Optics and mechanics of mirror benders

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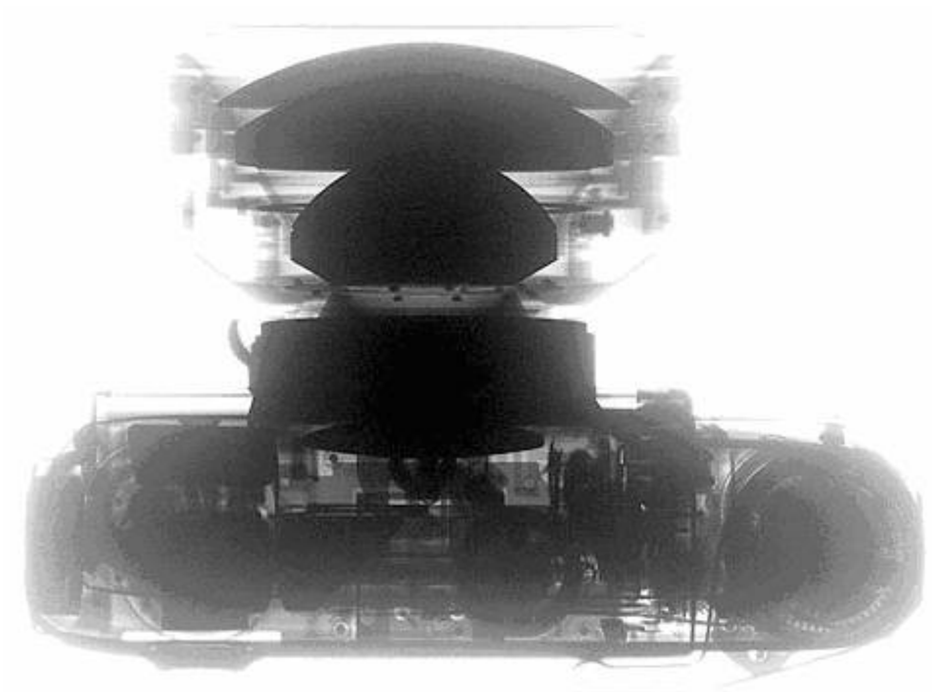
MECHANICAL ENGINEERING DESIGN OF SYNCHROTRON RADIATION EQUIPMENT AND INSTRUMENTATION

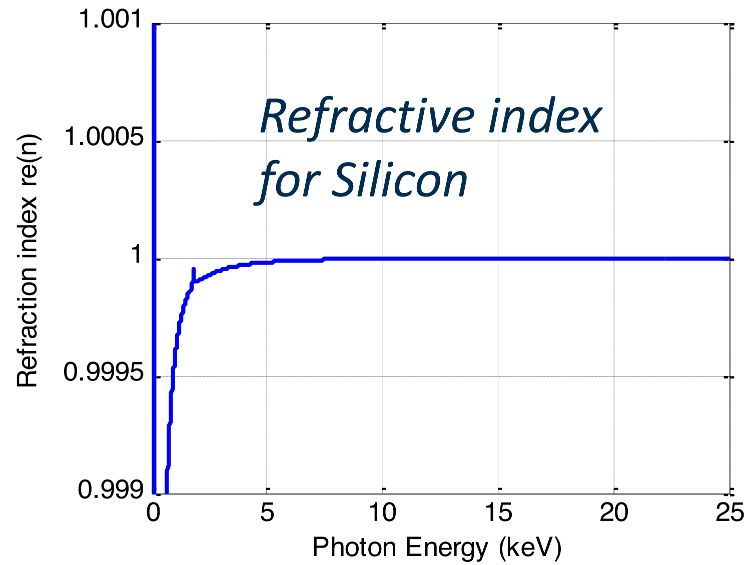
1. Basics of x-ray mirror optics
2. Characteristics of elliptic mirrors
3. Mirror bender calculations and metrology
4. Mechanical design of mirror benders

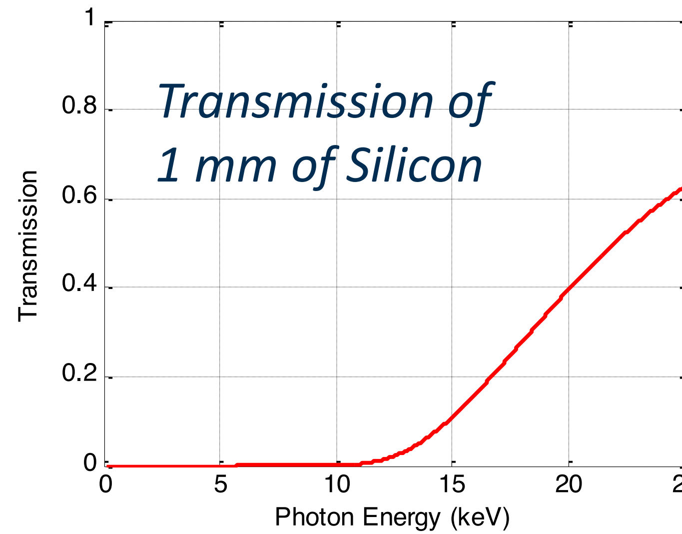
1. Basics of x-ray mirror optics

- *Focusing with reflective optics*
- *Surface error, aberration*

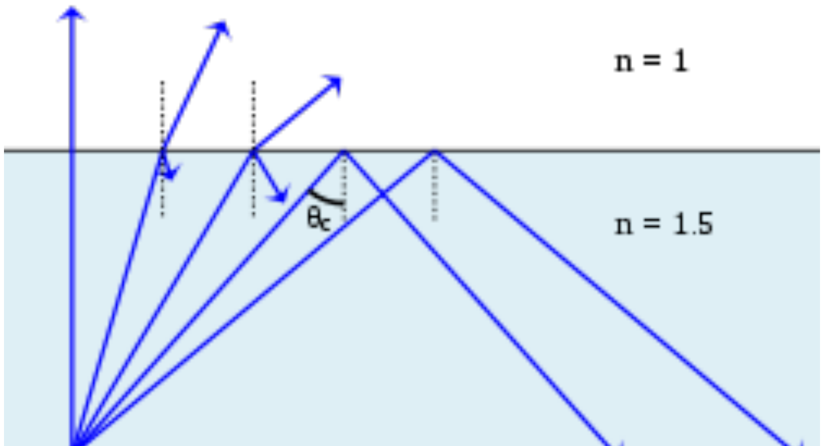
Refractive index of x-rays



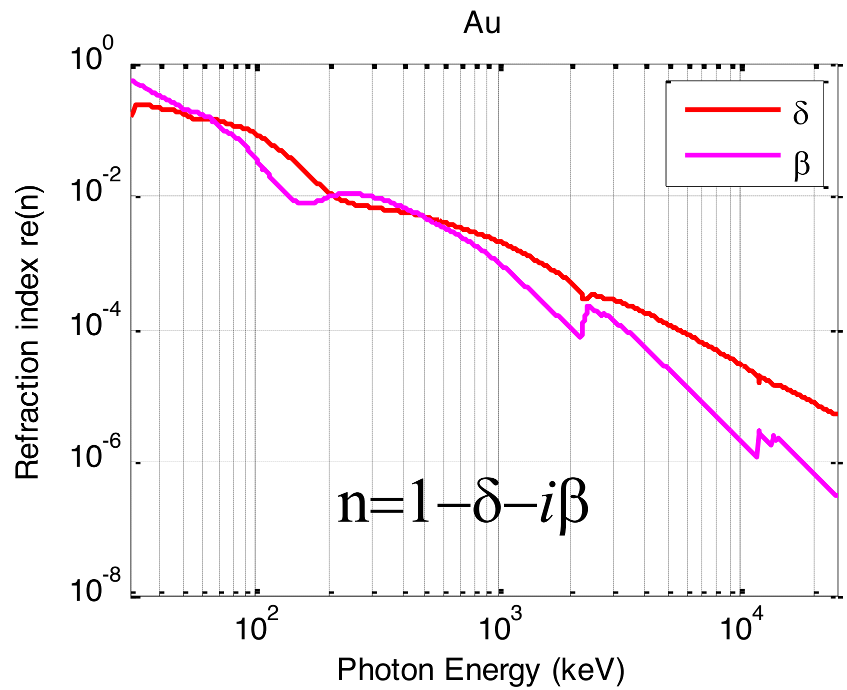




Total reflection

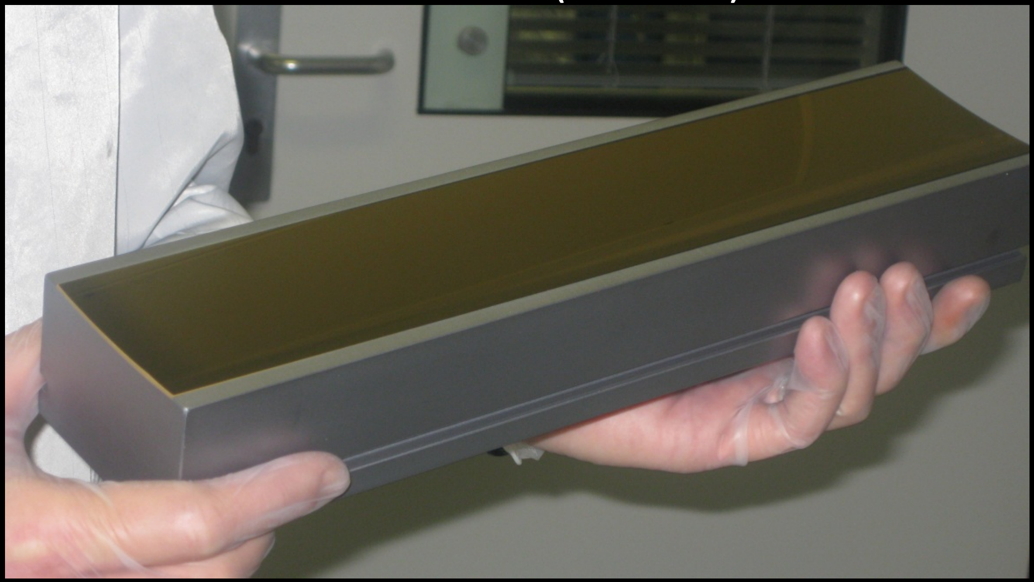


$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

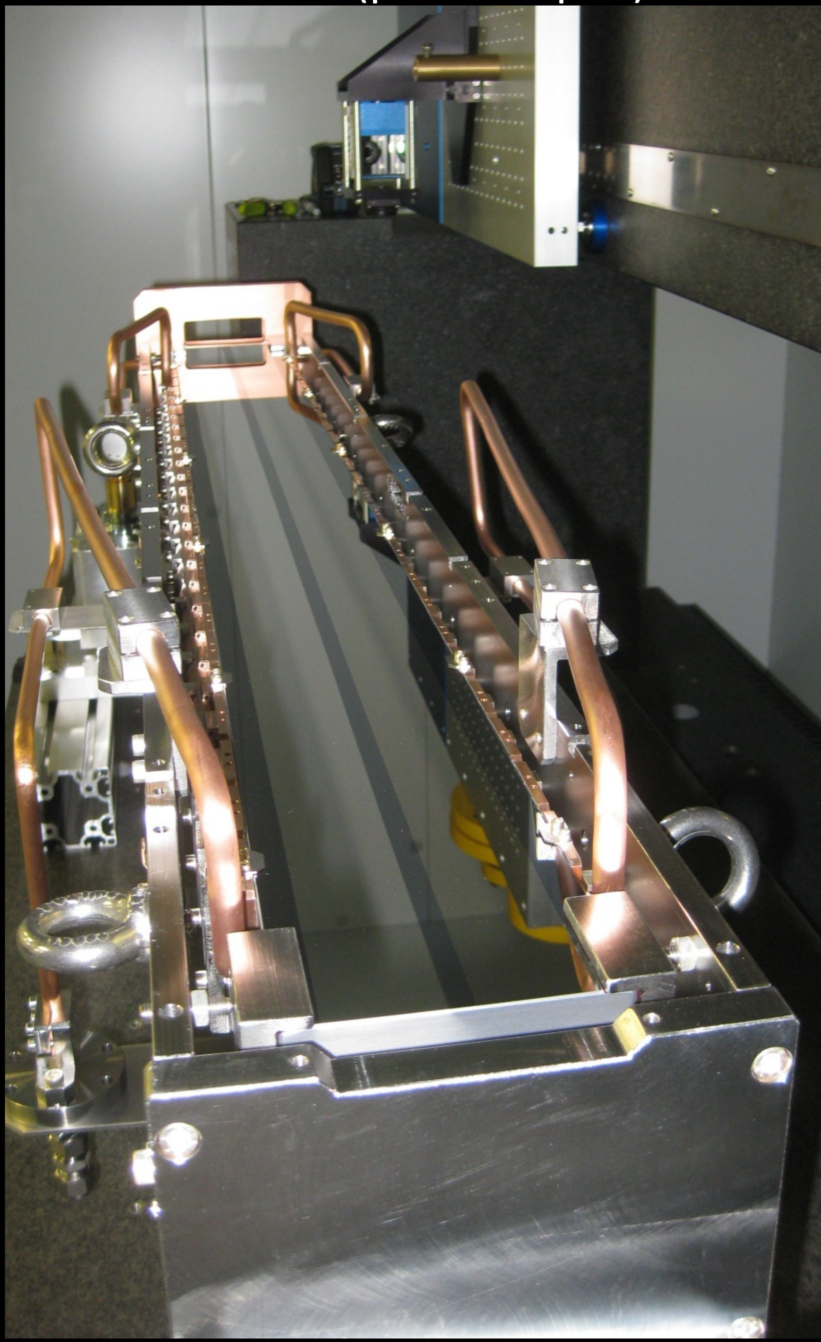


$$\theta_C = \sqrt{2\delta}$$

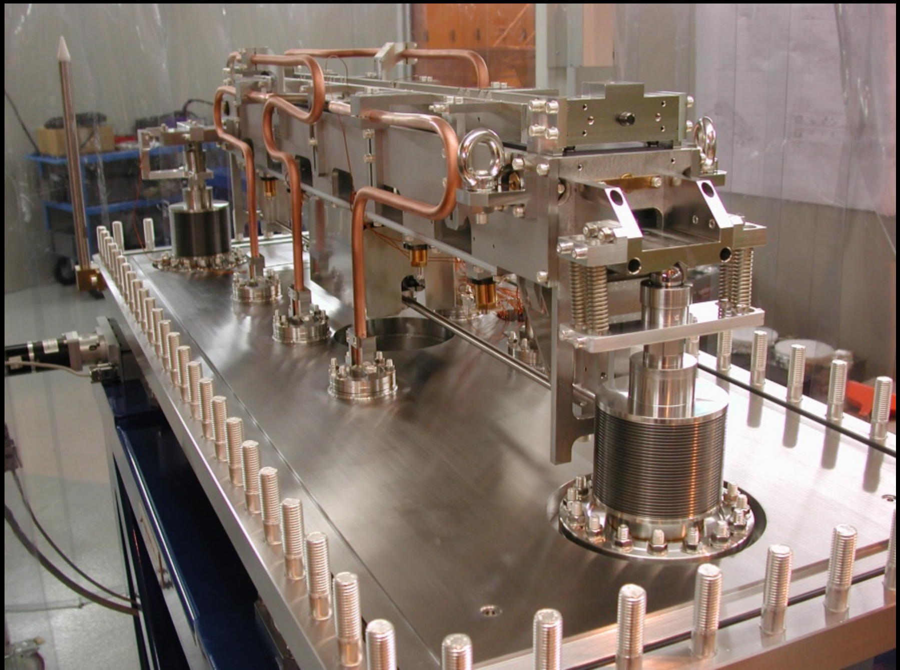
Mirror 400 mm (toroidal)



