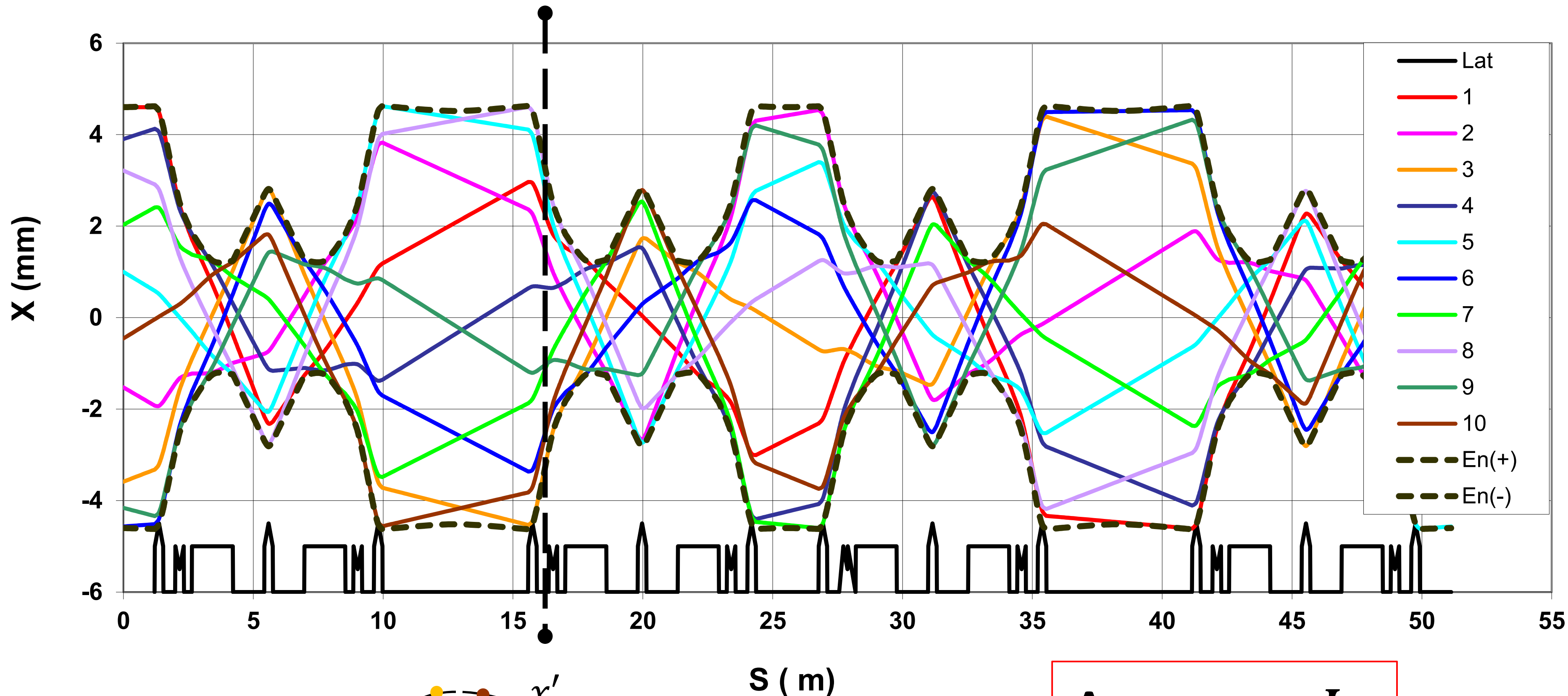


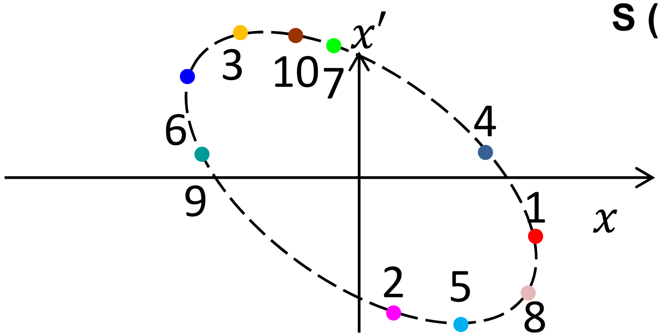


Area = πJ

Focused location: $\alpha = 0$

Beam optics

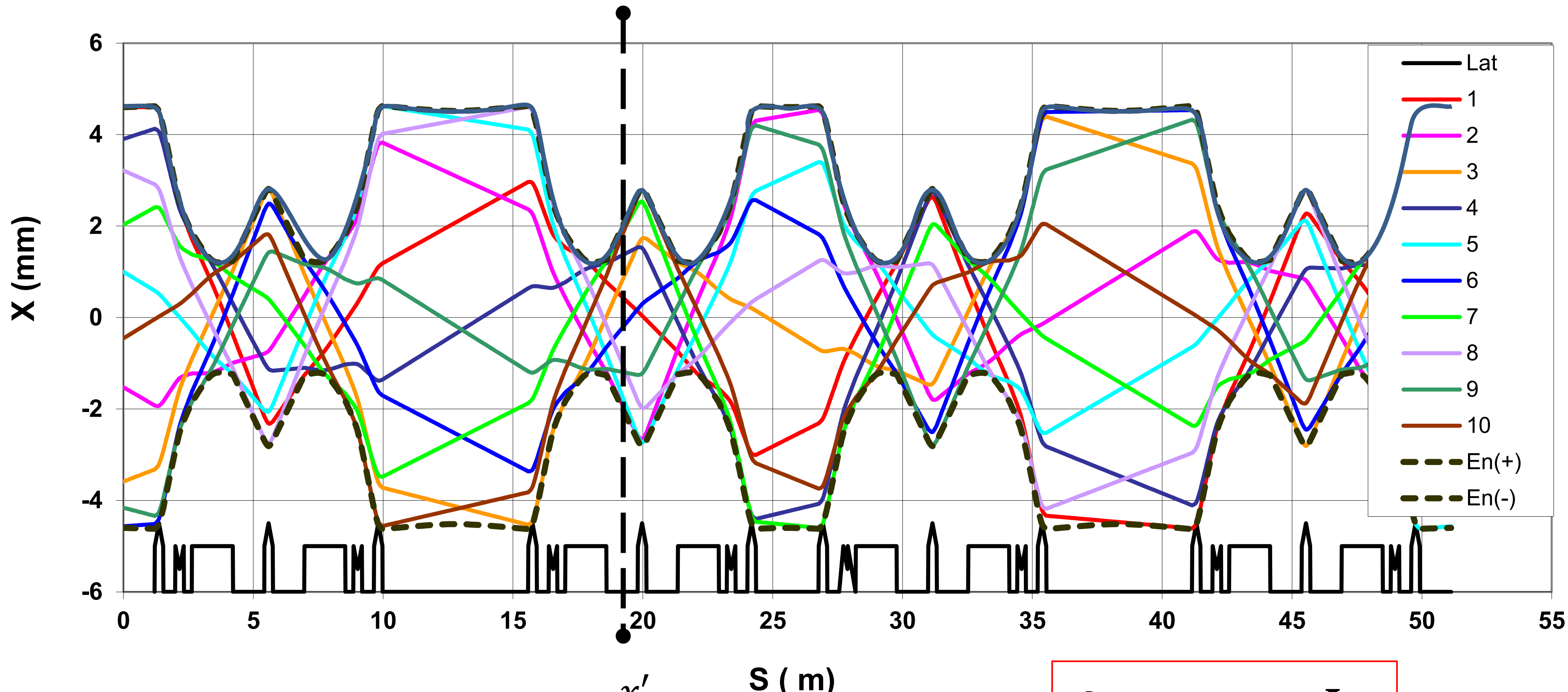


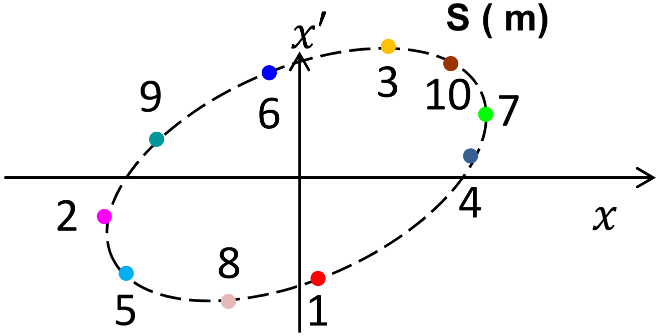


Area = πJ

Focusing location: $\alpha > 0$

Beam optics





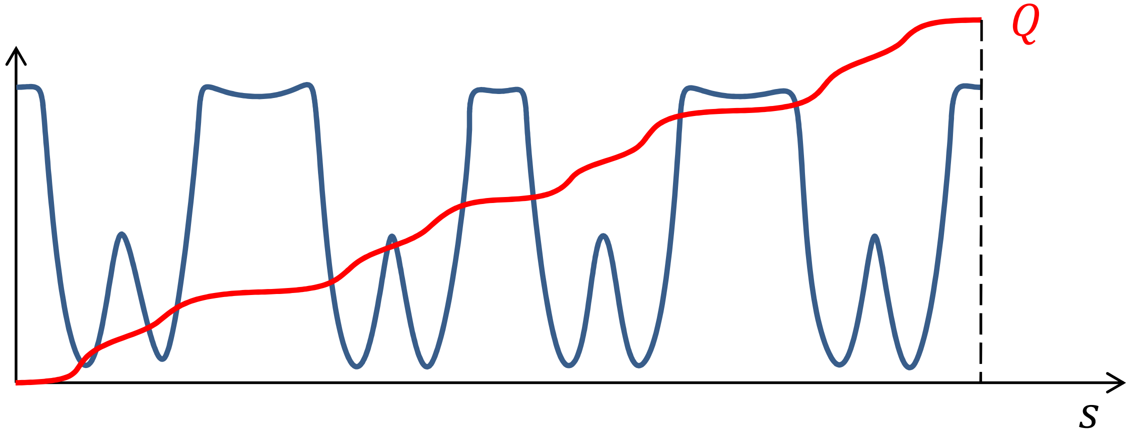
Area = πJ

Defocusing location: $\alpha < 0$

The tune

The total phase variation over 2π in one turn is called the tune (aka number of oscillations):

$$Q = \mu(L)/2\pi$$

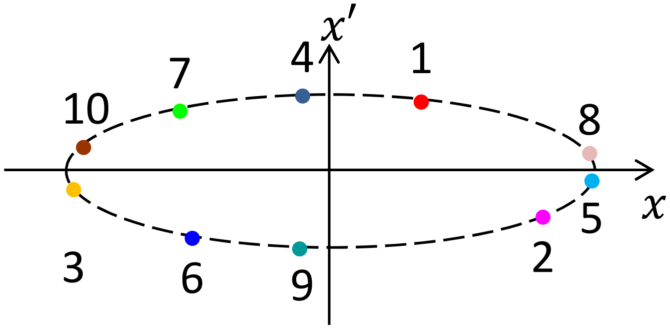


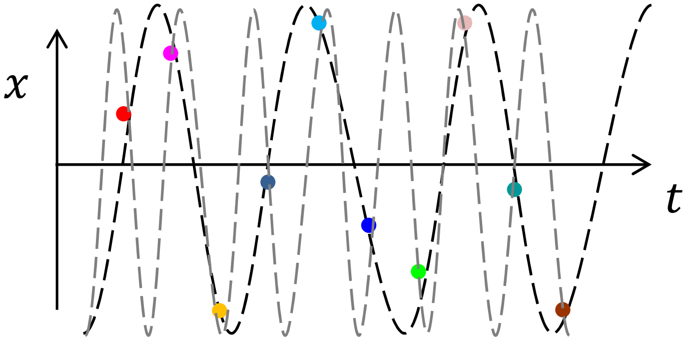
$$\mu(s) = \int_0^s \frac{ds}{\beta(s)}$$

Since $\beta(s)$ is periodic: $Q = \big(\mu(L+s) - \mu(s)\big)/2\pi$, then the tune is **constant**.