Mirror 1200 mm (plane elliptic)

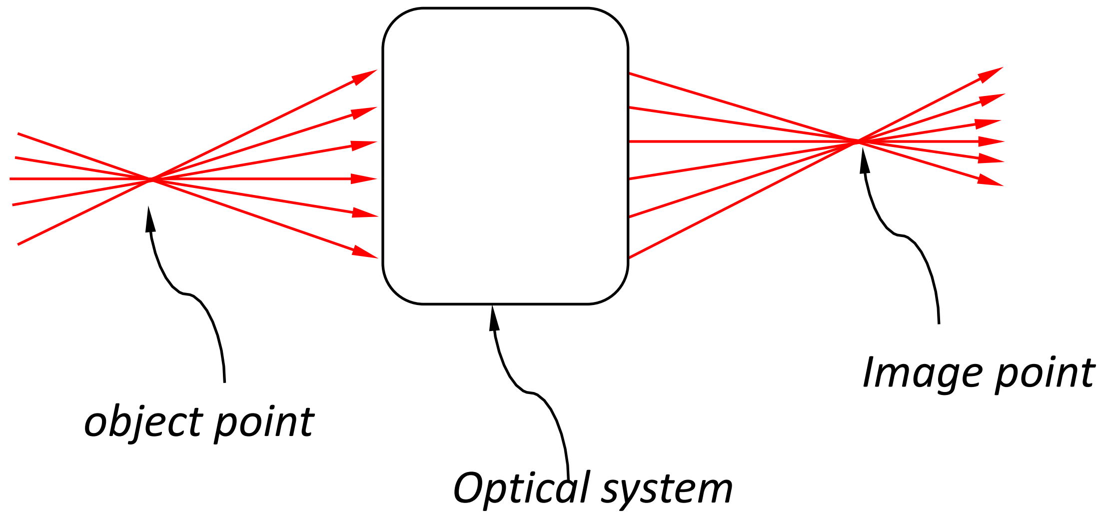


A mirror in its vessel



To focus (v.)

*An **optical system** images (stigmatically) an **object point A**, onto an **image point A'** if all the rays that depart from A, and reach the optical system, gather at A', <u>independently</u> of their direction.*

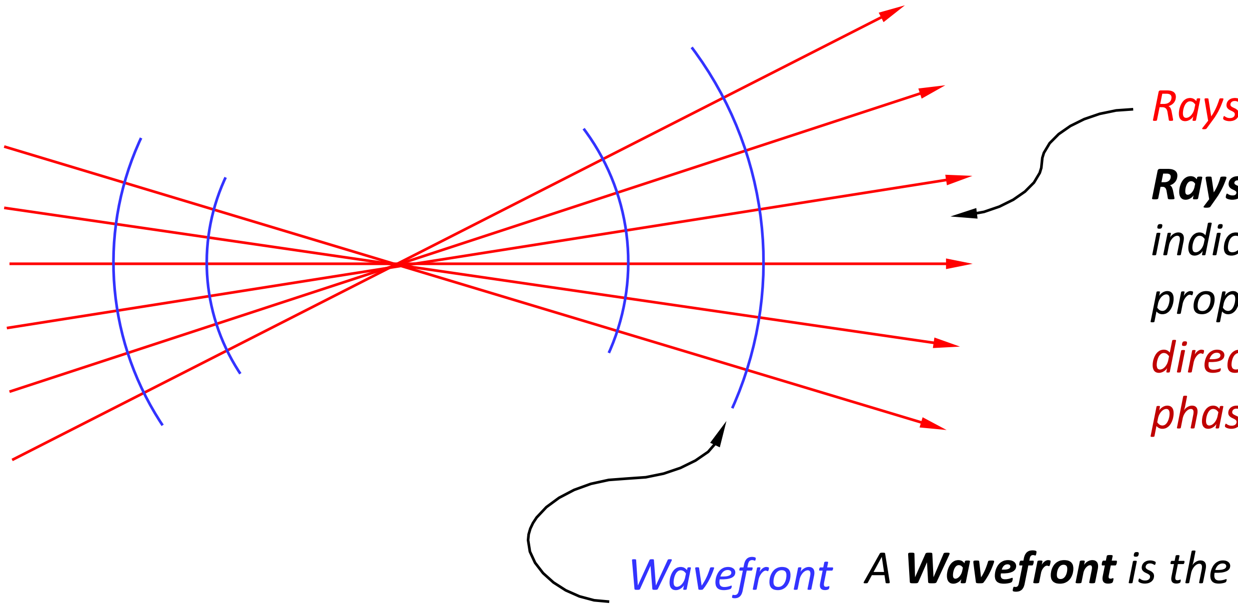


Examples of focusing systems are:
A pinhole, lenses (refraction), mirrors (reflection), Fresnel lenses (diffraction). In (almost) all the cases the focusing element deviates the direction of the incoming rays.

Rays vs Waves

X-Rays are waves (and photons are their quanta), and to be precise one should do ***wavefront*** *propagation.*

Geometrical optics, is an approximation, based on propagation of ***rays****.*

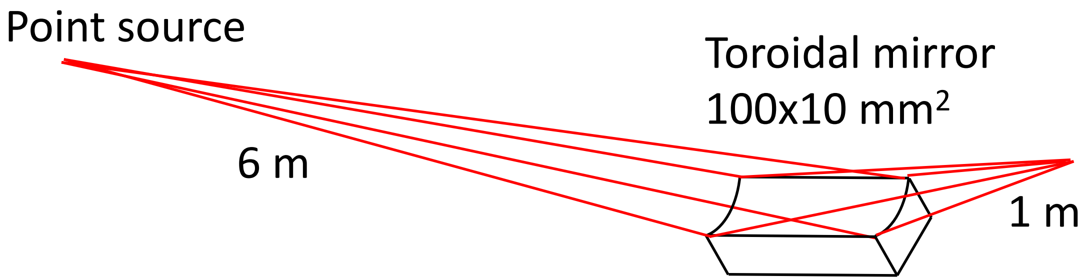


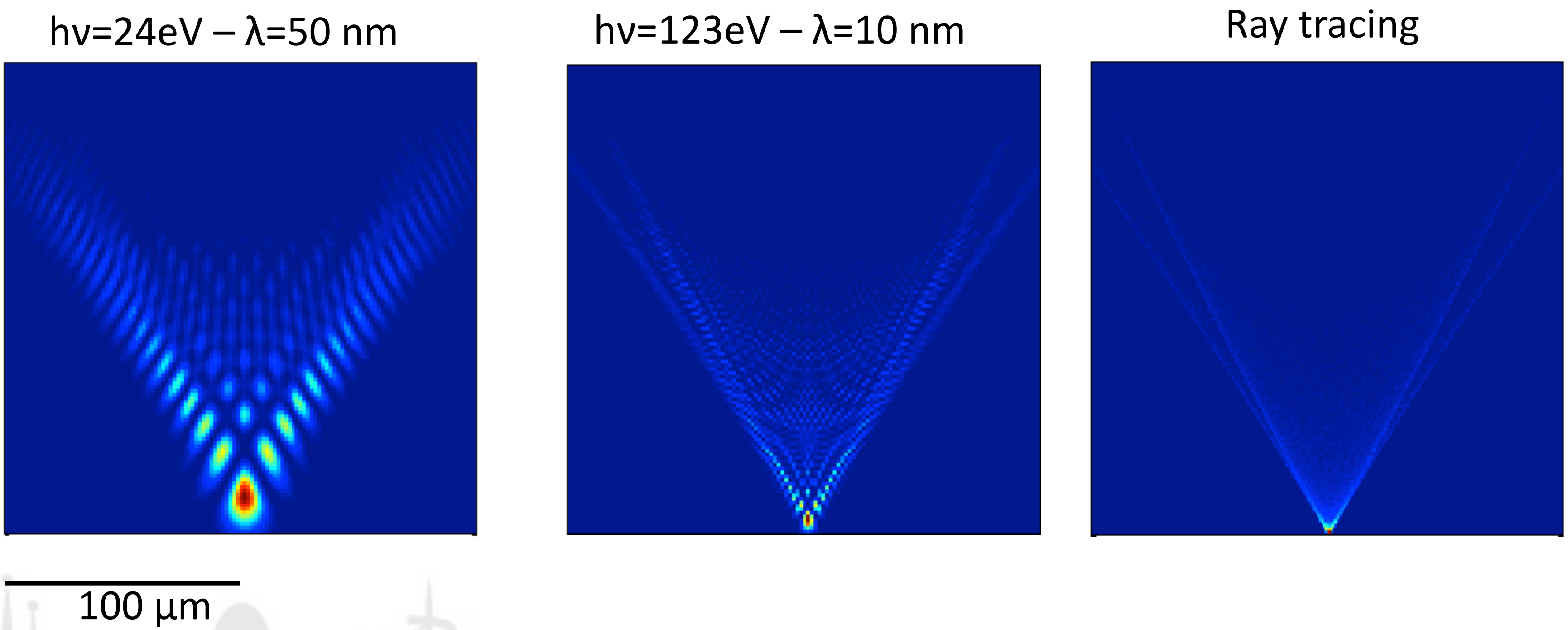
Rays *are the lines that indicate the direction of propagation of energy, direction of variation of phase*

A ***Wavefront*** *is the surface of points at the same* ***optical path distance*** *from the emission point. (for us, at the same distance), i.e. phase is constant across a surface*

"Rays are perpendicular to wavefronts"

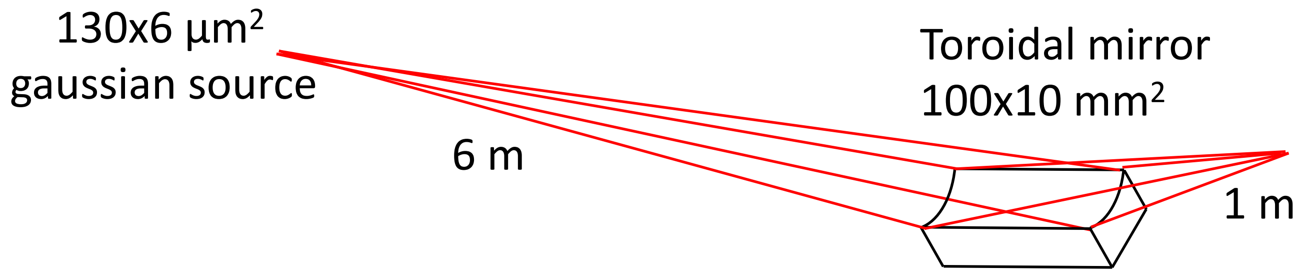
Ray tracing vs wavefront propagation

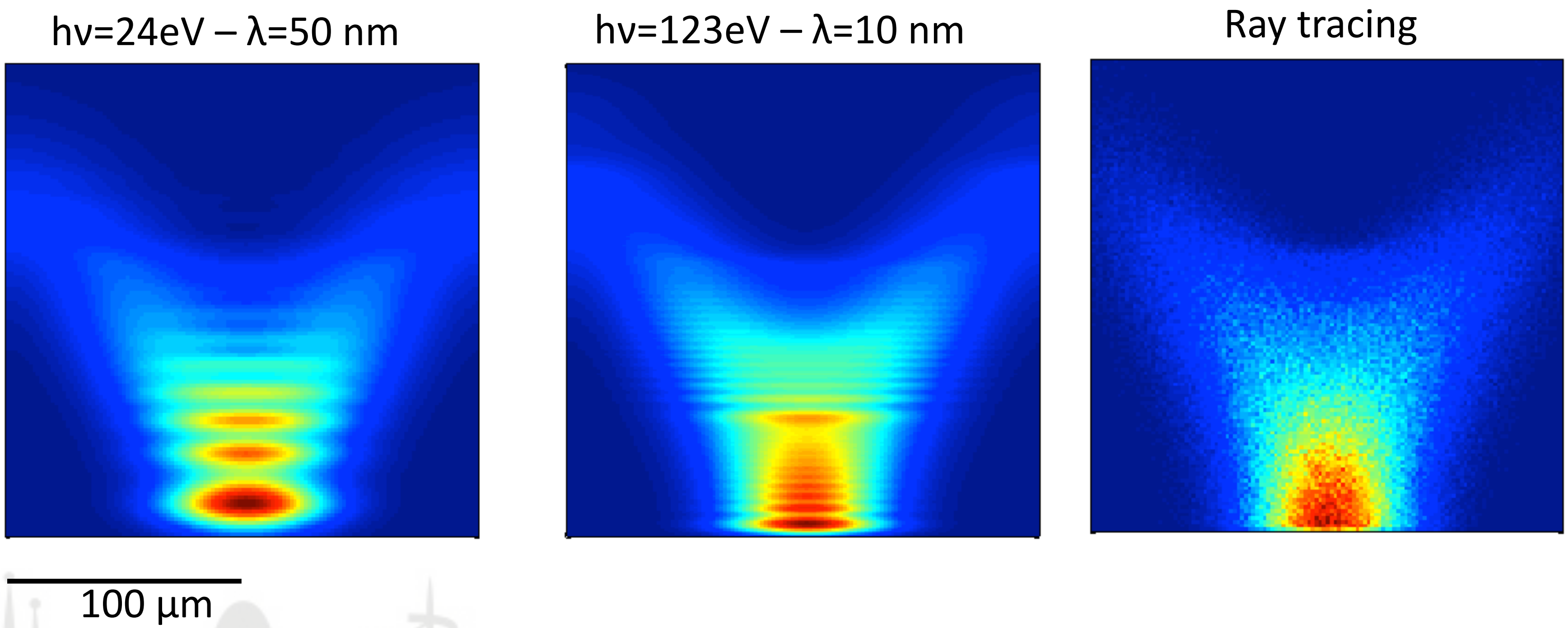




Although raytracing cannot account for diffraction or interference effects, it reproduces accurately the images if energy is high enough (short wavelength)

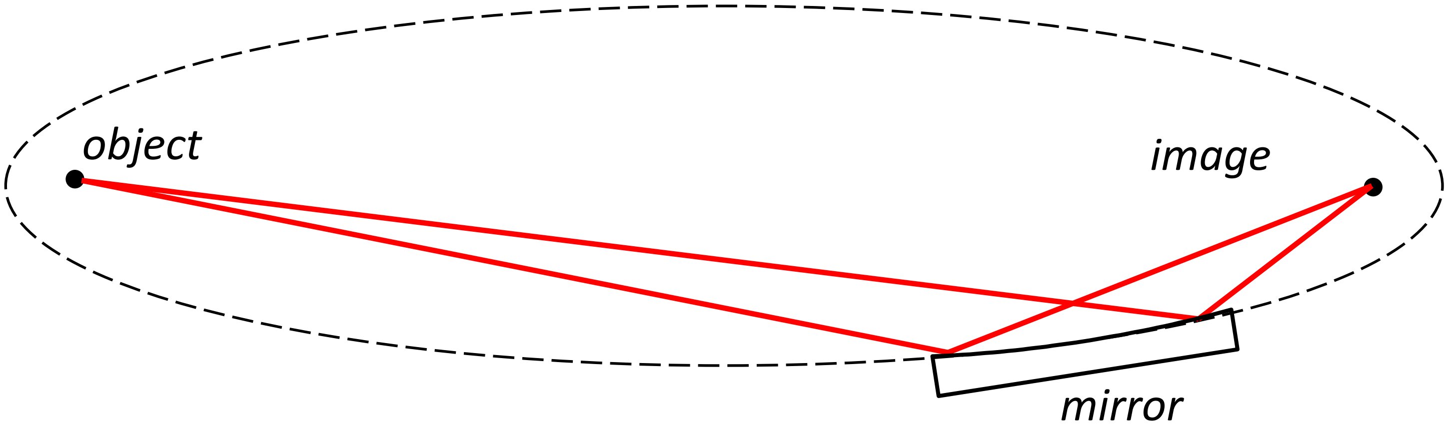
Effect of the source size



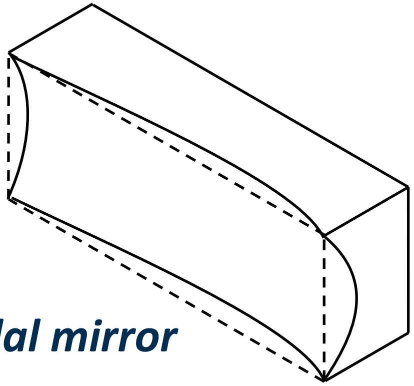


The source size of the electron beam reduces the coherence of the beam, and reduces the contrast of wave effects.

Focusing with mirrors



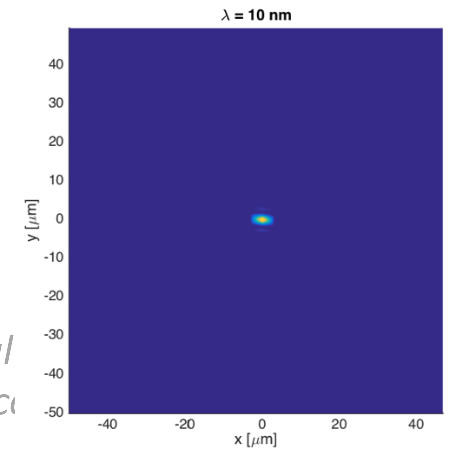
"The sum of the distances to the two foci is constant for all the points of an ellipse"



Ellipsoidal mirror

Stigmatic imaging

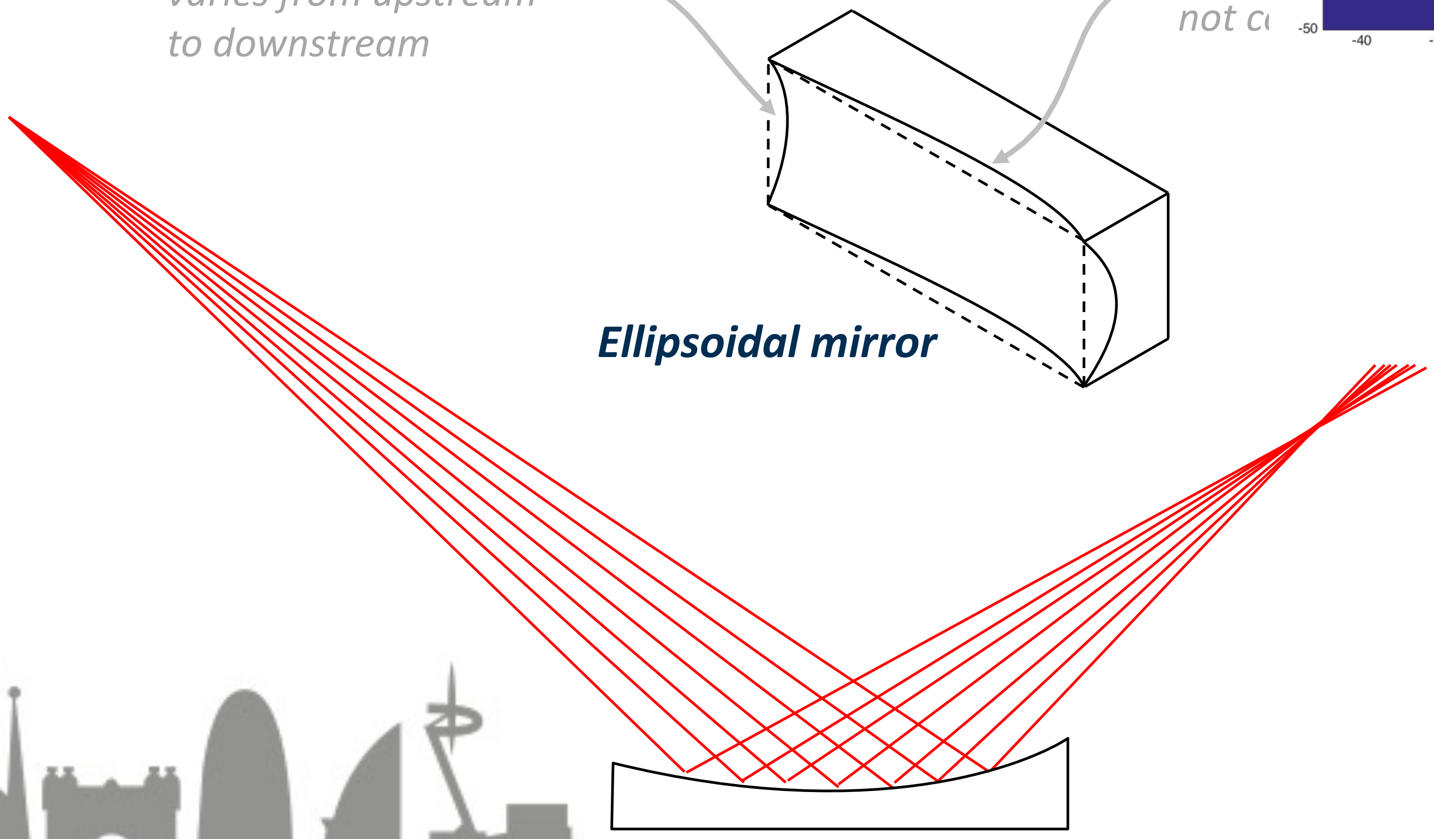
An ellipsoidal mirror focuses point-to-point ***stigmatically (or anastigmatically)***



Sagittal curvature varies from upstream to downstream

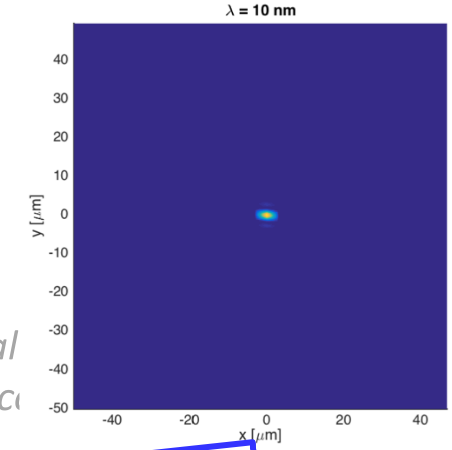
Local not c

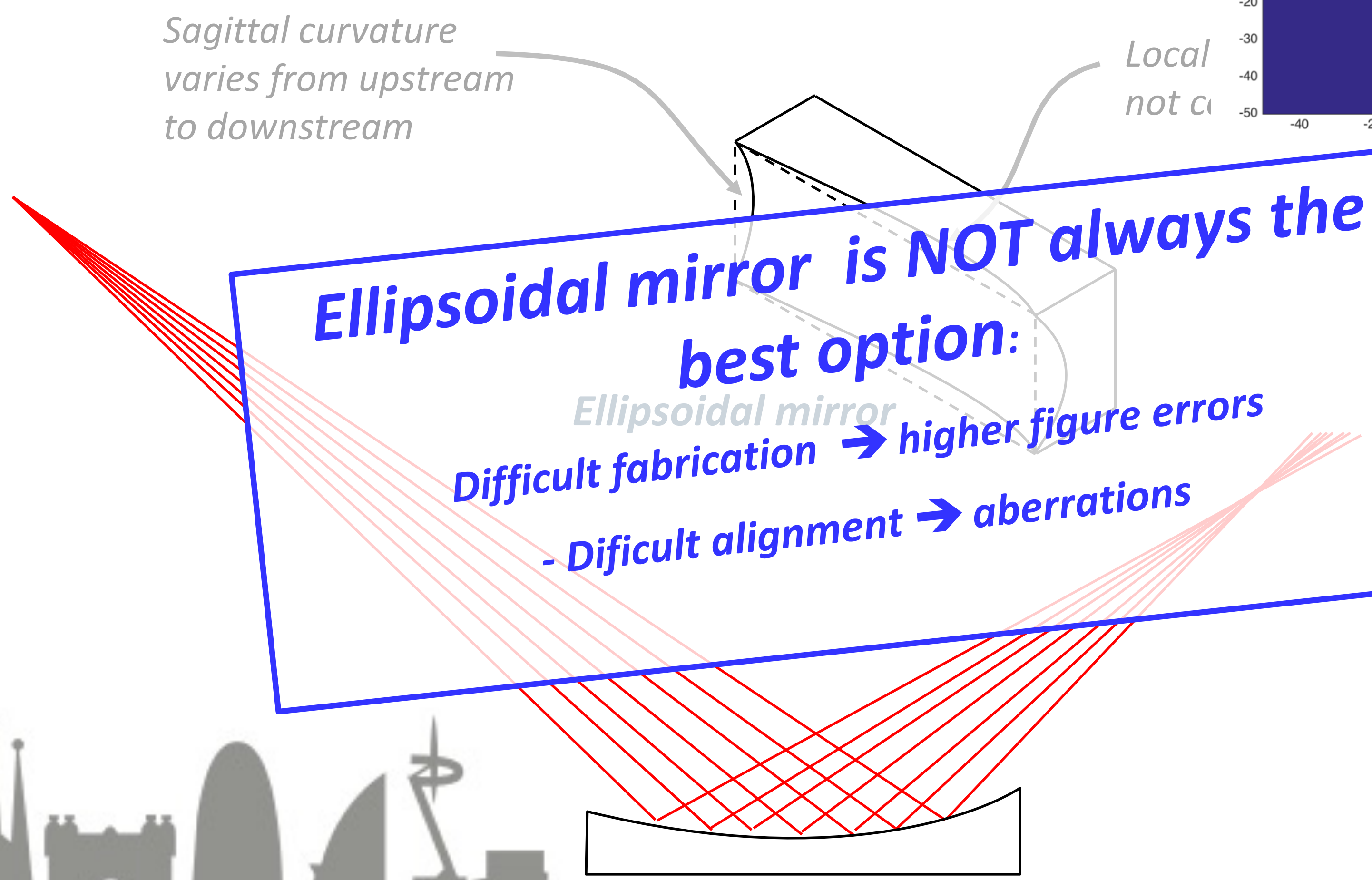
Ellipsoidal mirror



Stigmatic imaging

An ellipsoidal mirror focuses point-to-point ***stigmatically (or anastigmatically)***





Ellipsoidal mirror is NOT always the best option:

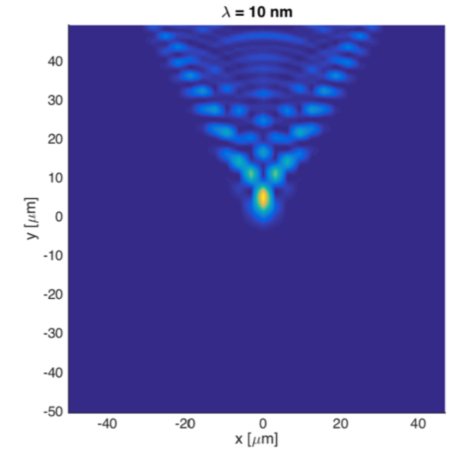
Difficult fabrication ➔ higher figure errors