The tune

At a given position in the ring only the non integer part of the tune is observable:

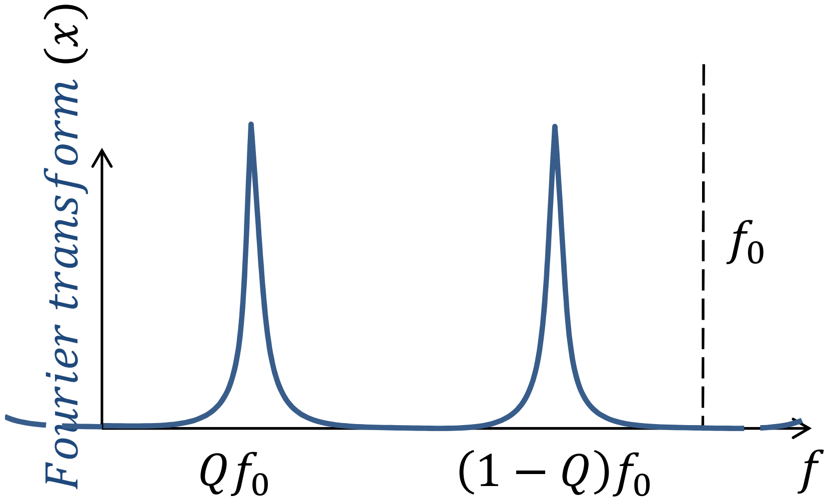




At a given position in the ring, the picture would be de same if the tune was $Q+1$ or $Q+2$...

The tune

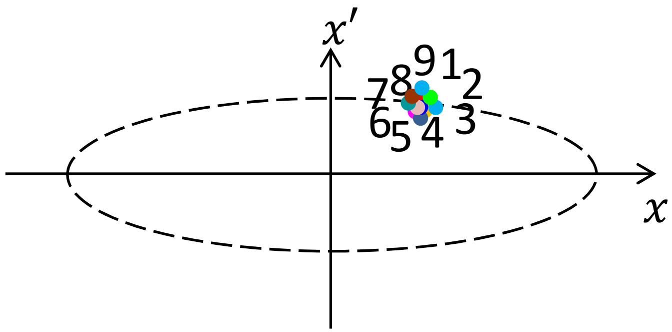
The tune times the revolution frequency (f_0) is the resonant frequency of the system…

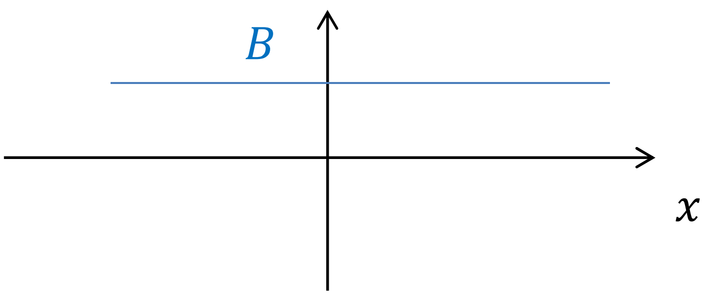


Exciting the beam at such frequency leads easily to a beam loss.

The tune

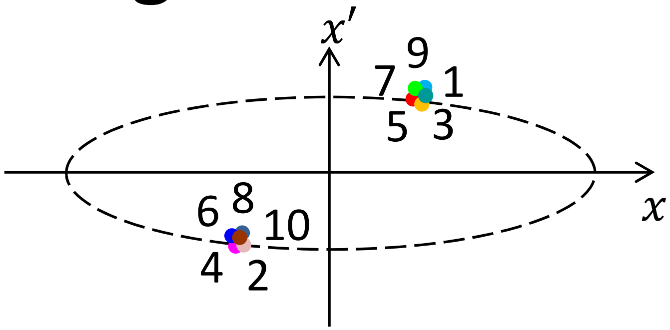
Also, the accelerator is unstable if the tune in **integer**:

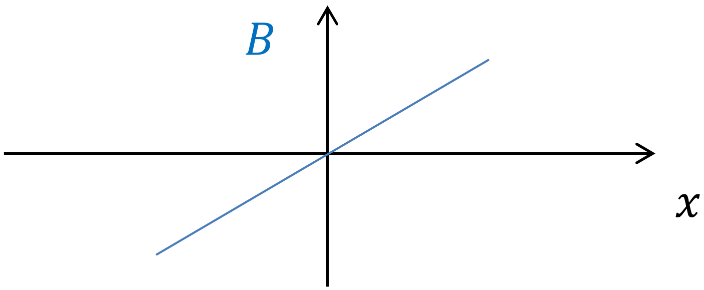




Any small dipole error would produce an orbit amplitude growth turn after turn:

Similarly, the accelerator is unstable if the tune is **half-integer** :

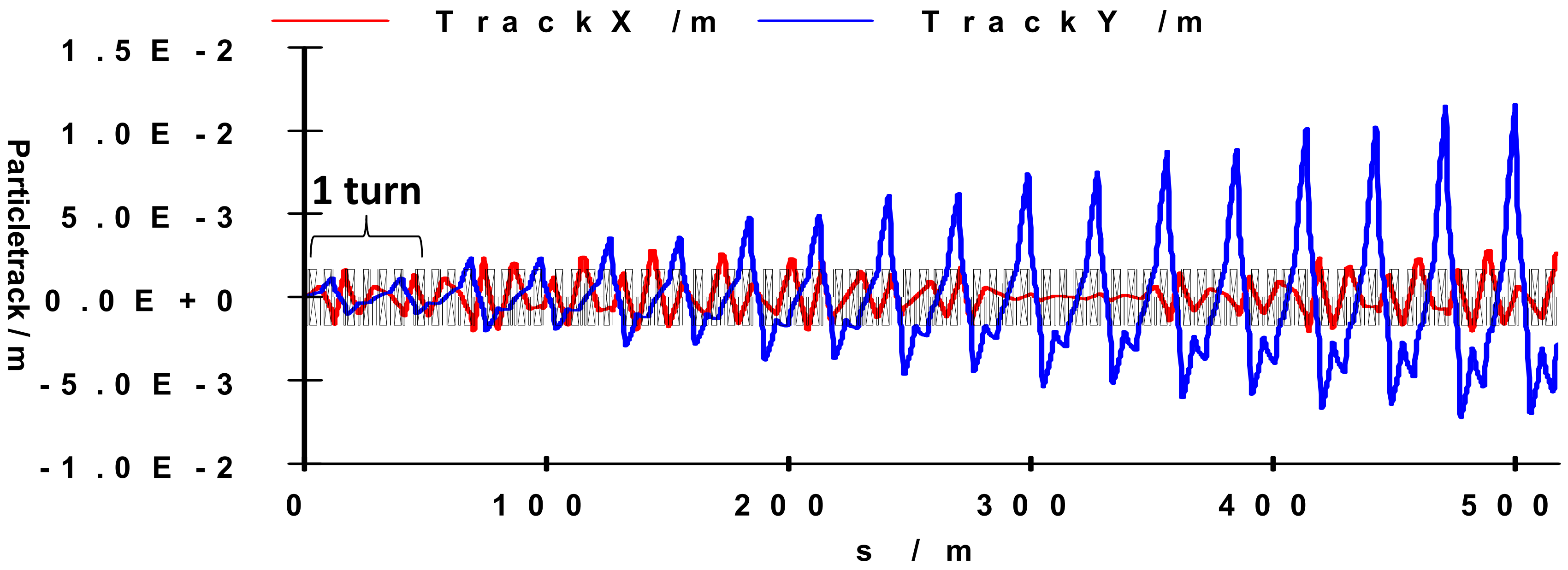




Any small quadrupole error would produce an orbit amplitude growth turn after turn:

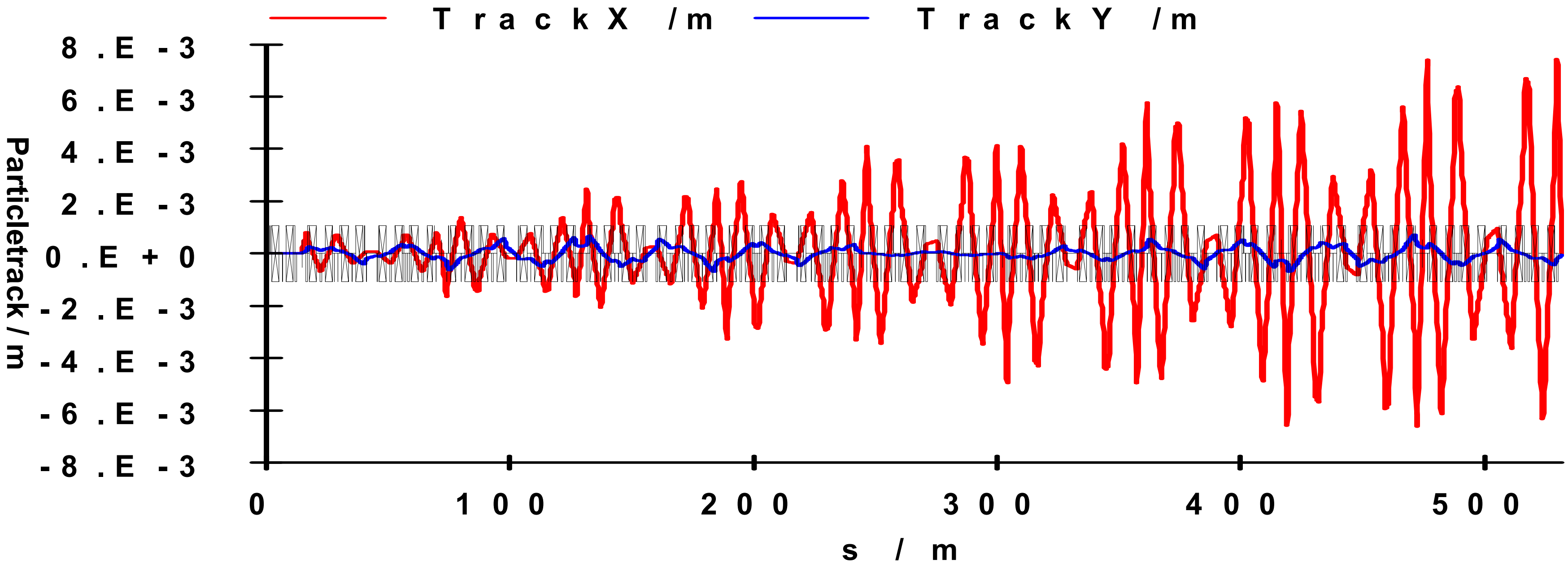
The tune

Trajectory Example with working point (Q_x ,Q_y) = (4.16 , 2.00) and small dipole error:



The tune

Trajectory Example with working point (Q_x ,Q_y) = (4.50 , 1.69) and small quadrupolar error:



The tune

More in general, circular accelerators must avoid **resonant lines**: $nQ_x + mQ_y = q$

Dipole resonantLines:

$Q_x = i$

$Q_y = j$

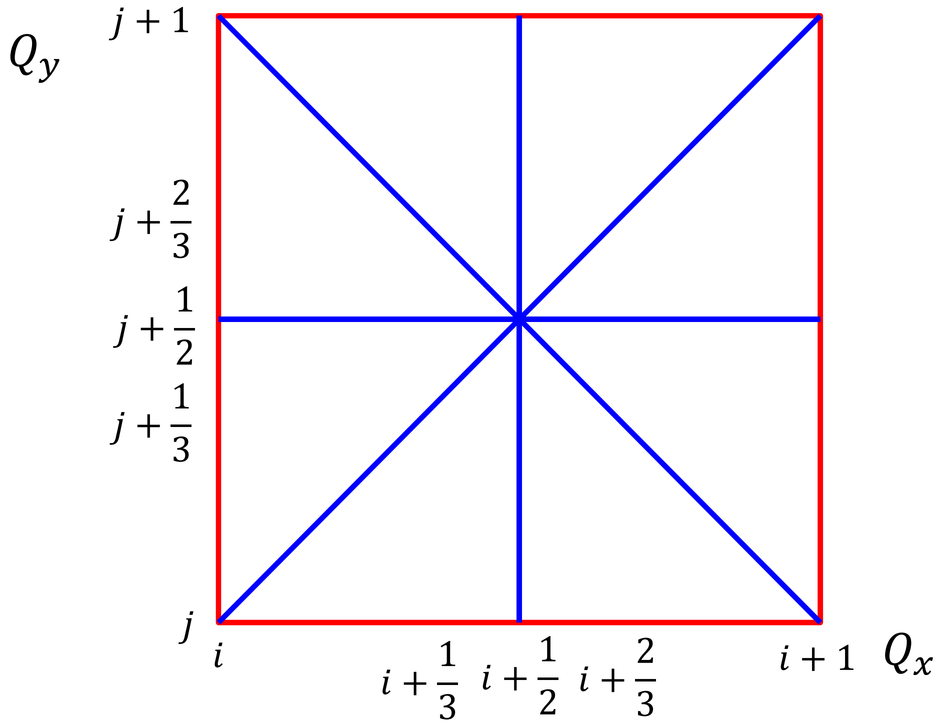
Quadrupole resonant lines

$Q_x = i + \frac{1}{2}$

$Q_y = j + \frac{1}{2}$

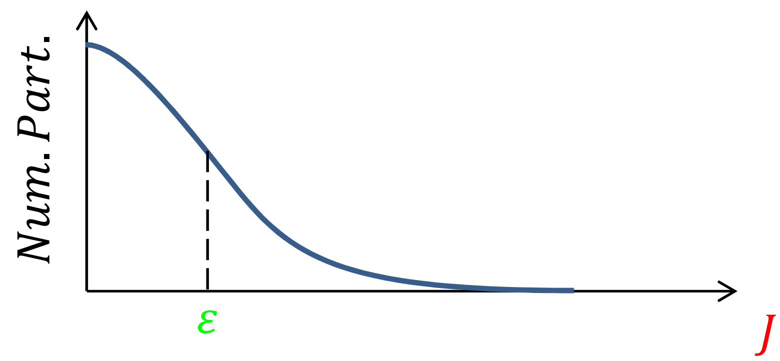
$Q_x - Q_y = i - j$

$Q_x + Q_y = i + j + 1$



Emittance

The tune Q is **constant** and all particles in the beam have more or less the same tune. All particles have different action J: there is a distribution of J values:



The RMS of the action distribution is called Emittance $\varepsilon = RMS(J) = \sigma_J$.

Emittance

The **beam size $\boldsymbol{\sigma_x}$** varies along the ring:

$$\boldsymbol{\sigma_x} = RMS(x) = \sqrt{\varepsilon_x \beta(s)}$$

at the same time:

$$\boldsymbol{\sigma'_x} = RMS(x') = \sqrt{\frac{\varepsilon_x}{\beta(s)}}$$

$\boldsymbol{\sigma}'\ \boldsymbol{x}\ \boldsymbol{xx}\ \boldsymbol{\sigma}'\ \boldsymbol{x}$ $=RRMMSS\ x'\ xx'\ x' =\ \ \varepsilon\, x\ \ \beta\, s\ \ \ \ \varepsilon\, x\ \ \beta\, s\ \ \ \varepsilon\, x\ \ \beta\, s\ \ \varepsilon\, x$
$\varepsilon\varepsilon\ \varepsilon\, x\ xx\ \varepsilon\, x\ \ \varepsilon\, x\ \ \beta\, s\ \ \beta\beta\ s\ ss\ s\ \ \varepsilon\, x\ \ \beta\, s\ \ \ \varepsilon\, x\ \ \beta\, s$

$\boldsymbol{\sigma}'\ \boldsymbol{x}\ \boldsymbol{xx}\ \boldsymbol{\sigma}'\ \boldsymbol{x}$ $=RRMMSS\ x'\ xx'\ x' =\ \ \varepsilon\, x\ \ \beta\, s\ \ \ \ \varepsilon\, x\ \ \beta\, s\ \ \ \varepsilon\, x\ \ \beta\, s\ \ \varepsilon\, x$
$\varepsilon\varepsilon\ \varepsilon\, x\ xx\ \varepsilon\, x\ \ \varepsilon\, x\ \ \beta\, s\ \ \beta\beta\ s\ ss\ s\ \ \varepsilon\, x\ \ \beta\, s\ \ \ \varepsilon\, x\ \ \beta\, s$

$\boldsymbol{\sigma}'\ \boldsymbol{x}\ \boldsymbol{xx}\ \boldsymbol{\sigma}'\ \boldsymbol{x}$ at the same time:

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$$\boldsymbol{\sigma}'_{\boldsymbol{x}} = RMS(x') = \sqrt{\frac{\varepsilon_x}{\beta(s)}}$$

Emittance

Also, emittance is a key parameter for beamlines because they want:

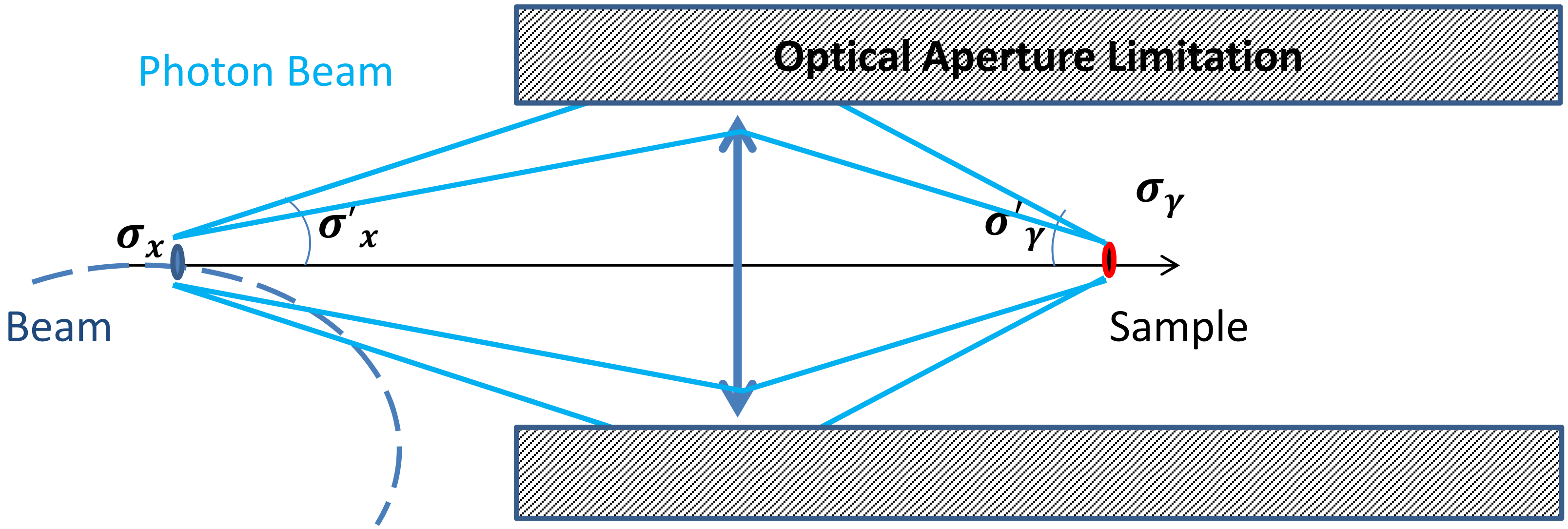
- **Resolution**: determined mostly by $\boldsymbol{\sigma_x}$.
- **Flux of photons**: determined mostly by $\boldsymbol{\sigma'_x}$.

The emittance contains both:

$$\varepsilon_x = \boldsymbol{\sigma_x \sigma'_x}$$

Emittance

Generally speaking, beamlines focalize at the sample ($\boldsymbol{\sigma_x}$), and for a given optical aperture they want as much photons as possible ($\boldsymbol{\sigma'_x}$):



Still, **beam size** is the first requirement.

Emittance

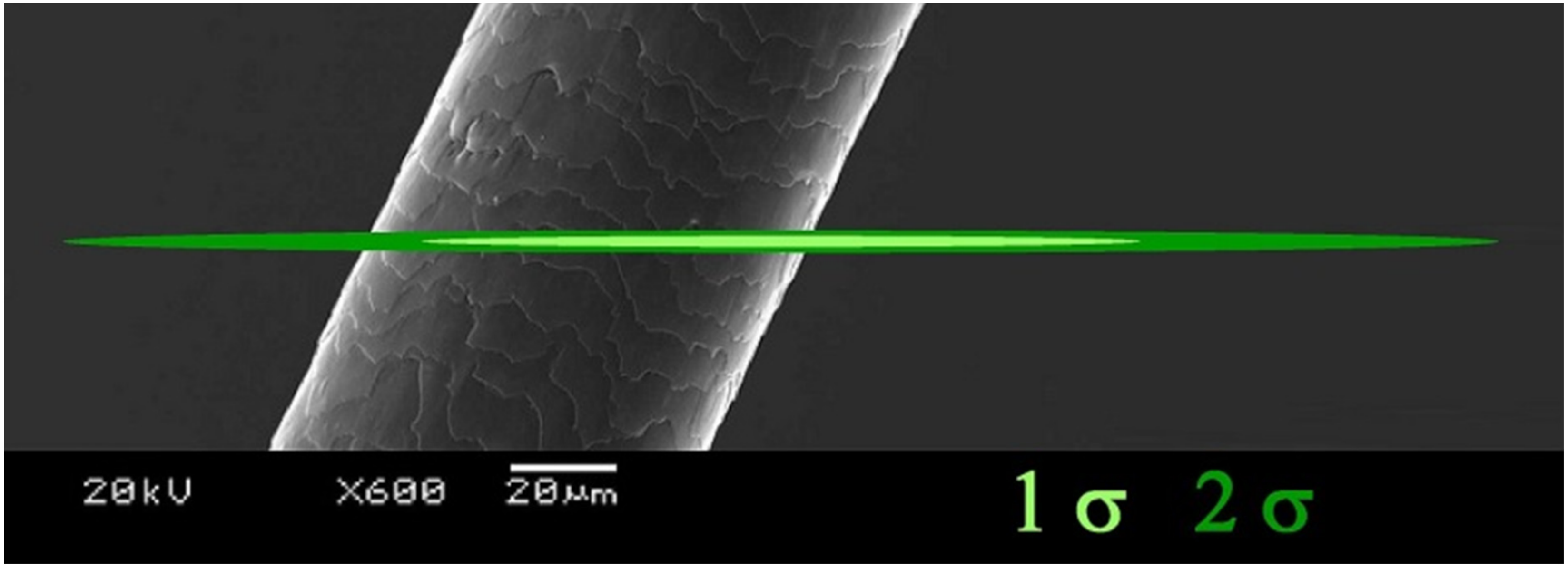
Emittance values for some Light Sources:

	Elettra	ALBA	Diamond	ESRF	APS	Spring-8
Energy (GeV)	2	3	3	6	7	8
Circumference (m)	259	269	562	845	1104	1436
Lattice	DBA	DBA	DBA	DBA	DBA	DBA
Emittance (nmrad)	7.4	4.4	2.7	4	3.1	3.4

Emittance of few *nmrad* means roughly 100μm× 100μrad $\sigma_x \times \sigma'_x$.

Emittance

In modern light sourced (picture from SLS), beam sizes are of the order of a hair thickness:



Lattice design

Quadrupoles must be placed and powered so that they produce **small beam sizes** where desired.

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3. Straight sections space (diagnostic, RF,IDs…).

Lattice design

Quadrupoles must be placed and powered so that they produce **small beam sizes** where desired.

Other practical details to take into account in Lattice design:

1. Phase advance between elements.
2. injection scheme.
3. Straight sections space (diagnostic, RF,IDs…).
4. Maximum beam size.