- Dificult alignment ➔ aberrations

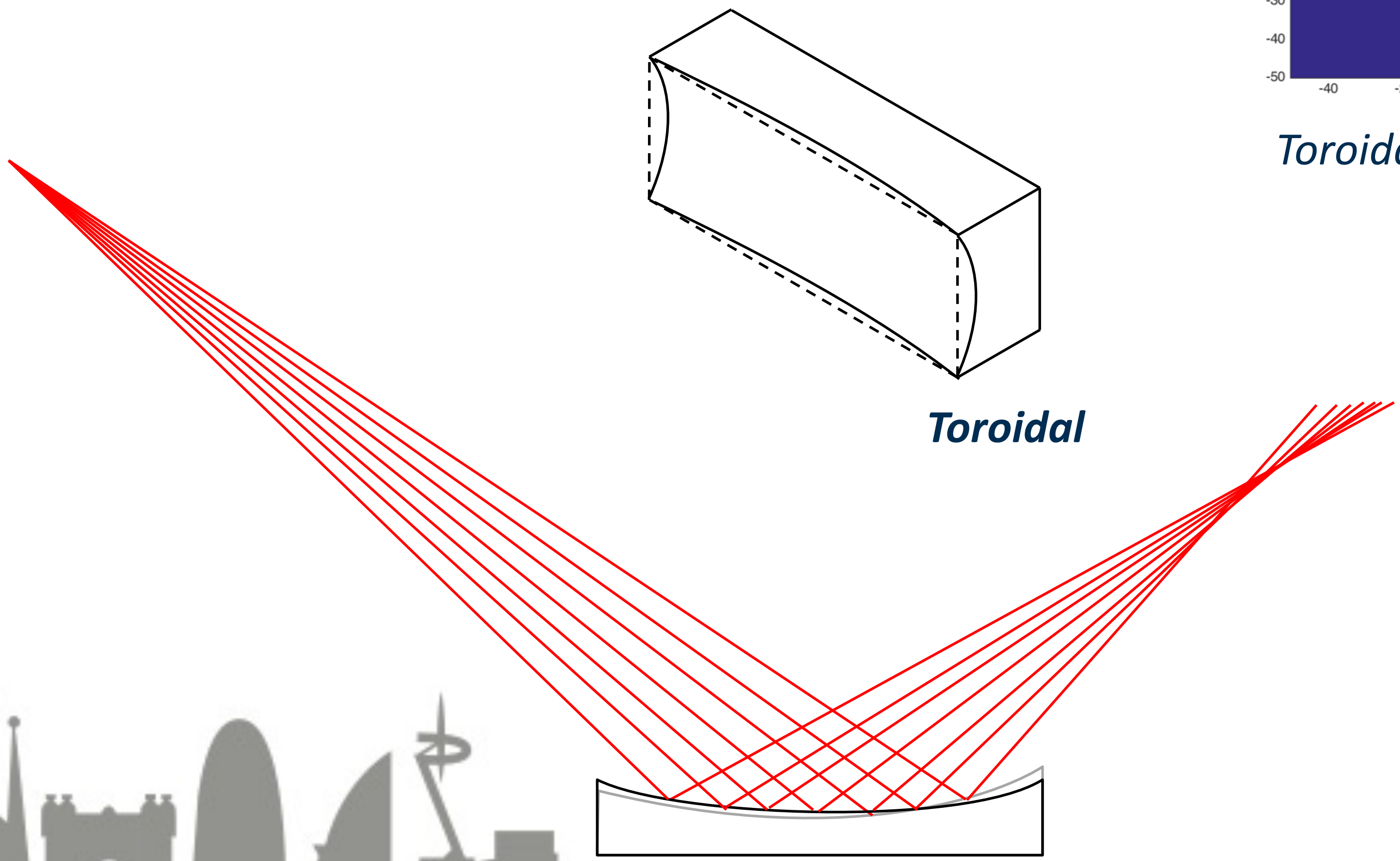
Aberrations

The approximation of the toroidal mirror to the ellipsoid is accurate only in the center of the mirror.

Rays far from it are not properly focused.

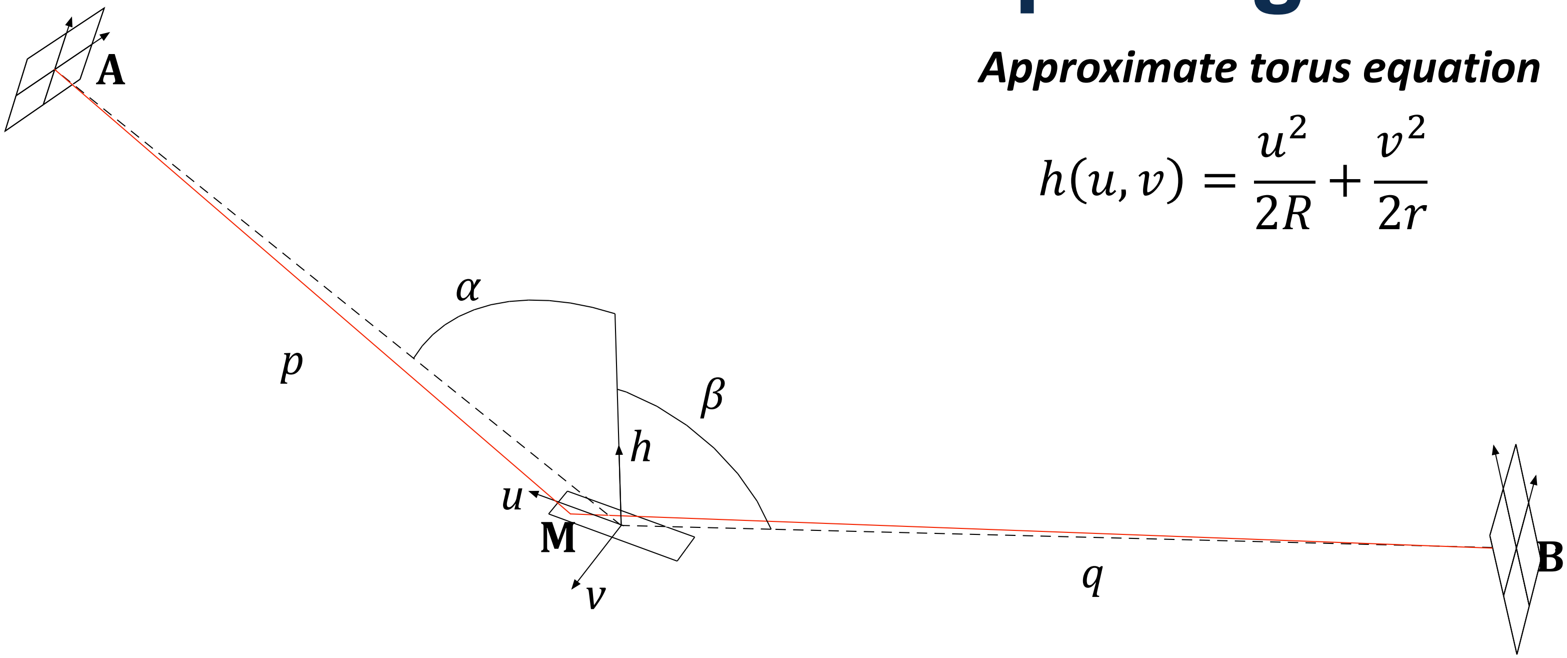


Toroidal aberration



Toroidal

Aberration computing



Approximate torus equation

$$h(u,v) = \frac{u^2}{2R} + \frac{v^2}{2r}$$

Optical path difference

$$OPD(u,v) = \|\mathbf{M} - \mathbf{A}\| + \|\mathbf{B} - \mathbf{M}\|$$

Image condition

We need to compute the optical distance between object and mirror, and between mirror and image.

$$\|\mathbf{A}-\mathbf{M}\| = \sqrt{(A_x - M_x)^2 + \left(A_y - M_y\right)^2 + \left(A_y - M_y\right)^2}$$

It can be expressed as a Taylor series

$$F_{AB}(u, v | p, q, \alpha, \beta, r_i) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{nm}(p, q, \alpha, \beta, r_i) u^n v^m$$

$$F_{nm}(p, q, \alpha, \beta, R_i) = \frac{1}{n!\, m!} \frac{\partial^{n+m} F}{\partial u^n \partial v^m}\bigg|_{(0,0|p,q,\alpha,\beta,r_i)}$$

Main aberrations

- *F_{20} meridional defocusing**

$$F_{20} = \frac{\cos^2\alpha}{2}\left(\frac{1}{p}+\frac{1}{q}-\frac{2}{R\cos\alpha}\right)$$

- *F_{02} sagittal defocusing*

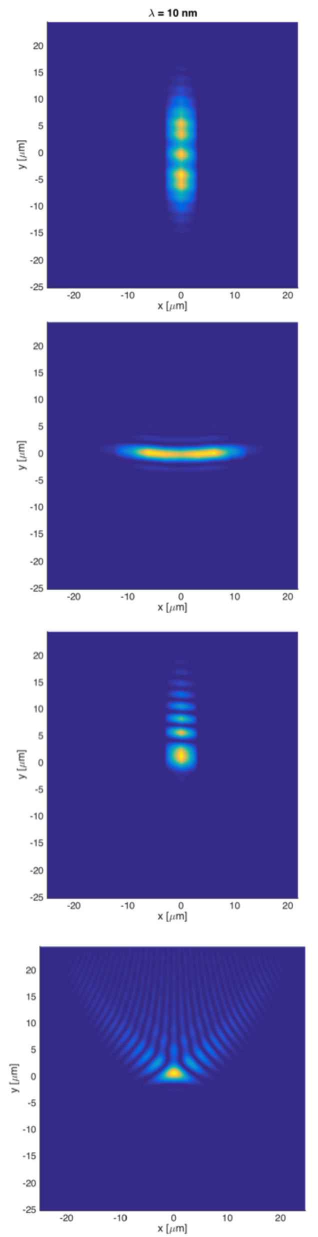
$$F_{02} = \frac{1}{2}\left(\frac{1}{p}+\frac{1}{q}-\frac{2\cos\alpha}{r}\right)$$

- *F_{30} primary coma**

$$F_{30} = \frac{\sin\alpha\cos^2\alpha}{2}\left(\frac{1}{p}-\frac{1}{q}\right)\left(\frac{1}{p}+\frac{1}{q}-\frac{1}{R\cos\alpha}\right)$$

- *F_{12} astigmatic coma*

$$F_{12} = \frac{\sin\alpha}{2}\left(\frac{1}{p}-\frac{1}{q}\right)\left(\frac{1}{p}+\frac{1}{q}-\frac{\cos\alpha}{r}\right)$$



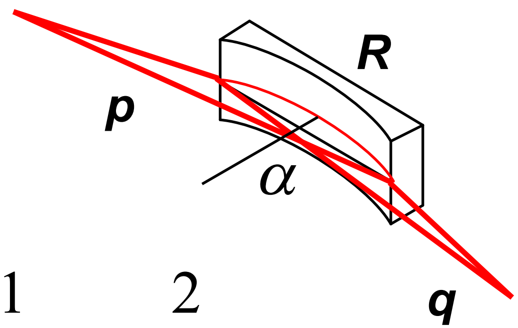
F_{20} F_{30} Will appear again on the tutorial

Varying the incidence angle

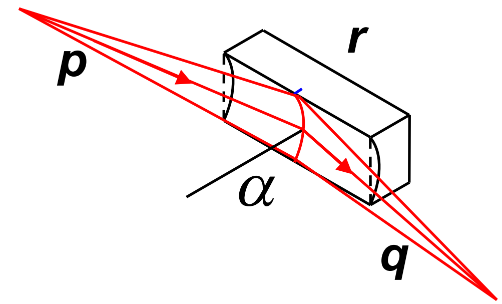
The basic equation for paraxial optics is the focus condition

Meridional focusing

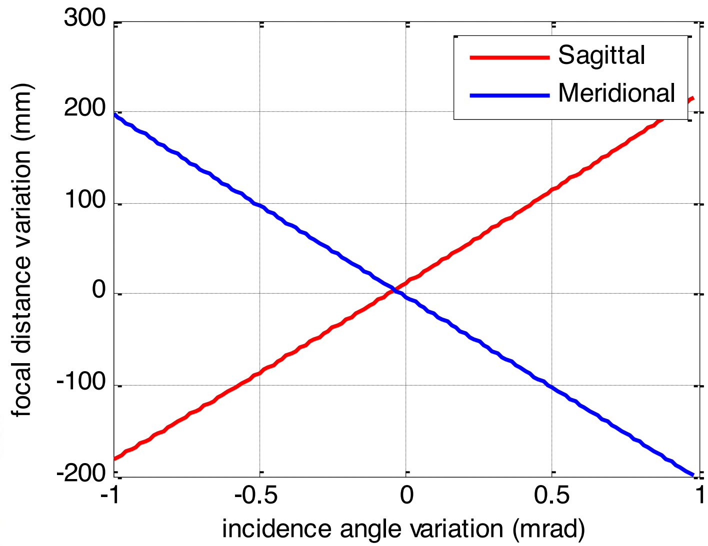
Sagittal focusing



$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R\cos\alpha}$$



$$\frac{1}{p}+\frac{1}{q}=\frac{2\cos\alpha}{r}$$



Varying the incidence angle

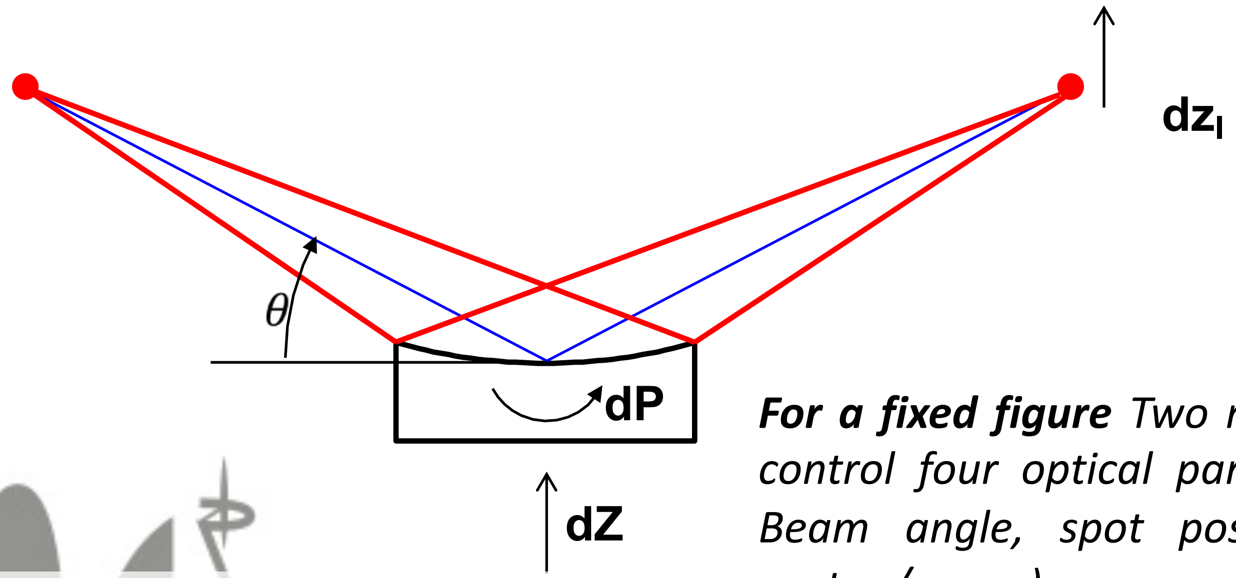
When one changes the pitch of a focusing mirror, one steers the beam, but also changes the incidence angle (therefore the focusing).