Lattice design

Design p repetitive structures (cells) is simpler and cheaper, and also, allows to avoid resonances:

$$nQ_x + mQ_y = q$$

Lattice design

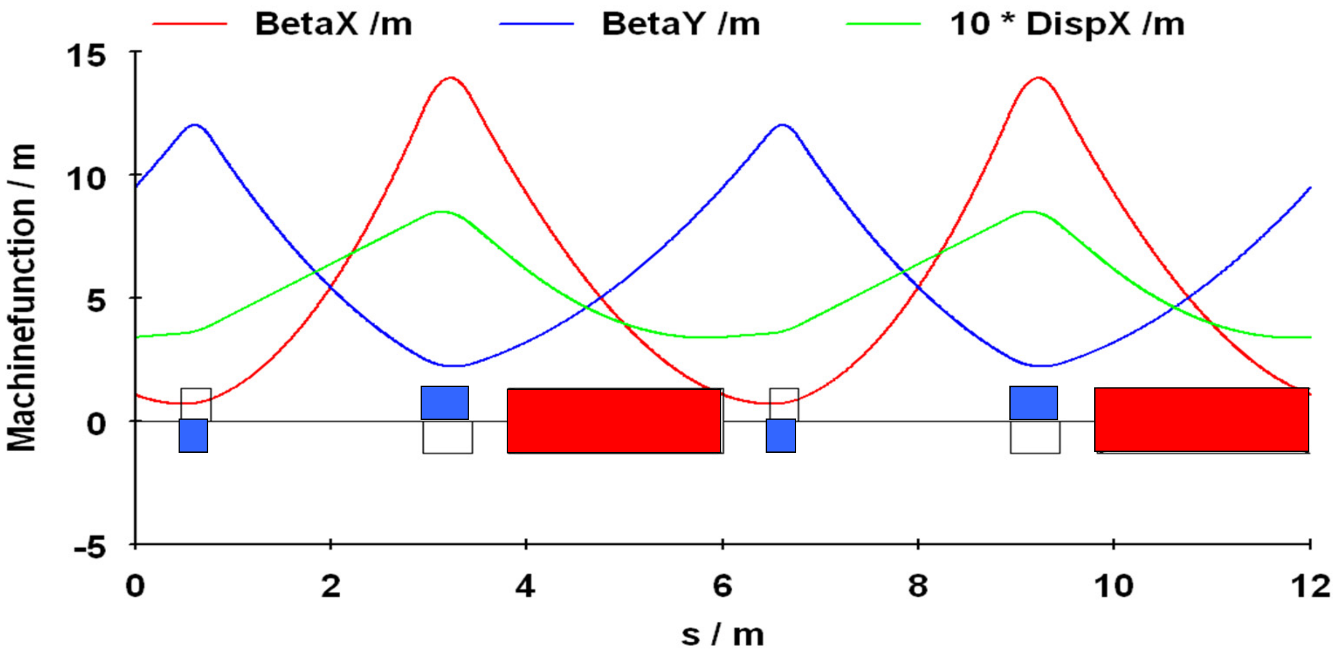
Design p repetitive structures (cells) is simpler and cheaper, and also, allows to avoid resonances:

$$npQ_x + mpQ_y = q$$

The most simple cell (the unit block to build a lattice) consists of alternating FOcusing and De-fOcusing quadrupoles (FODO).

Lattice design

Nowadays, the FODO cell is typically used at booster synchrotrons, but third generation storage rings need more complex lattices.



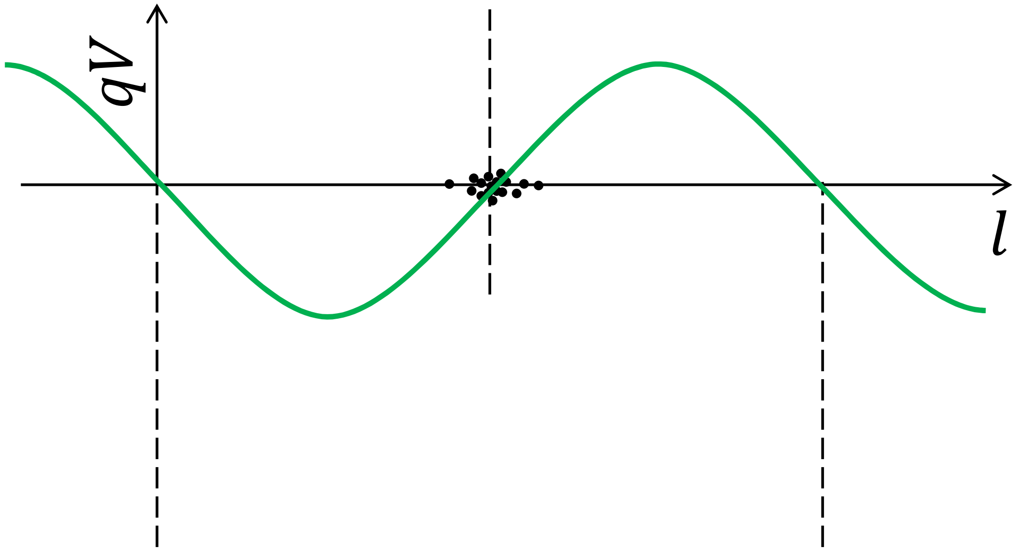
Energy deviations

So far we have assumed that the energy of the particles does **not change** and it is **the same** for all of them. But…

- RF cavities: change the energy depending on arrival time.
- Radiation: produces energy loss at every bending magnet.
- Particle interactions: produce energy exchange with residual gas particles or among them.

Energy deviations

The RF cavities produce longitudinal focusing:



Energy deviations

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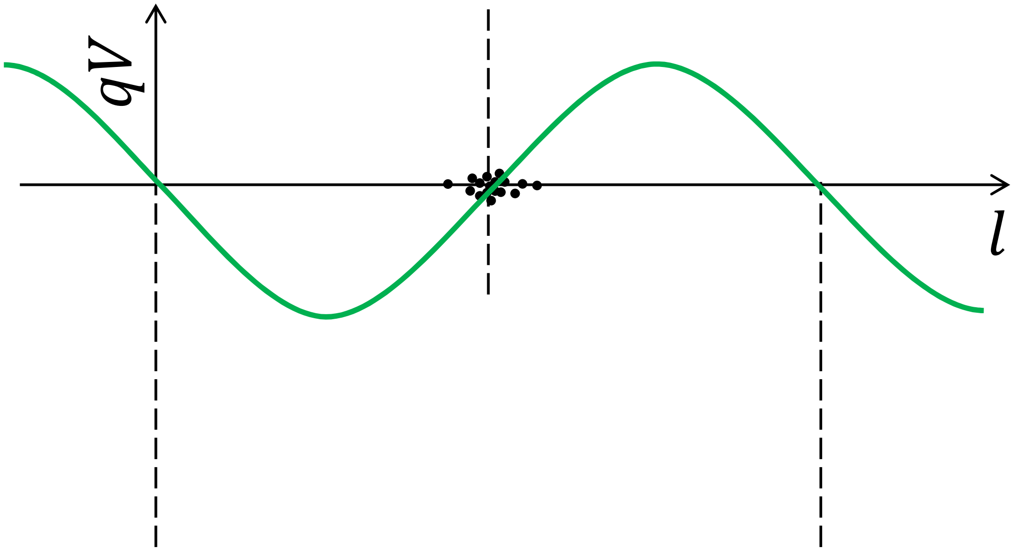

We will see that more energetic particles take more time to go around, they get retarded but then the RF removes energy from them…

Energy deviations

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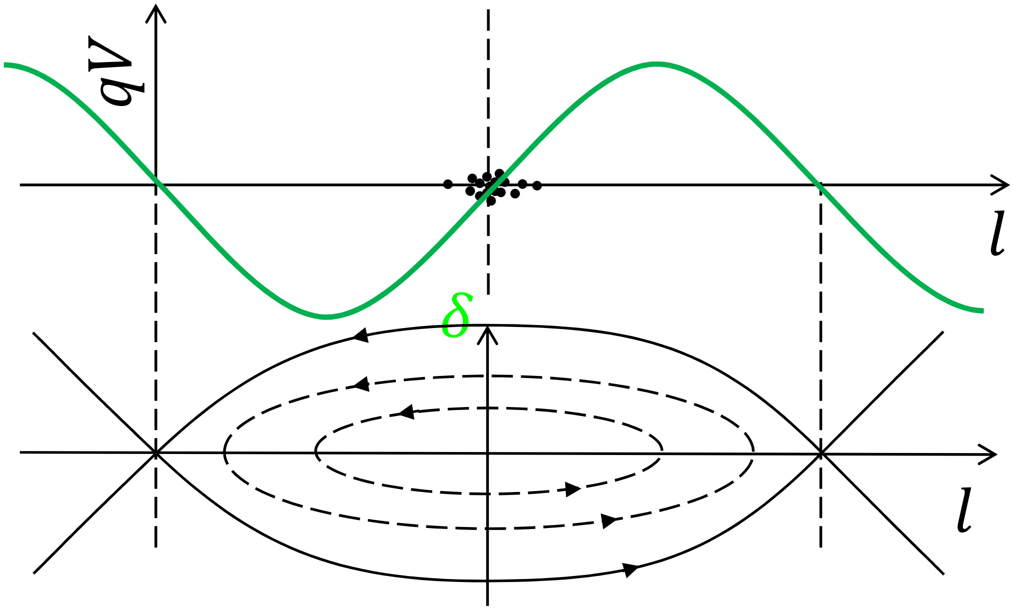
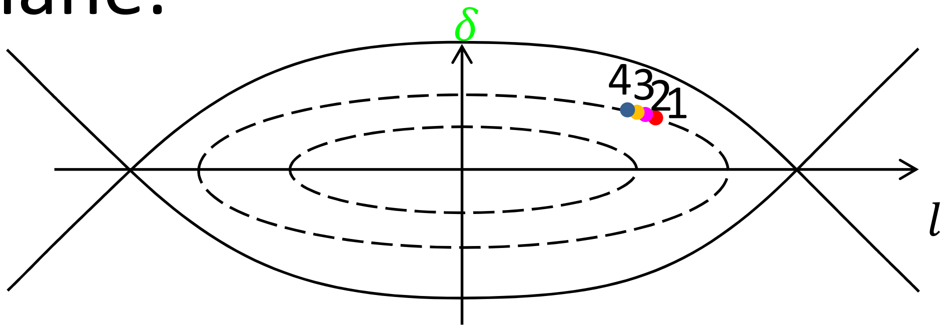


We will see that more energetic particles take more time to go around, they get retarded but then the RF removes energy from them…

Energy deviations

However, the dynamics in the longitudinal plane (synchrotron oscillations) is much slower than the transverse plane:

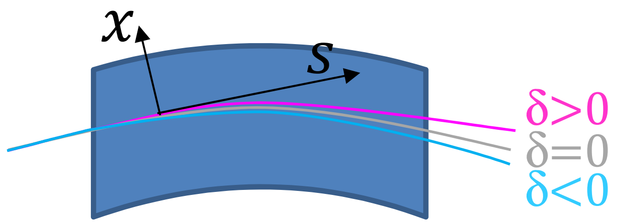


The energy deviation with respect to the reference particle $\delta = \frac{\Delta E}{E}$ is assumed to be constant. Which is a good approximation in many cases.

Energy deviations

If a particle has an energy deviation, it follows circles with different radius:

$$\rho = \frac{E}{qcB} = \rho_0(1+\delta)$$



Both dipoles and quadrupoles will have different effect, but the first order effect comes from the dipoles and adds a term to the **horizontal plane** equation:

$$\frac{d^2x}{ds^2} + \left(k(s) + \frac{1}{{\rho_0}^2(s)}\right)x = \frac{\delta}{\rho_0(s)}$$

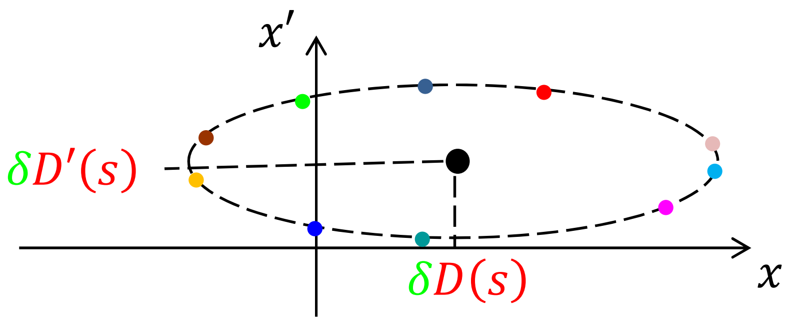
Energy deviations

An energy deviation produces a dispersion orbit around the machine equal to $\delta D(s)$.

$$\frac{d^2 D}{ds^2} + \left(k(s) + \frac{1}{{\rho_0}^2(s)}\right) D = \frac{1}{\rho_0(s)}$$