$$dz_I = 2qdP + \left(1 - \frac{q}{p}\right)dZ$$

$$d\theta = +dP - \frac{dZ}{p}$$

$$dP = \frac{1}{p+q}dz_I + \frac{p-q}{p+q}d\theta$$

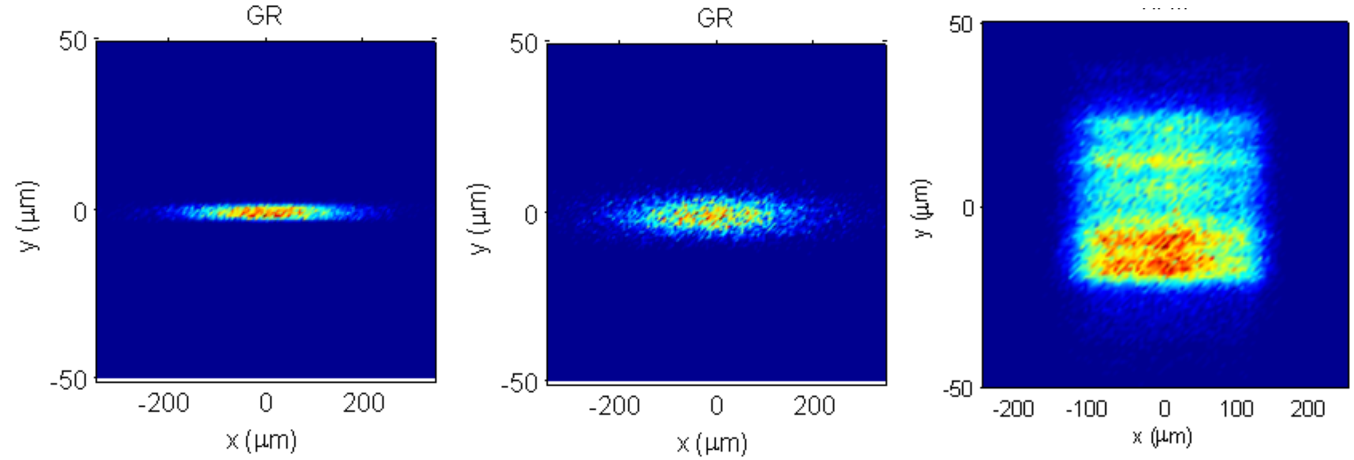
$$dZ = \frac{p}{p+q}dz_I - \frac{2pq}{p+q}d\theta$$



For a fixed figure *Two motors (pitch, z) control four optical parameters (focus, Beam angle, spot position, footprint center / coma)*

Special cases. 1:1 magnifiction. Flat mirrors

Figure error: slope



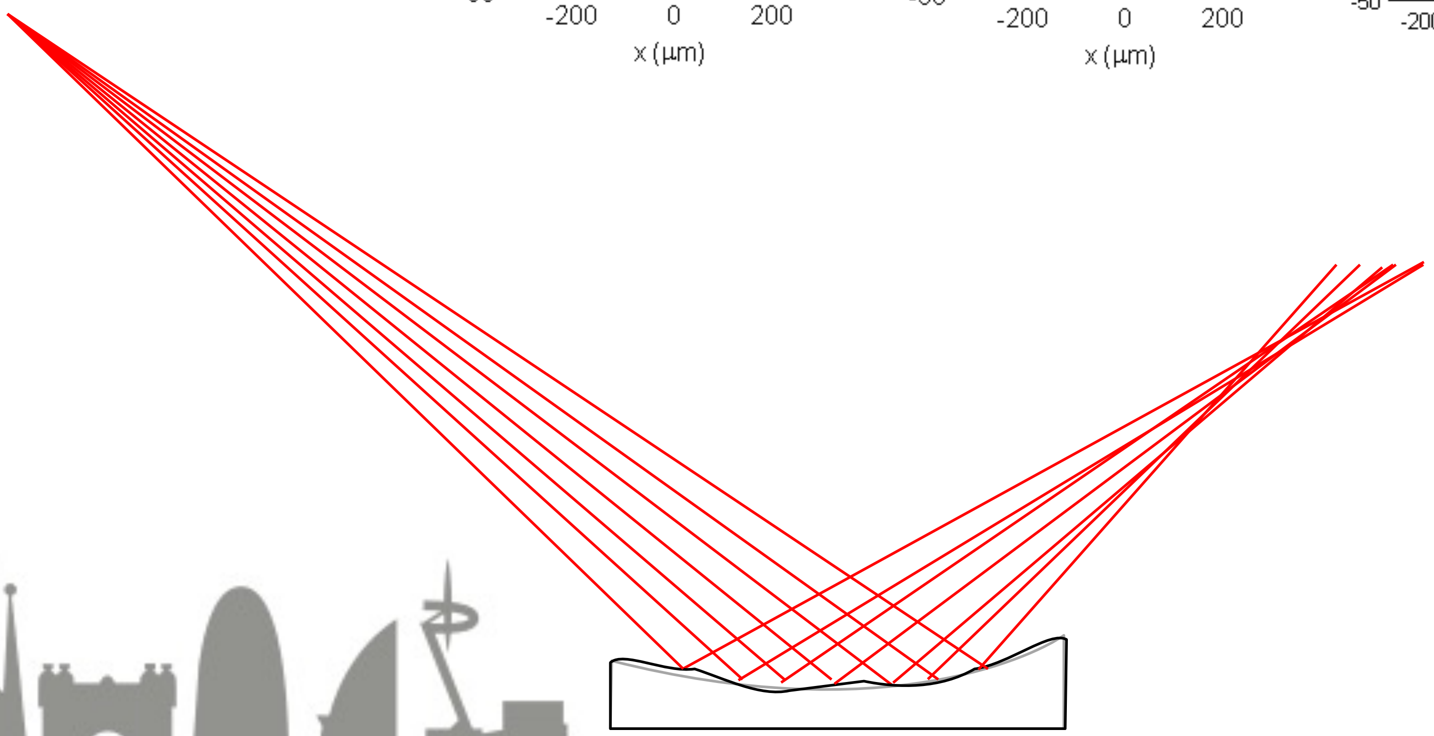


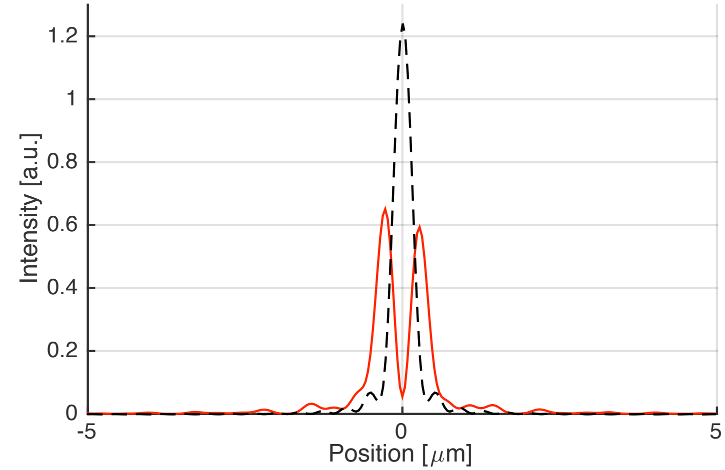
Figure error for coherent sources (a case)

*For diffraction limited imaging (**FELs**) , one needs to control the **wavefront distortion**, which is given by the residual **height error.***

Just consider the case in which the mirror is perfect almost everywhere except for a step in the center.

-slope error is very small,

*- There is **no light on axis** due to destructive interference between the two sides of the wavefront*



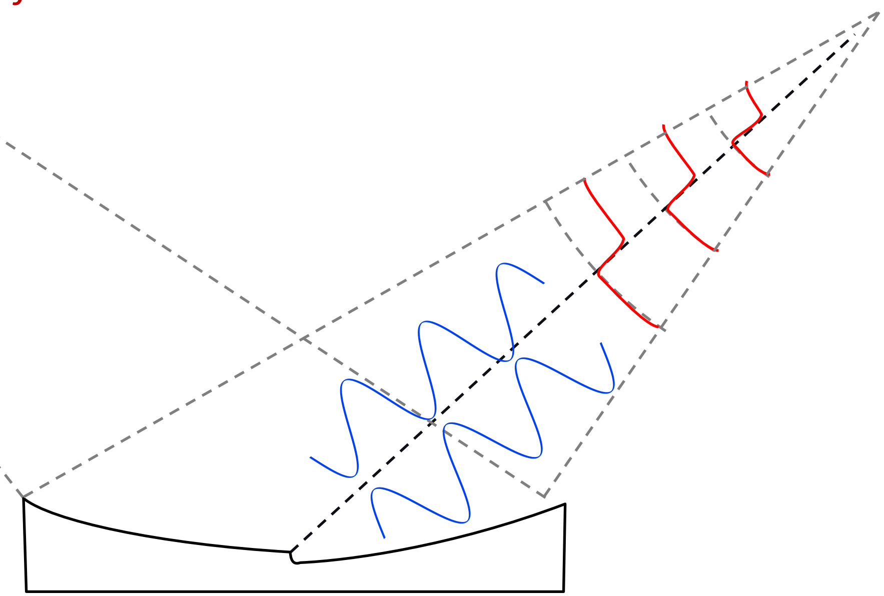
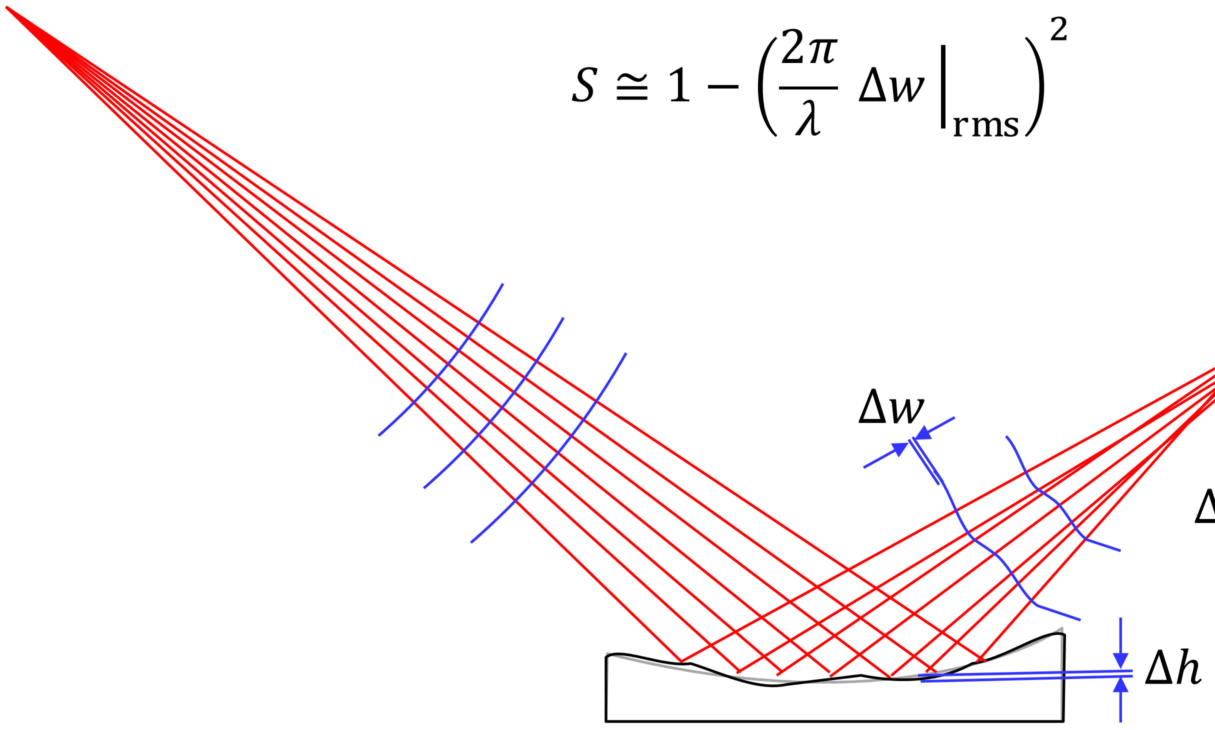


Figure error: height error

Interference effects in image formation are given by the Strehl ratio (actual peak intensity to the error free peak intensity)

$$S \cong 1 - \left(\frac{2\pi}{\lambda}\,\Delta w\,\Big|_{\mathrm{rms}}\right)^2$$

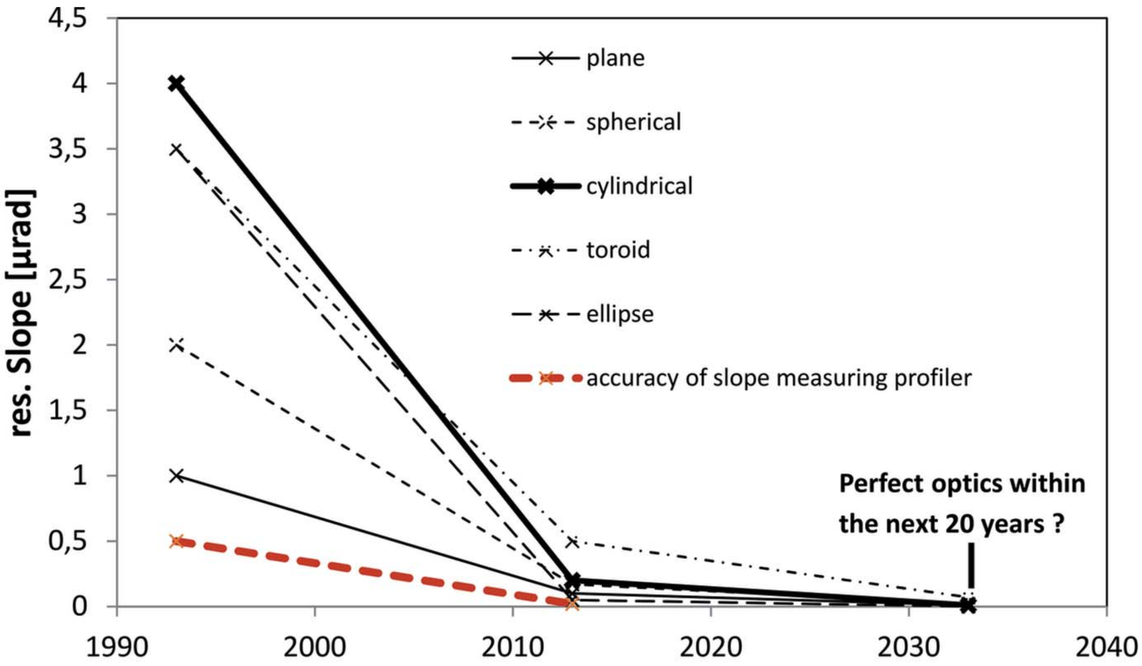


$$\Delta w = 2\Delta h \cos\alpha$$

$$\Delta h\,\Big|_{\mathrm{rms}} \leq \lambda\,\frac{\sqrt{1-S}}{4\pi\,\cos\alpha}$$

Example
LCLS II – S = 0.97 - 2.1 mrad -13 keV ➔ 0.4 nm rms

Some reference values of figure error



Siewert et al. JSR 21,968-975 (2014)

Quickly evolved in the recent past:

- *Development of mirror metrology*
- *Deterministic polishing techniques:*

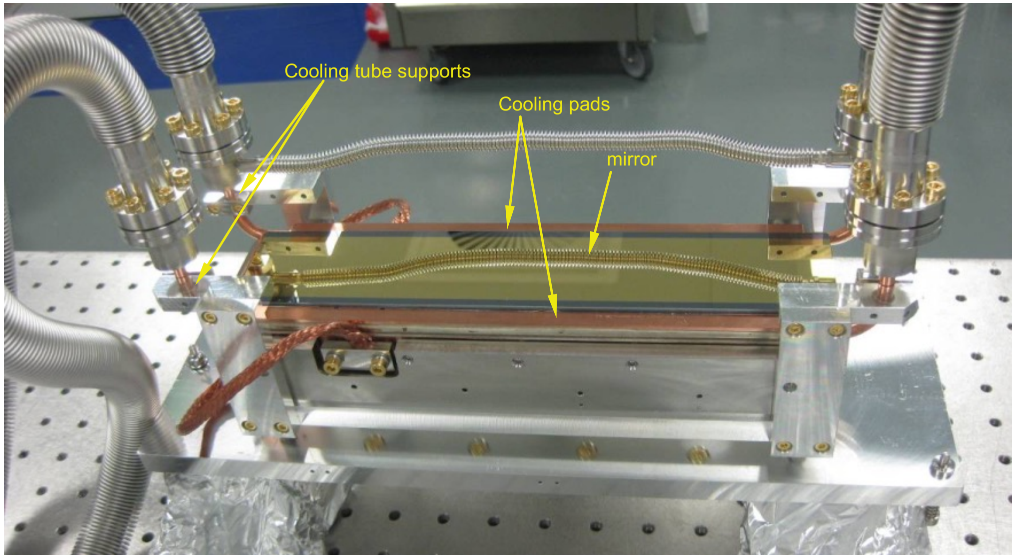
Most (usual) companies can deliver mirrors better than 0.5 µrad rms, on whatever figure and length.

For flats error is sub-nanometer, and slope error below 100 nrad, cost and delivery time.

Sagittal curvature always difficult

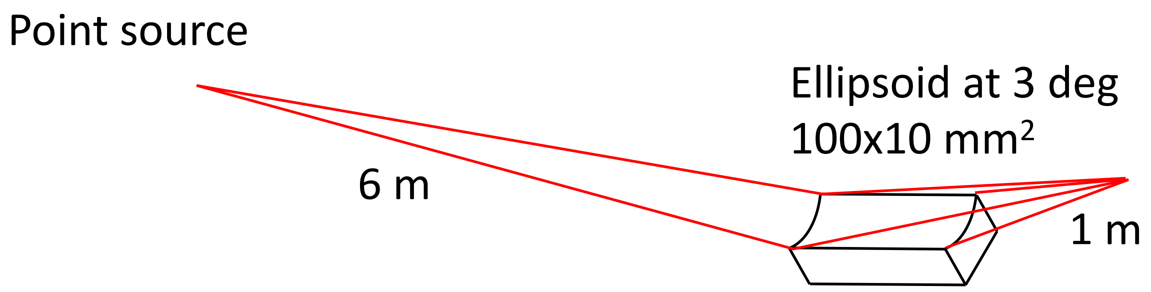
Causes of figure error

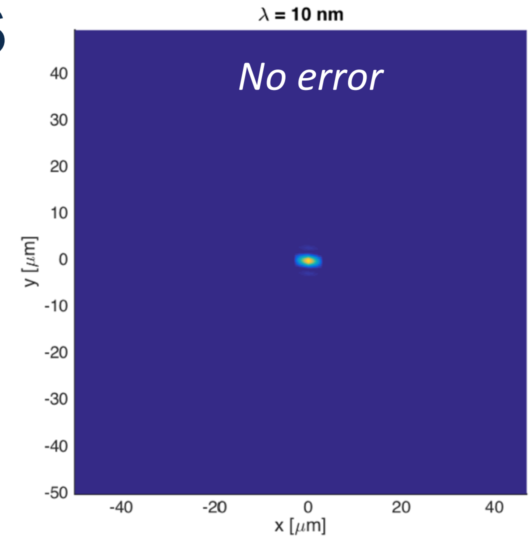
- ***Fabrication error (figuring, polishing metrology) : well below 0.5 μrad rms***
- ***Gravity sag.***
- ***Clamping mechanics.***
- ***Stress induced by cooling scheme.***
- ***Thermal deformations.***
- ***Bender errors, ellipse approximatino errors***
- ***Coating stress! Bakeout developed stresses (?)***

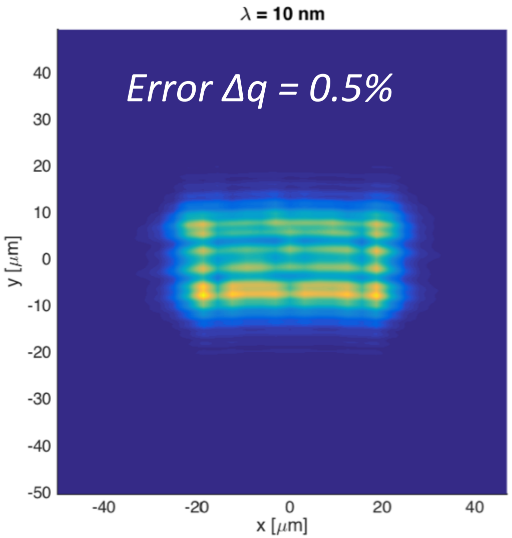


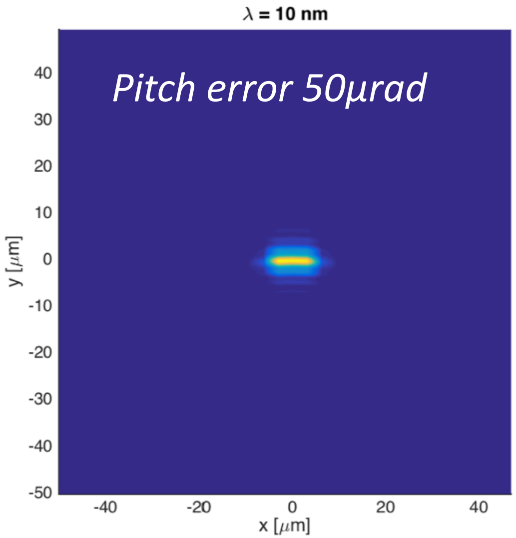
Mirror alone: R = 331.039 m / 0.163 μrad RMS
In holder: R = 338.303 m / 0.533 μrad RMS
(First iteration)

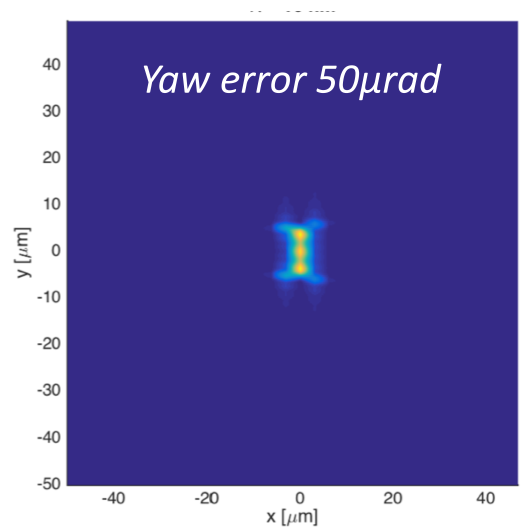
Misalignments







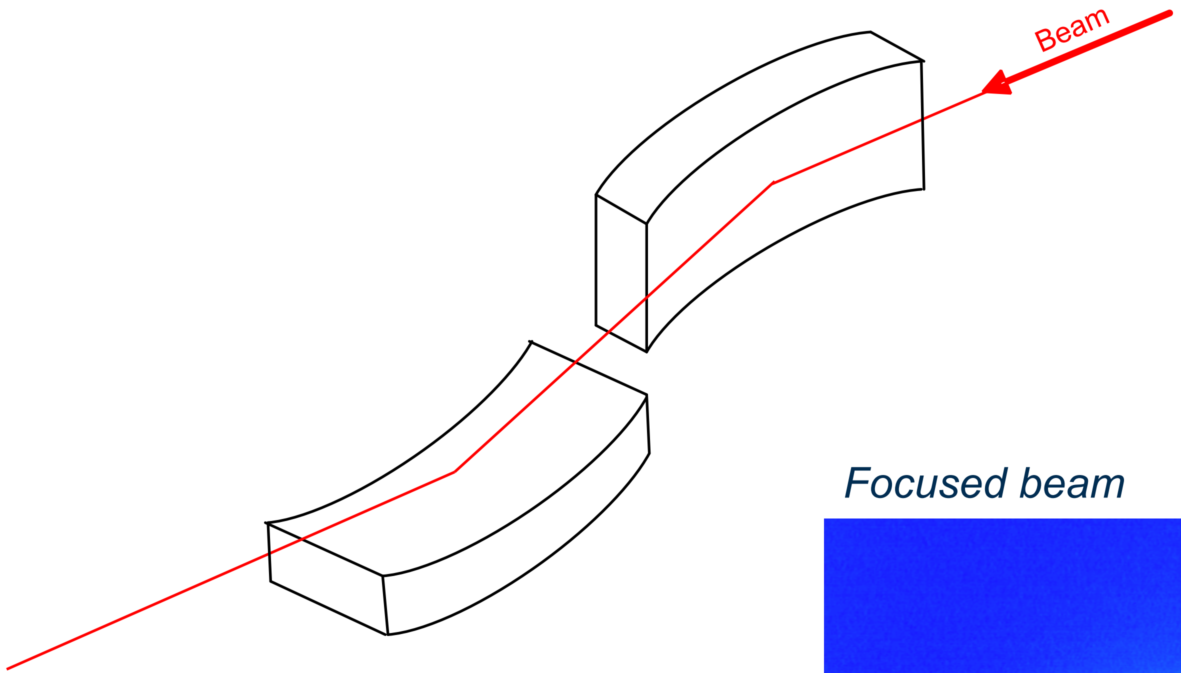




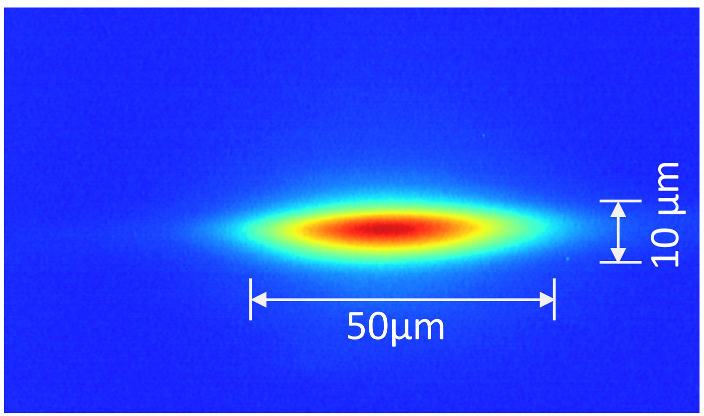
FOV is 100 μm

Kirkpatrick-Baez configuration

By decoupling vertical and horizontal focusing, one can obtain aberration free focusing by using high quality mirrors ***polished flat****, with low slope error, and* ***mechanically bent*** *onto plano-elliptic figure.*