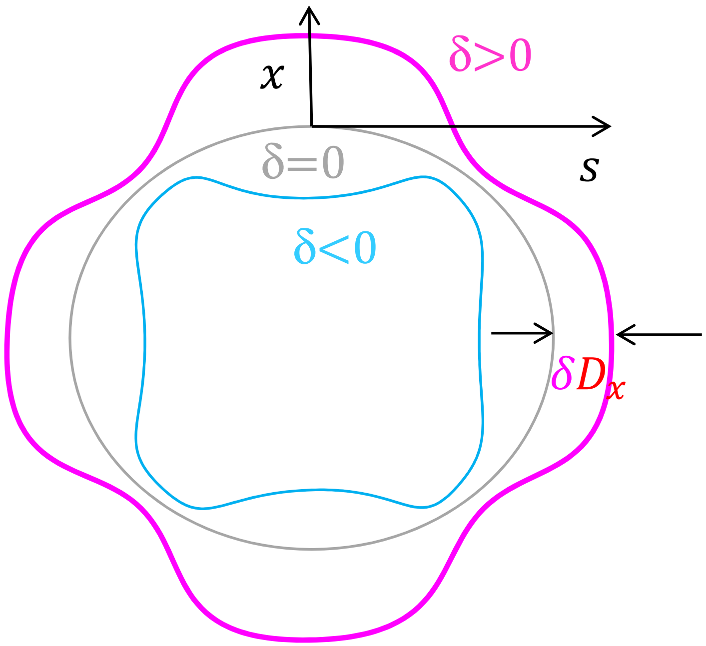
The **betatron oscillations** go around $\delta D(s)$:

$$x(s) = \sqrt{J\beta(s)} cos(\mu(s) - \boldsymbol{\mu_0}) + \delta D(s)$$



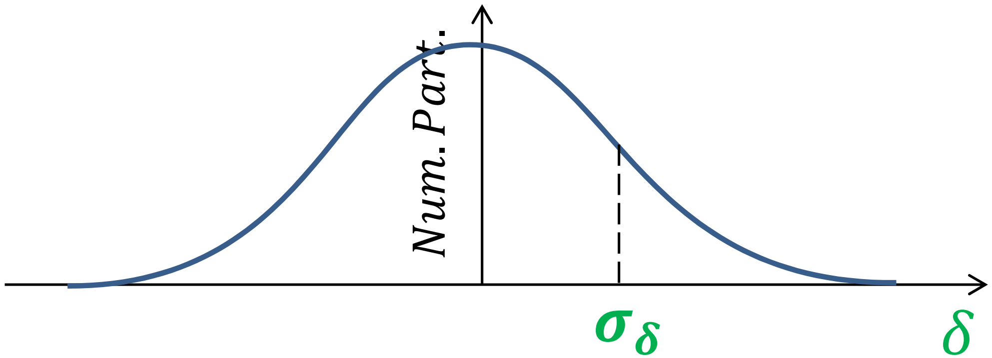
Energy deviations

In general the dispersion is positive ($D_x(s)$>0), so, particles with more energy ($\delta > 0$) take a longer way and take more time to do one turn!



Energy deviations

The beam contains particles with a distribution of energies, the RMS value $\boldsymbol{\sigma_\delta} = RMS(\delta)$:



Assuming that it is Gaussian and not correlated with the action J distribution:

$$\boldsymbol{\sigma_x} = \sqrt{\varepsilon_x \beta(s) + \sigma_\delta{}^2 D^2(s)}$$

$$\boldsymbol{\sigma'_x} = \sqrt{\frac{\varepsilon_x}{\beta(s)} + \sigma_\delta{}^2 D'^2(s)}$$

Chromaticity

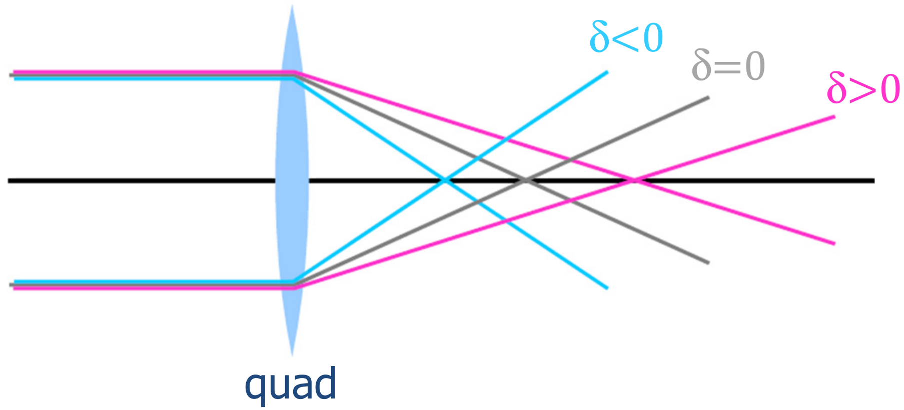
Additionally, depending on δ the quadrupoles have also a different focusing strength.

Energy deviations causes the beta function, the phase and also the tune to change accordingly. The chromaticity ξ is defined as:

$$Q_x = Q_{x,0} + \delta\xi_x$$
$$Q_y = Q_{y,0} + \delta\xi_y$$

Chromaticity

The chromaticity effect can be **visualized** as a different focal length for every quadrupole:



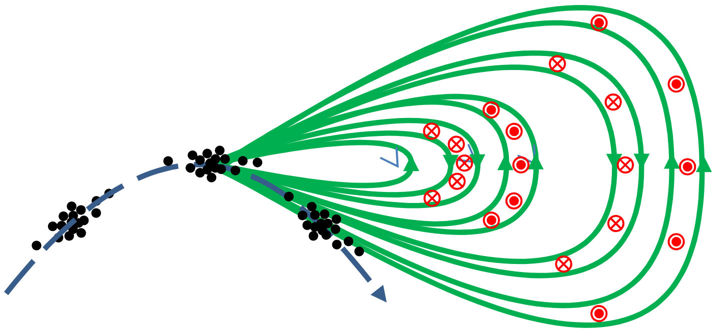
The chromaticity due to dipoles and quads (also called natural chromaticity) is always negative ranging from -30 to -100 (unitless) in both planes.

Radiation Energy Loss

The synchrotron radiation takes energy U_0 every turn:

$$U_0 = \frac{q^2\gamma^4}{3\epsilon_0\rho}$$

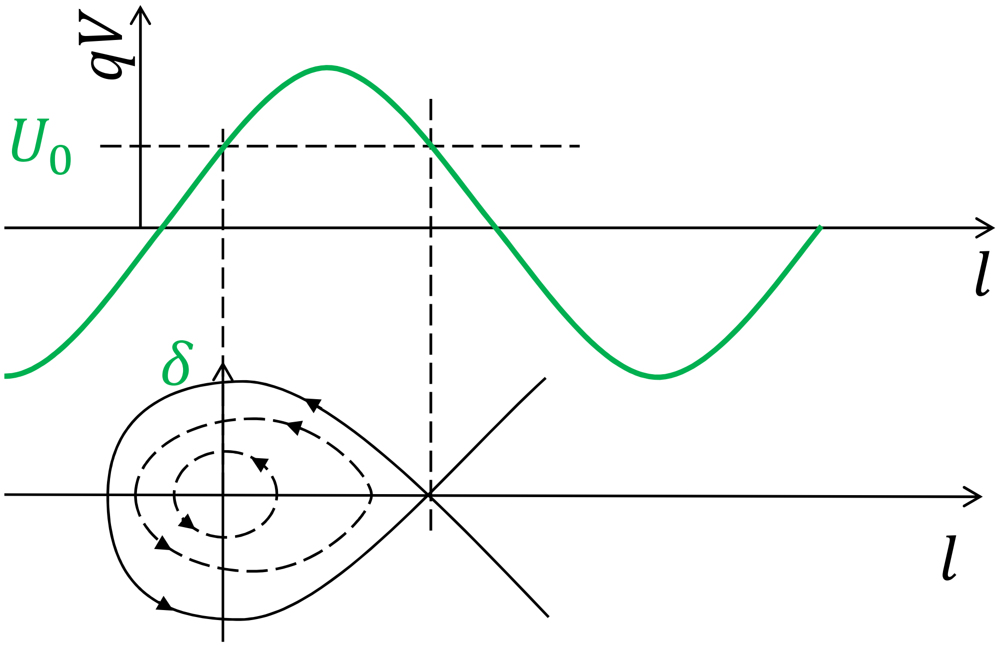
$$\gamma = \frac{E}{mc^2}$$



The proton has a mass around 1GeV, while electrons' is 0.5keV. For present accelerators, protons are assumed to almost no radiate.

Radiation Energy Loss

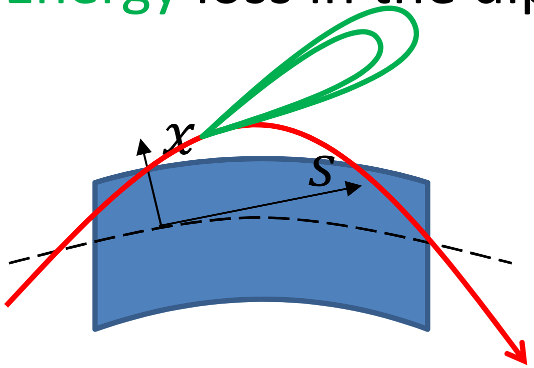
The RF cavities need to provide at least the energy loss per turn U_0.



Radiation Damping

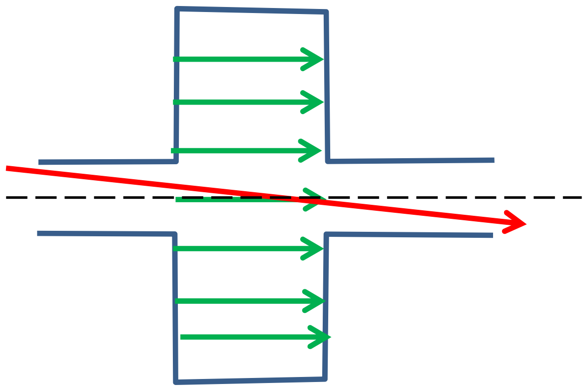
However, the energy compensation in the RF changes the momentum direction of the particle towards the **design momentum**!

Energy loss in the dipole



Loss in a random direction

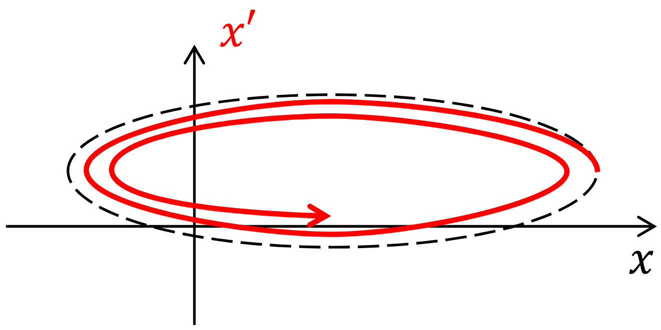
Energy gain in the RF

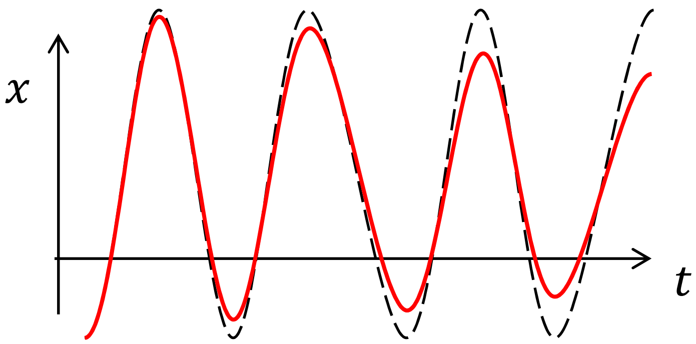


Gain in a predefined direction

Radiation Damping

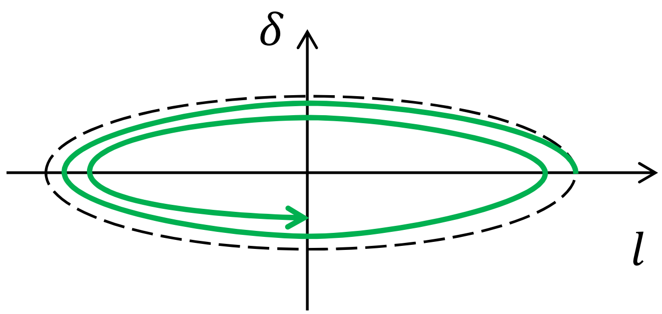
As a result, turn after turn, the trajectory angle x' gets reduced:

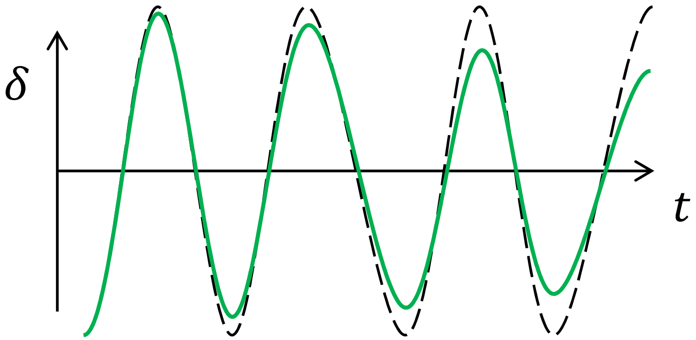




As the path gets closer to design value, the revolution time also gets closer to the design value.