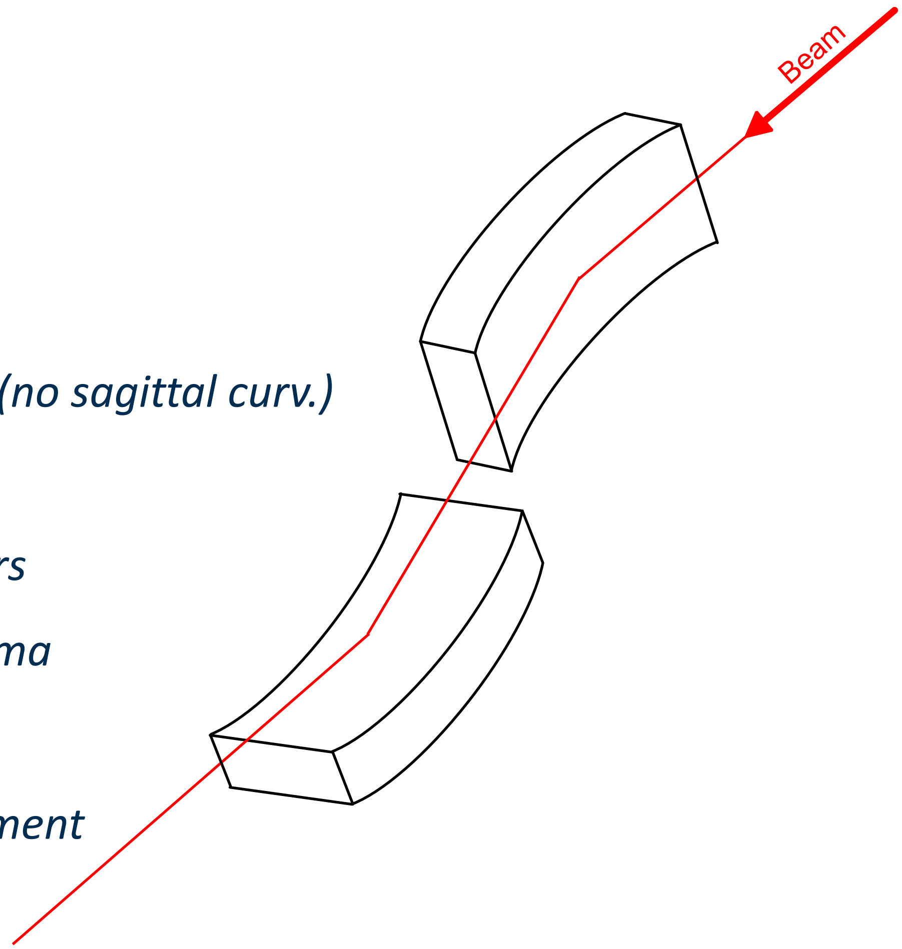


Focused beam



Some KB advantages

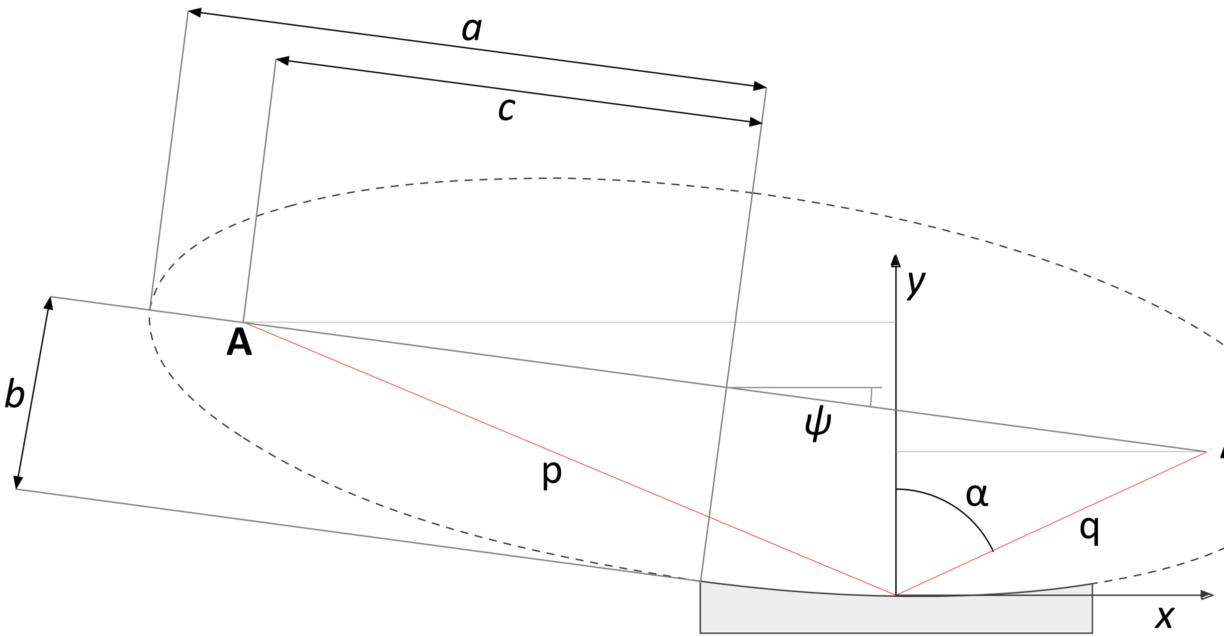
- ***Decouple horizontal and vertical planes***
  - *Astigmatic sources or images*
- ***No sagittal curvature***
  - *Allow different stripes*
  - *More accurate metrology*
  - *Relaxes alignment requirements (no sagittal curv.)*
- ***If bender***
  - *Start from flat or spherical mirrors*
  - *Adaptive focus and primary comma*
    - *Several foci*
    - *Focus independent of alignment*



2. Characteristics of elliptic mirrors

- *Geometry of the ellipse.*
- *Height, slope, curvature.*
- *Polynomial approximation.*

Description of an elliptic mirror



Major semi-axis

$$a = \frac{p+q}{2}$$

Half distance between foci (linear eccentricity)

$$c = \frac{1}{2}\sqrt{p^2 + q^2 - 2pq\cos 2\alpha}$$

Minor semi-axis

$$b = \sqrt{pq}\cos\alpha$$

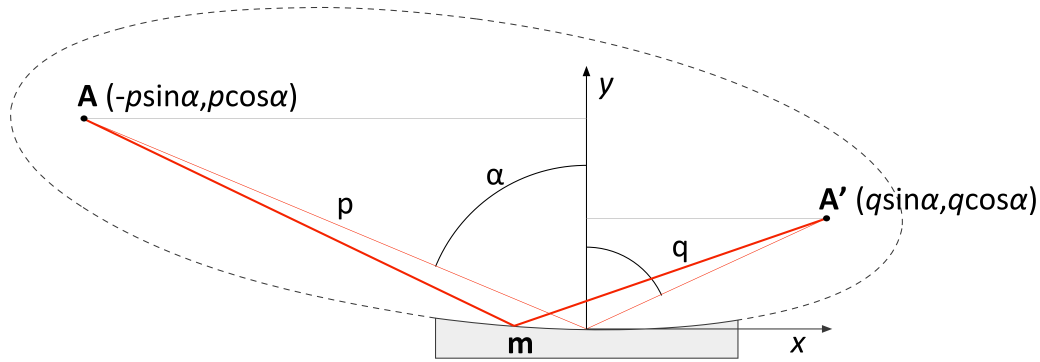
eccentricity

$$e = \sqrt{1 - \left(\frac{2}{p+q}\right) pq\cos^2\alpha}$$

Orientation

$$\tan\psi = \frac{q-p}{q+p}\cot\alpha$$

Equations of the ellipse



Vector equation of the ellipse

$$\|\mathbf{r}_{\mathrm{A}} - \mathbf{r}_m\| + \|\mathbf{r}_{\mathrm{A}'} - \mathbf{r}_m\| = p + q$$

Implicit equation of the ellipse (as a function of coordinates)

$$\sqrt{(x + p\ \sin\alpha)^2 + (y - p\ \cos\alpha)^2} + \sqrt{(x - q\ \sin\alpha)^2 + (y - q\ \cos\alpha)^2} = p + q$$

Explicit equations of the ellipse

Concave branch of the explicit solution of the ellipse

$$y(x) = -\frac{x(p^2-q^2)\sin 2\alpha - 4(p+q)\cos\alpha\left(pq - \sqrt{pq}\sqrt{pq - x(p-q)\sin\alpha - x^2}\right)}{p^2+6pq+q^2-(p-q)^2\cos 2\alpha}$$

- *Note that y(0)=0*
- *This is an **exact** solution!*

Slope function of the ellipse

$$\frac{dy}{dx}(x) = -\frac{2(p+q)\cos\alpha\left((p-q)\sin\alpha - \frac{\sqrt{pq}\left((p-q)\sin\alpha + 2x\right)}{\sqrt{pq - x(p-q)\sin\alpha - x^2}}\right)}{p^2+6pq+q^2-(p-q)^2\cos 2\alpha}$$

- *Note that y'(0)=0*
- *This is an **exact** solution!*

Curvature and imaging condition

Curvature of the ellipse

$$\frac{1}{R(x)} = \frac{d^2y}{dx^2}(x) = \frac{\sqrt{pq}(p+q)\cos\alpha}{2[pq - x(p-q)\sin\alpha - x^2]^{3/2}}$$

- *Note that it is not constant*

Curvature at the center of the ellipse

$$\frac{1}{R_0} = \frac{(p+q)\cos\alpha}{2pq}$$

$$\frac{2}{R_0\cos\alpha} = \frac{1}{p} + \frac{1}{q}$$

- *It is the focusing condition (mer)*

Taylor expansion of the ellipse

Definition

$$y(x) = \sum_{n=0}^{\infty} E_n x^n \qquad \textit{with} \qquad E_n = \frac{1}{n!}\frac{d^n y}{dx^n}\bigg|_{x=0}$$

In the case of an ellipse, the coefficients are functions of p,q and α

$$E_2 = \frac{\cos\alpha}{4}\left(\frac{1}{p}+\frac{1}{q}\right) = \frac{1}{2R_0} \qquad E_3 = \frac{\sin 2\alpha}{16}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)$$

$$E_4 = \frac{\mathrm{E}_3^2}{4\mathrm{E}_2}(5-\csc^2\alpha) + \mathrm{E}_2^3\sec^2\alpha \qquad E_5 = \frac{\mathrm{E}_3^3(7-3\csc^2\alpha)}{4\mathrm{E}_2^2} + 3\mathrm{E}_2^2\mathrm{E}_3\sec^2\alpha$$

$$E_6 = \frac{\mathrm{E}_3^4(21-14\csc^2\alpha+\csc^4\alpha)}{8\mathrm{E}_2^3} + \mathrm{E}_2\mathrm{E}_3^2(7-\csc^2\alpha)\sec^2\alpha + 2\mathrm{E}_2^5\sec^4\alpha$$

Aberrations, revisited

Defocus is given by the quadratic term of the ellipse polynomial

$$F_{20} = \frac{\cos^2\alpha}{2}\left(\frac{1}{p} + \frac{1}{q} - \frac{2}{R\cos\alpha}\right) \Longleftrightarrow F_{20} = 2\cos\alpha\left[\frac{\cos\alpha}{4}\left(\frac{1}{p} + \frac{1}{q}\right) - \frac{1}{2R}\right]$$

Remember: $E_2 = \frac{\cos\alpha}{4}\left(\frac{1}{p} + \frac{1}{q}\right)$

$$F_{20} = 2\cos\alpha\,\Delta E_2$$

Primary coma is given by the cubic term of the ellipse polynomial

If $F_{20}=0$

$$F_{30} = \frac{\sin\alpha\cos^2\alpha}{2}\left(\frac{1}{p} - \frac{1}{q}\right)\left(\frac{1}{p} + \frac{1}{q} - \frac{1}{R\cos\alpha}\right) \Longleftrightarrow F_{30} = -2\cos\alpha\frac{\sin 2\alpha}{16}\left(\frac{1}{q^2} - \frac{1}{p^2}\right)$$

Remember: $E_3 = \frac{\sin 2\alpha}{16}\left(\frac{1}{q^2} - \frac{1}{p^2}\right)$

$$F_{30} = -2\cos\alpha\,\Delta E_3$$

3. Mirror bender calculations and metrology

- *Euler-Bernoulli equation*
- *Rectangular footprint case*
- *Custom footprint case*

Elastic beam theory

Euler-Bernoulli equation

$$E \cdot I(x) \frac{d^2}{dx^2} z(x) = M(x)$$

Bending moments

$$M(x) = \sum_{i|x_i<x} (x - x_i) F_i$$

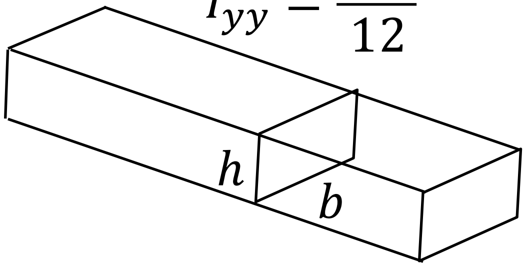
- *z(x) is the deformation induced by the load. [m]*
- *M(x) is the distribution of bending moments along the mirror*
- *E is the Young's modulus of the body.[N/m^2]*
- *I(x) is the second moment of inertia of the mirror substrate section. [m^4]*

Second moment of inertia

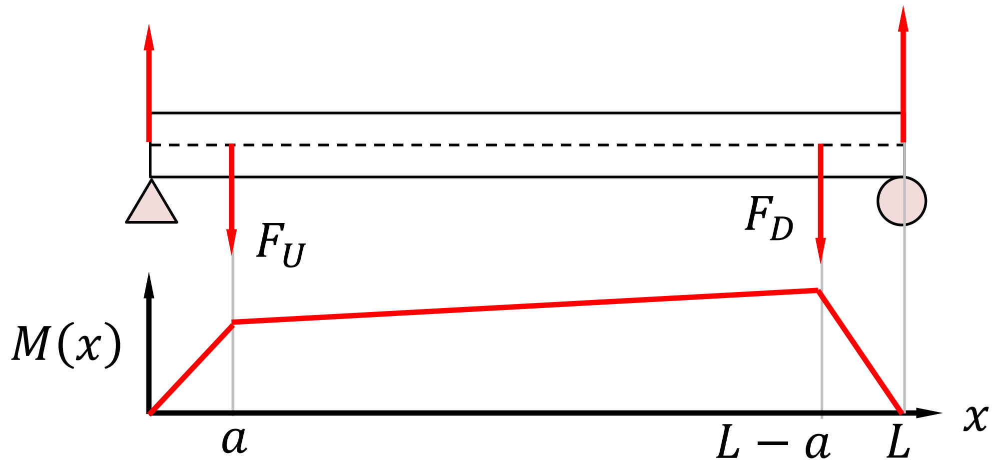
$$I(x) = \iint_{Section} z^2 dx dz$$

For a rectangular section

$$I_{yy} = \frac{bh^3}{12}$$



Integration (rectangular mirror footprint)



General Solution to Euler-Bernoulli' equation

$$z(x) = c_o + c_1 x + \int_{-L/2}^{x} \int_{-L/2}^{x"} \frac{M(x')}{E\,I(x')}\, dx'\, dx"$$