As the revolution time gets closer to the design value, the RF gives less extra energy to that particle:





In all directions, an initially off energy and/or off center particle tends to end in the design orbit and energy.

Radiation Damping

This effect is called Radiation Damping, it is present in any particle accelerator, but almost only is noticeable in a reasonable time scales in lepton machines:

$$U_0 = \frac{q^2\gamma^4}{3\epsilon_0\rho}$$

Here we will deal mainly with electron accelerators, where the energy loss per turn can be a considerable part (0.01%) of the particle energy.

Equilibrium Beam Size

However, the beam size is not zero around the closed orbit.

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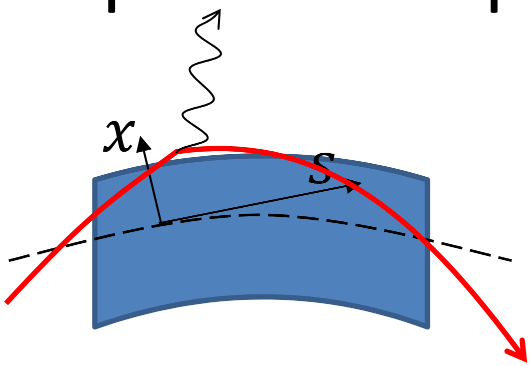
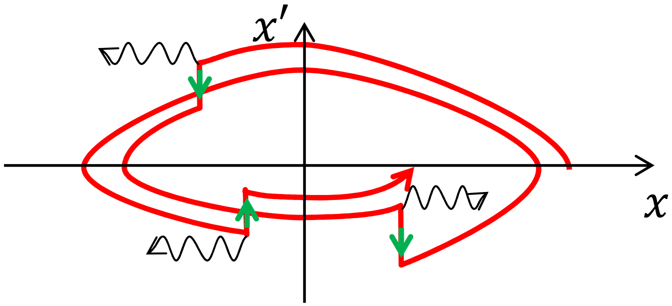
At some point the radiation quantization effects interrupt the damping:

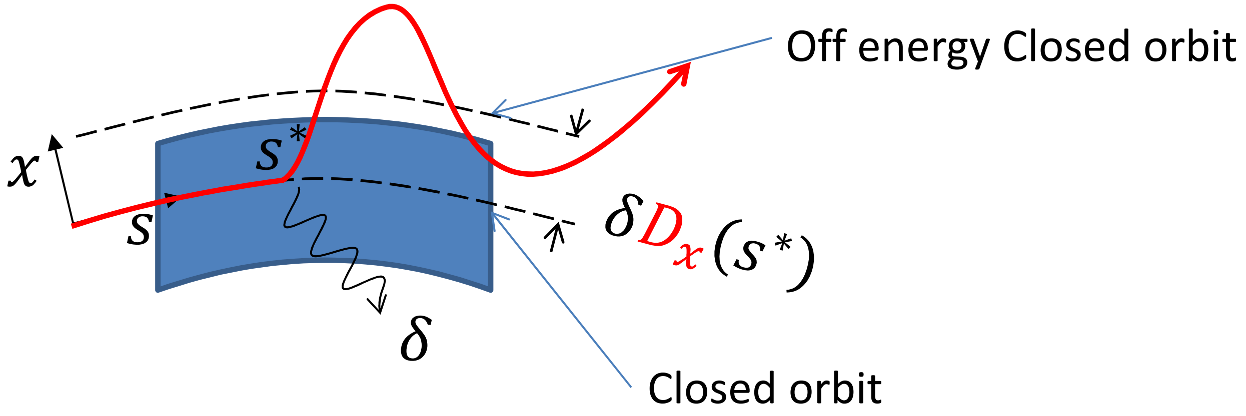




Equilibrium Beam Size

The damping is a result of the dipole+RF effect, it damps in the 3 planes.

The quantum exitation happens at the dipoles. The effect in the transverse plane depends on the dispersion at that point:



Equilibrium Beam Size

The equilibrum emittance can be expressed as a function of the dispersion and the bending radius:

$$\varepsilon_x = C_q \frac{\gamma^2}{P_x} \frac{\oint \frac{\mathcal{H}_x(s)}{\rho^3{}_x(s)} ds}{\oint \frac{1}{\rho^2{}_x(s)} ds}$$

Anti damping

Damping

C_q is a constant, P_x is the horizontal partition number and $\mathcal{H}_x$ is the dispersion invariant:

$$\mathcal{H}_x(s) = \beta(s) D_x'^2(s) + 2\alpha(s) D_x(s) D_x'(s) + \gamma(s) D_x{}^2(s)$$

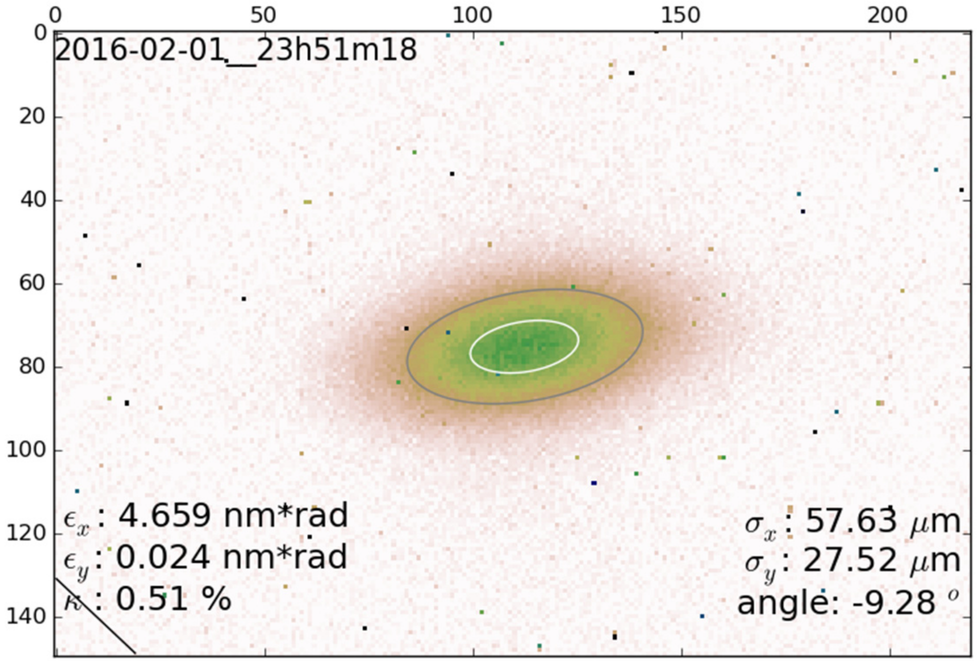
Equilibrium Beam Size

This makes a big difference in stability and in the beam size limitation: ε_x, ε_y and σ_δ :

- Hadrons synchrotrons (Protons and Ions): where beam size is determined by the injected beam.
- Lepton synchrotrons (electrons mainly): beam size does NOT depend on the initial beam distribution.

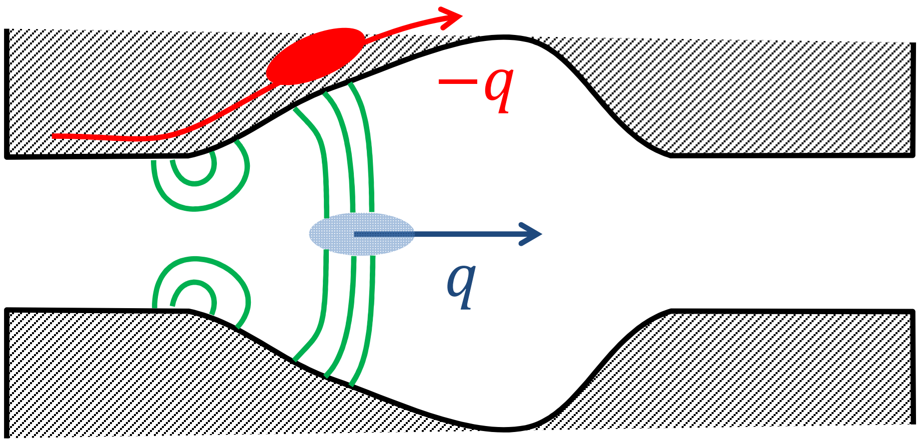
Equilibrium Beam Size

Wait a minute... when we look at the beam size in our CCD we are actually seeing a cloud of electrons performing quantum jumps?... Yes!



Instabilities

The beam electromagnetic field can reflect in abrupt transitions of the vacuum chamber. Or from a different point of view, image current radiates!

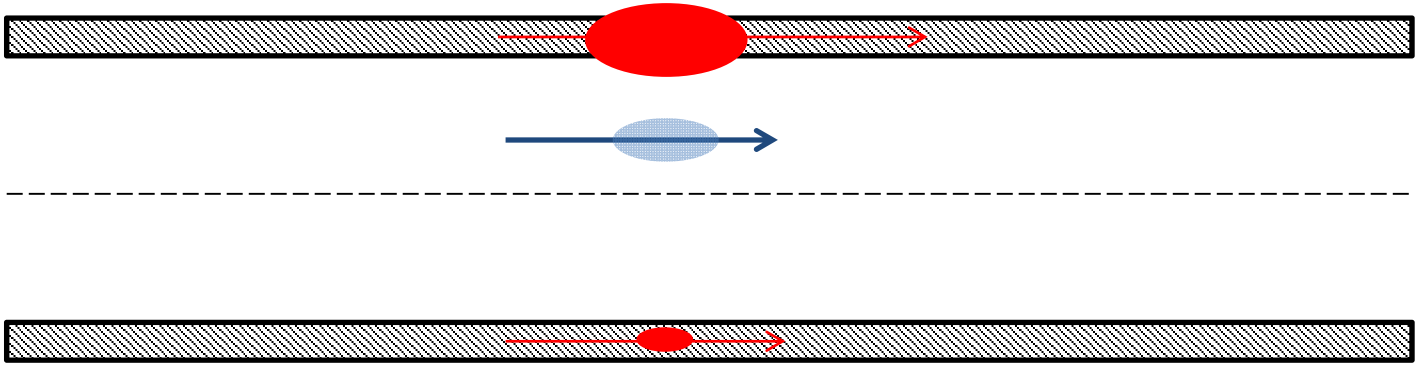


These are called wake-fields. The more particles in the bunch the bigger the wake-field.

Transverse Instabilities

Hence, the wake-fields may cause instabilities at big currents.

If the beam is centered, no transverse fields are generated, but if it moves transversally, it will kick transversally in the same direction.



Head tail instability

It takes some time for the electromagnetic field to reflect in the chamber. Hence, the particles in the bunch will be influenced by the wake field but with a delay.

The head of the bunch will affect the tail…

Head tail instability

Transverse Head Tail Instability:

A warning:

Beam instabilities is a complicated thing to study, if you only can remember one thing from all this, let it be: chromaticity must be corrected to positive values, **otherwise…**

Head tail instability

Transverse Head Tail Instability:

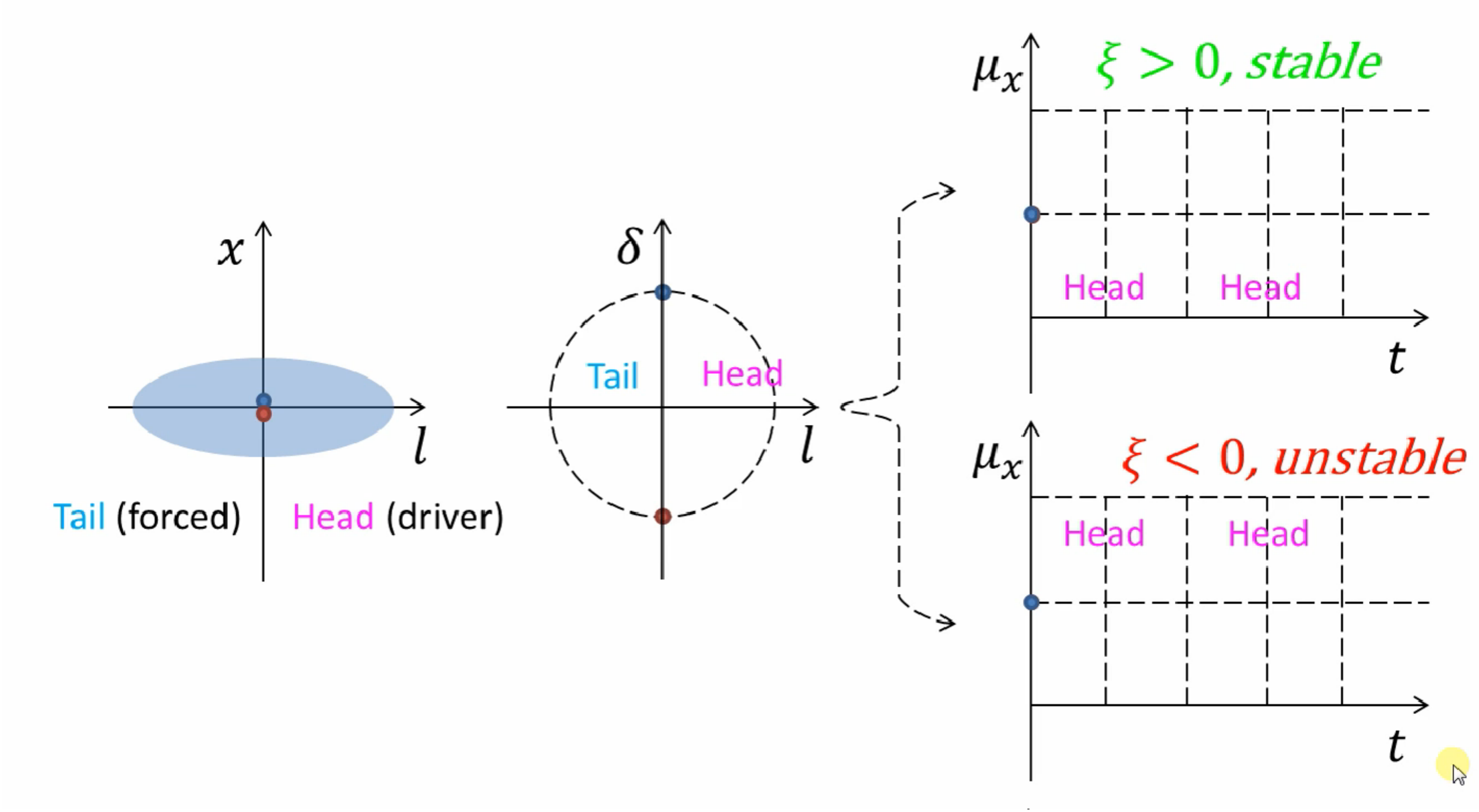
Head tail instability

Transverse Head Tail Instability:

The Head transverse betatron oscillations produce a force that increases or damps the Tail transverse oscillations. After half synchrotron oscillation the roles are inverted!

Head tail instability

Transverse Head Tail Instability:



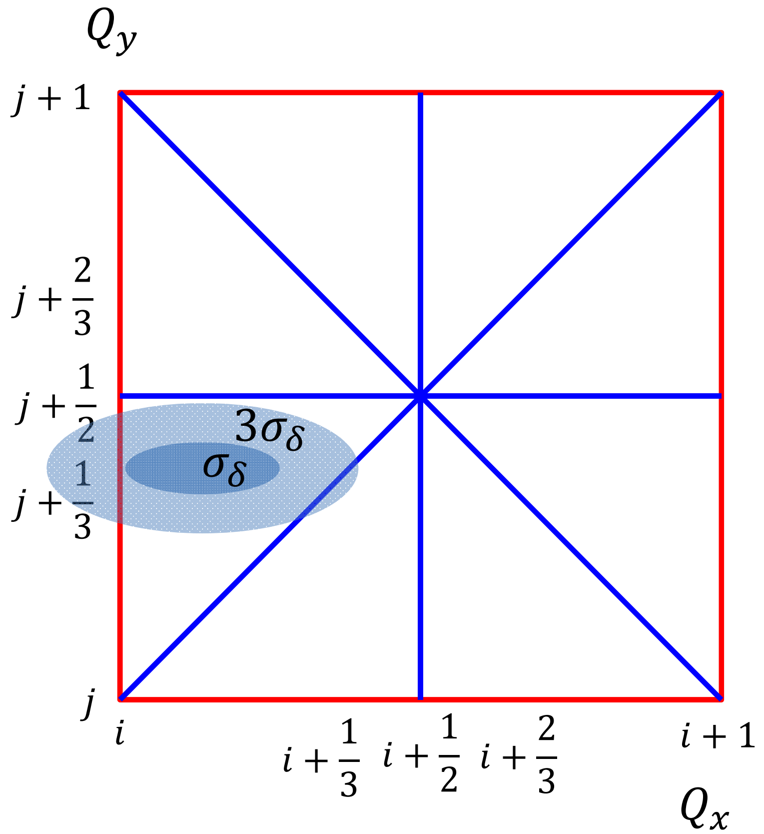
Instabilities

Head tail is just one type of instability. A mathematically rigorous analysis shows that to avoid the more important instability sources, **chromaticity** should be small and positive.

However, in a storage ring with quadrupoles **chromaticity** is big and negative!

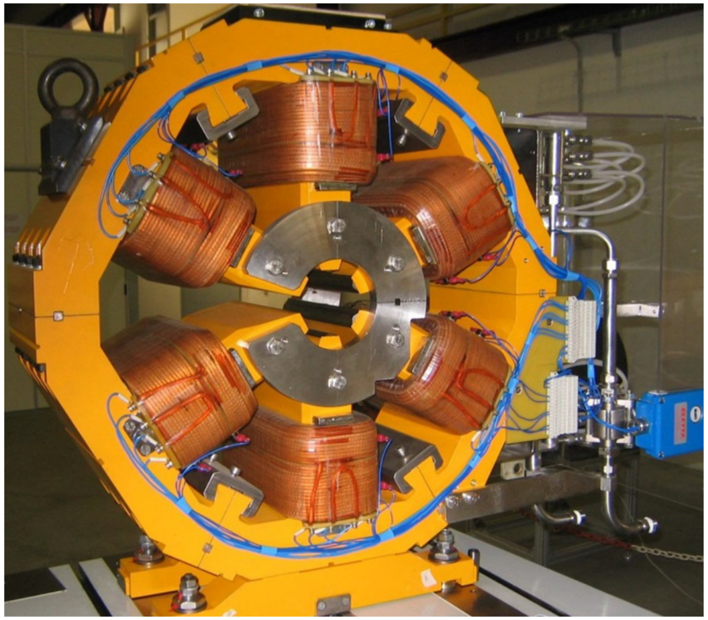
Chromaticity

On top of that, large chromaticity has other harmful effects. Since typical values for σ_δ are 10^{-3} and natural chromaticities are around -100, this makes the beam to have a big tune spread so that it can hit harmful resonances.



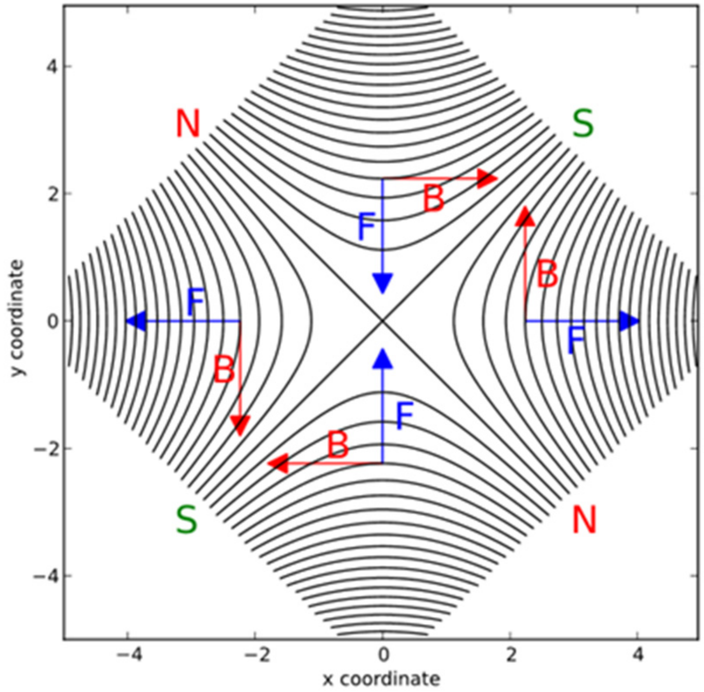
Chromaticity

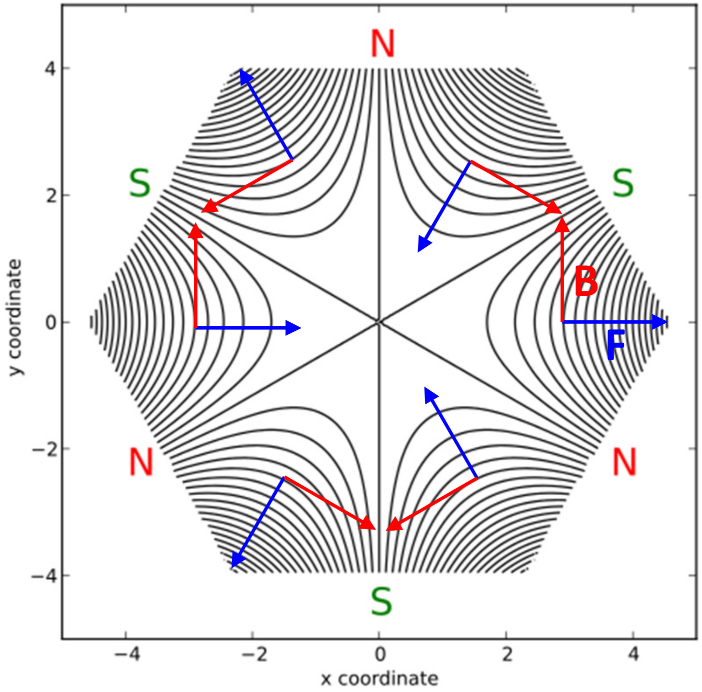
Indeed, **chromaticity** is corrected in any accelerator. To do so, sextupole magnets are used. They are similar to quadrupoles, but have 6 poles instead:



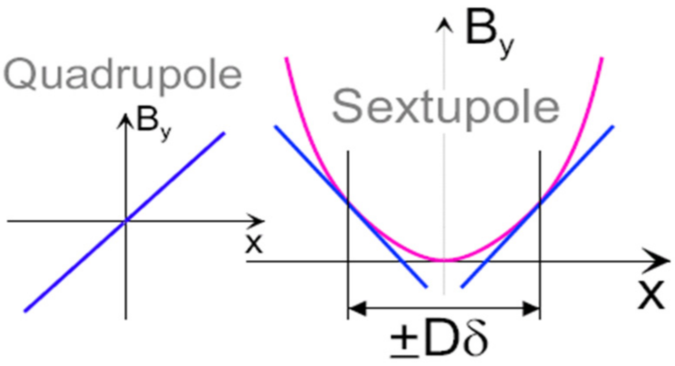
Chromaticity

Like quadrupoles, the force (field) is zero at the center, but in the sextupole it has the **same sign** at both sides:

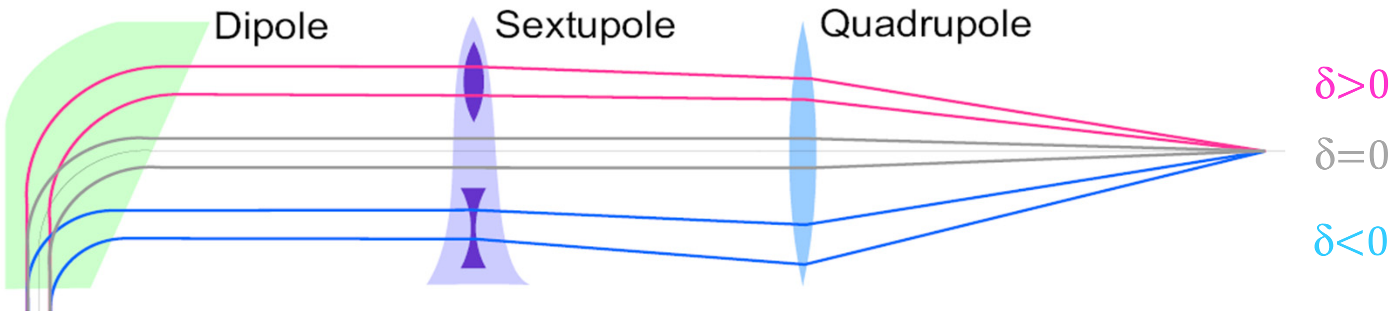




Sextupole have parabolic magnetic field on axis:



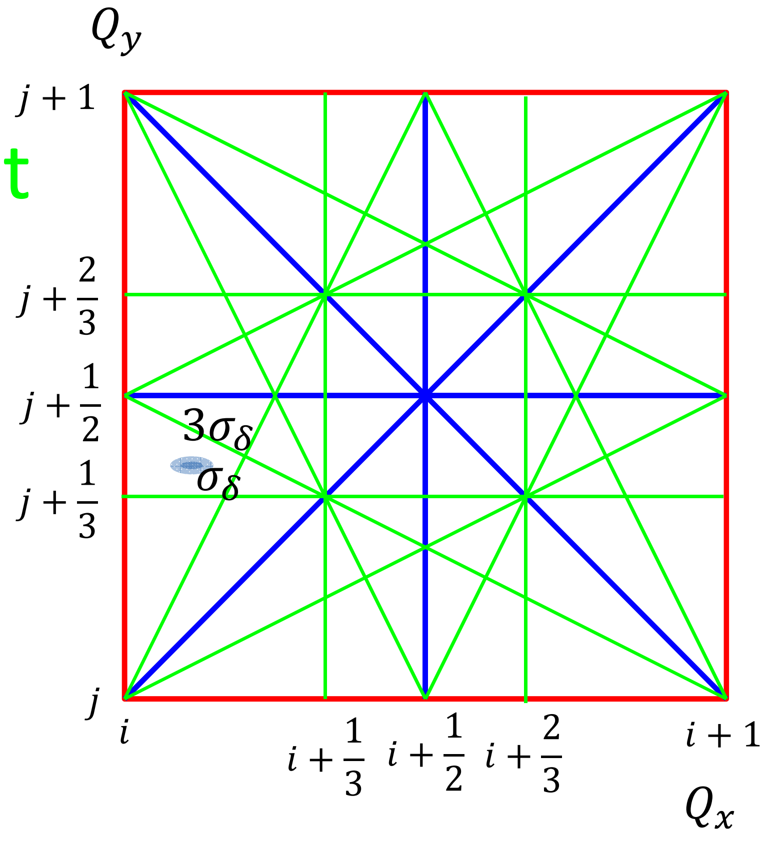
Placing sextupoles at locations with dispersion $D(s)$ allows to correct the chromaticity:



Chromaticity

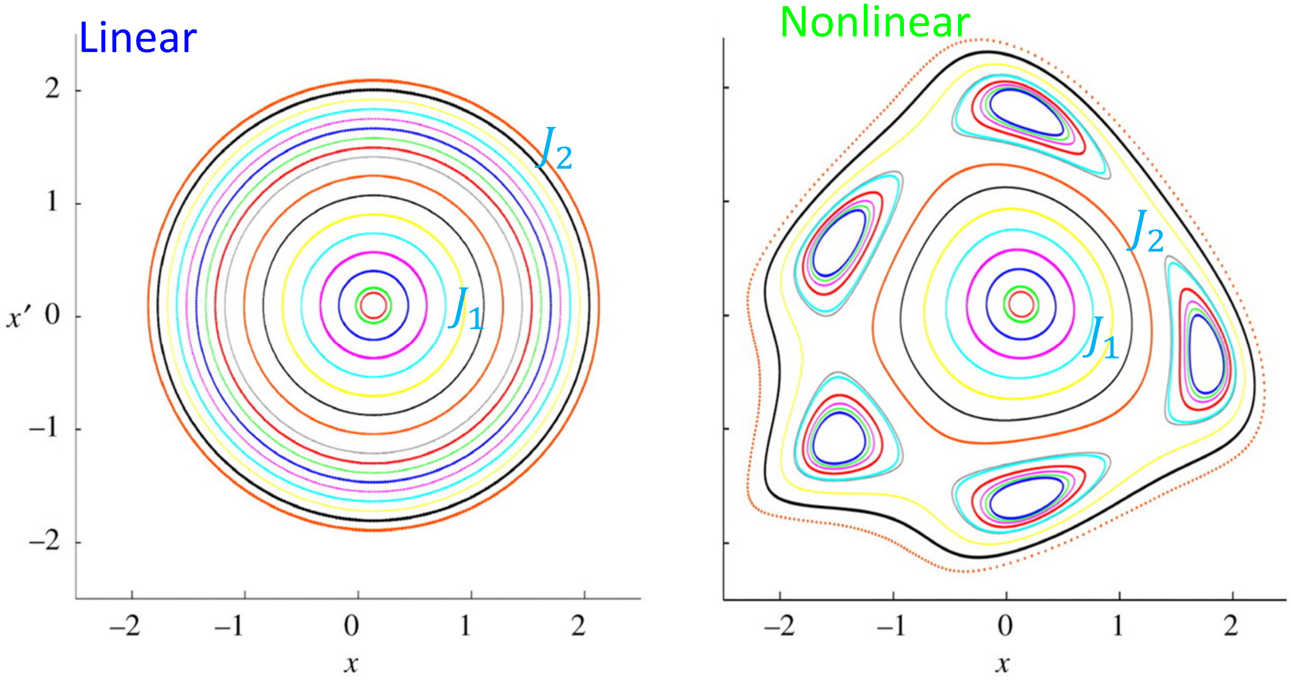
Sextupoles allow for stable beams with small tune shifts with energy.

As a pay off, more resonant lines appear and the system becomes non linear: stability is not guaranteed at big amplitudes.

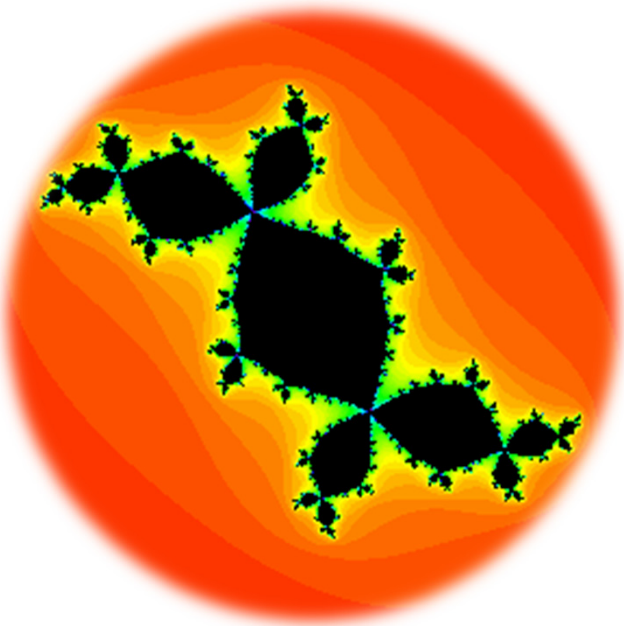


Nonlinear Lattice design

Linear systems behave independently of the amplitude (action). Nonlinear systems don't:



You all know other nonlinear systems:



The rabbit fractal:

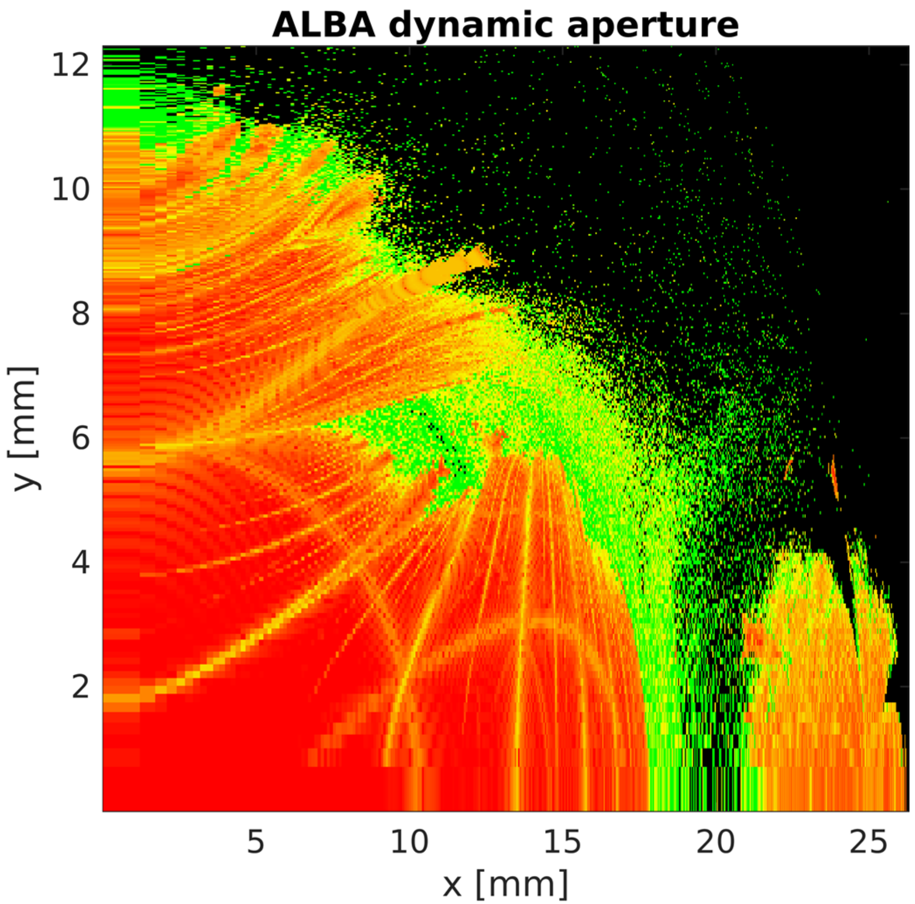
$$z_{n+1} = z_n^{2} + c$$
$$c = -0.123 + 0.745i$$

Nonlinear

Here colors indicate how divergent are the points…

Nonlinear Lattice design

In accelerators we try to see which particles are likely **not** to diverge...



The accelerator map:

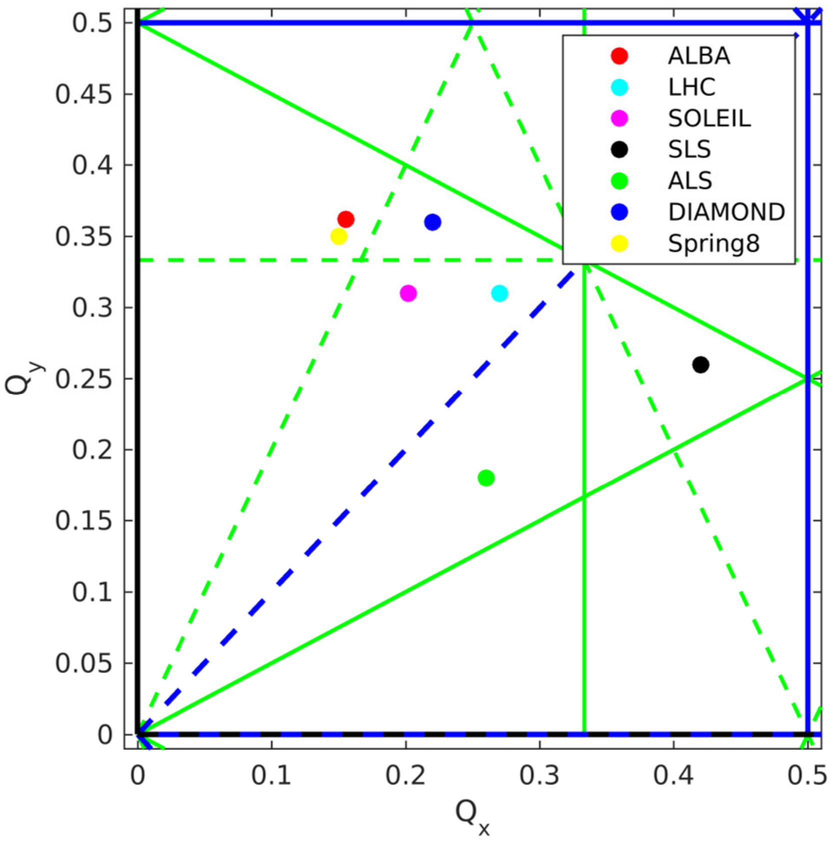
$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \\ y_{n+1} \\ y'_{n+1} \end{pmatrix} = Map \begin{pmatrix} x_n \\ x'_n \\ y_n \\ y'_n \end{pmatrix}$$

Nonlinear Lattice design

The non linear effects depend on how strong the sextupoles are and where they are located.

In modern light sources, with every time smaller emittances, and higher quadrupolar fields, stronger chromatic effects and stronger sextupolar fields, sextupoles is part of the lattice design.

The working point is chosen to avoid resonances, but the best location depends on the details of every synchrotron **lattice**:



The end

Thanks! Questions?