For constant section mirror the solution is a third degree polynomial

$$z(x) = c_o + c_1 x + E_2 x^2 + E_3 x^3$$

$$E_2 = \frac{3a}{Ebh^3}(F_U + F_D)$$

$$E_3 = \frac{2a}{Ebh^3 L}(F_D - F_U)$$

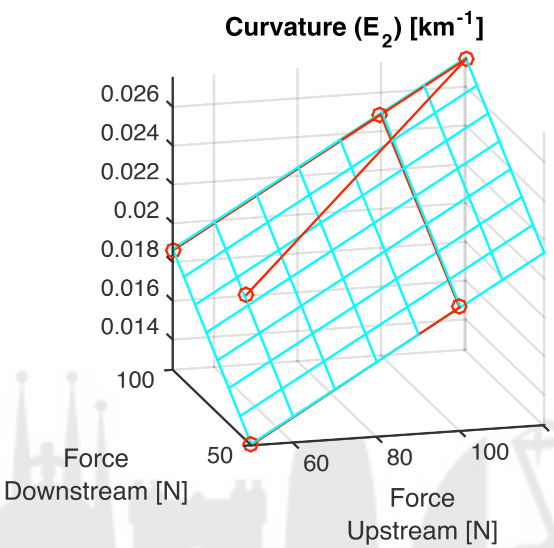
Rectangular mirror footprint

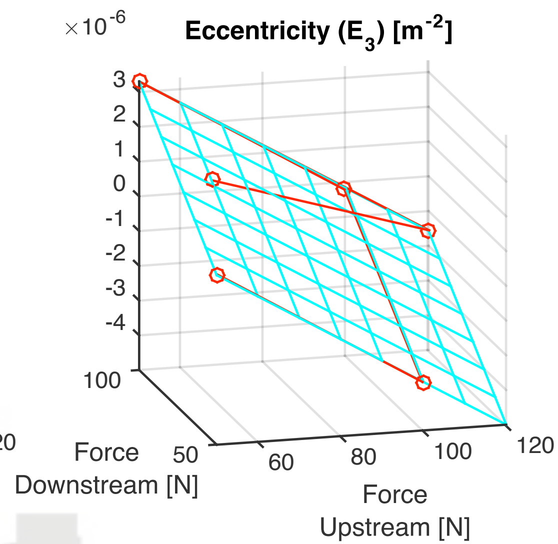
$$E_2 = \frac{3a}{Ebh^3}(F_U + F_D)$$

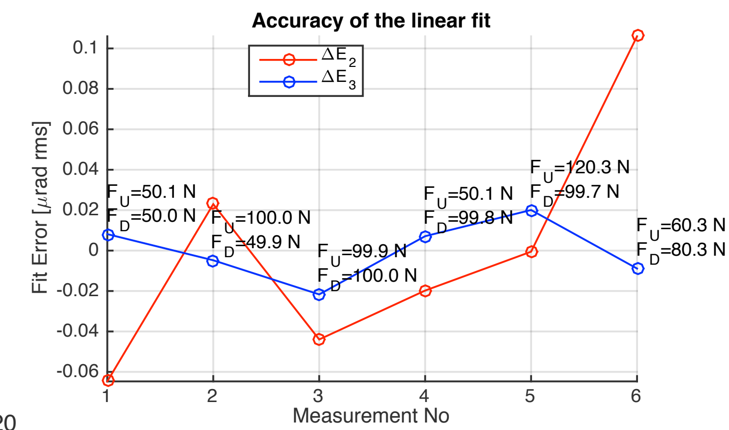
$$E_3 = \frac{2a}{Ebh^3L}(F_D - F_U)$$

- *The sum of forces **controls** the main curvature (and the defocusing)*
- *The difference between the forces **controls** the eccentricity (and primary coma)*

$$\begin{pmatrix} E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} E_{2,INI} \\ E_{3,\mathrm{INI}} \end{pmatrix} + \begin{pmatrix} E_{2U} & E_{2D} \\ E_{3U} & E_{3D} \end{pmatrix} \begin{pmatrix} F_U \\ F_D \end{pmatrix}$$







fit better than about 50 nrad rms (metrology limited – too quick)

High demagnification ellipses

The error of the cubic polynomial approximation is given by:

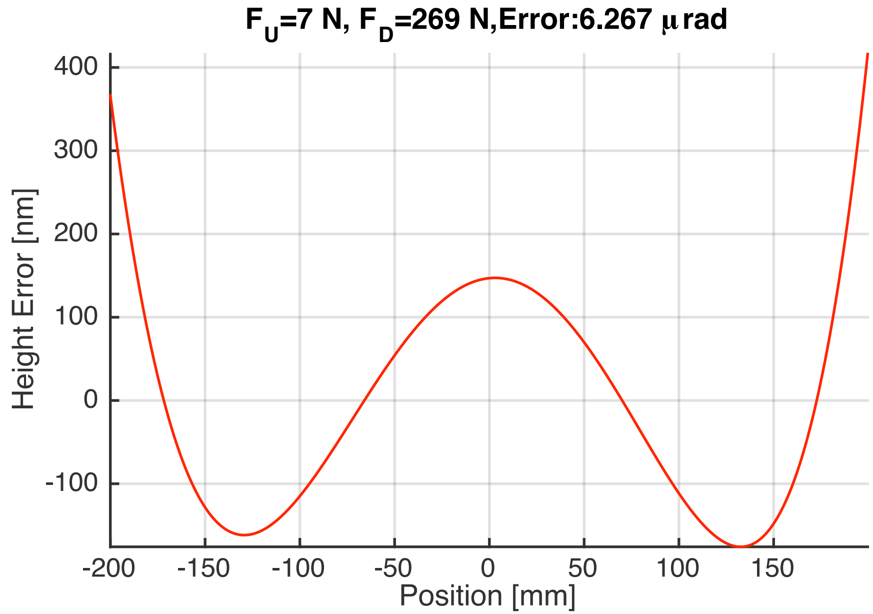
$$y_{ERROR}(x) = y_{EXACT}(x) - E_2x^2 - E_3x^3$$

$$y_{ERROR}(x) =$$

$$-\frac{x(p^2-q^2)\sin 2\alpha - 4(p+q)\cos\alpha\left(pq-\sqrt{pq}\sqrt{pq-x(p-q)\sin\alpha - x^2}\right)}{p^2+6pq+q^2-(p-q)^2\cos 2\alpha} - \frac{\cos\alpha}{4}\left(\frac{1}{p}+\frac{1}{q}\right)x^2 - \frac{\sin 2\alpha}{16}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)x^3$$

For high demagnification ellipses, higher order aberrations are significant

For ALBA's LOREA beamline HFM 10:1, 400 mm long, this error is in the order of 6.3 urad rms



Varied width mirrors

Deformation equation includes curvature:

$$\frac{Eb(x)h^3}{12}\frac{d^2y}{dx^2}(x) = M_0 + \Delta M\frac{2x}{L}$$

$$M_0 = \frac{1}{2}(a_1F_1 + a_2F_2)$$

$$\Delta M = \frac{1}{2}(a_1F_1 - a_2F_2)$$

By inserting the ***exact*** *curvature profile, we have the definition of the required mirror width.*

$$\frac{Eb(x)h^3}{12}\frac{\sqrt{pq}(p+q)\cos\alpha}{2[pq - x(p-q)\sin\alpha - x^2]^{3/2}} = M_0 + \Delta M\frac{2x}{L}$$

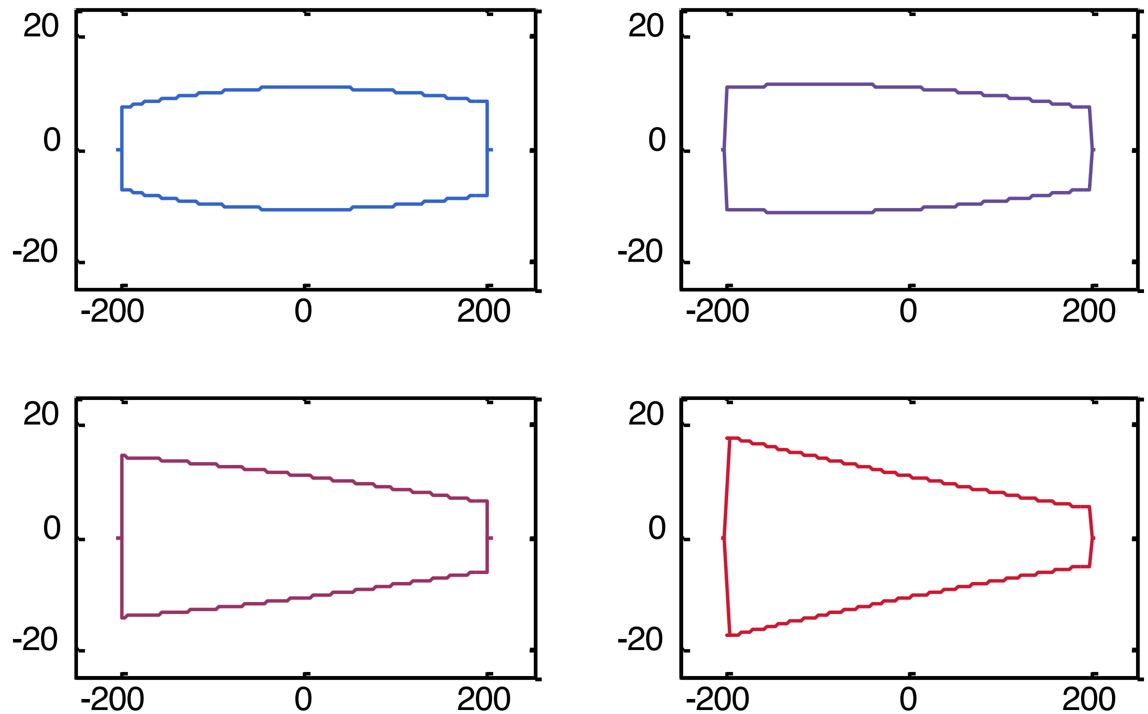
The solution depends on the forces one can apply.

$$b(x) = b_0\left(\frac{1}{b_0}M_0 + \frac{1}{b_0}\Delta M\frac{2x}{L}\right)\frac{24[pq - x(p-q)\sin\alpha - x^2]^{3/2}}{Eh^3\sqrt{pq}(p+q)\cos\alpha}$$

Often b(x) is just approximated by a low order polynomial variation

Examples of varied profile mirrors

Mirror width profile for a 10:1 demagnification mirror, for different bending moment combinations.



Model of deformation

*Deformation of the mirror, in this case, is **NOT a polynomial**.*

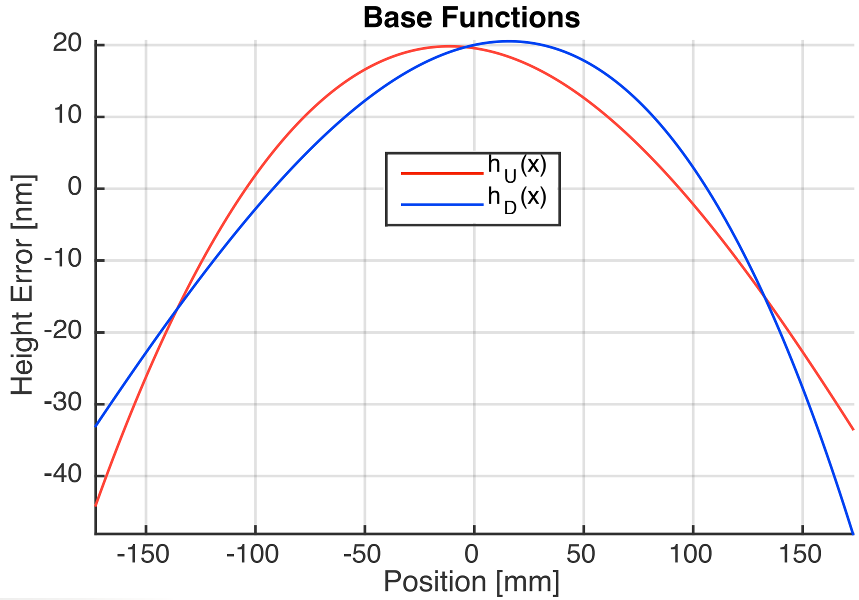
$$\frac{Eb(x)h^3}{12}\frac{d^2y}{dx^2}(x) = M_0 + \Delta M \frac{2x}{L}$$

$$\frac{Eb(x)h^3}{12}\frac{d^2y}{dx^2}(x) = a\left(\frac{x}{L} - \frac{1}{2}\right)F_U - a\left(\frac{x}{L} + \frac{1}{2}\right)F_D$$

*… but it is still a **linear combination** of two functions.*

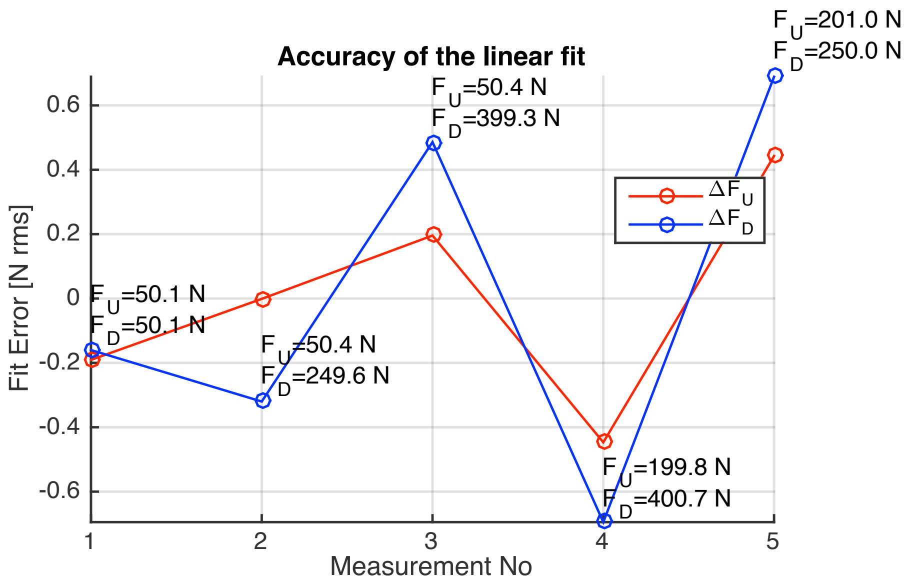
$$y(x) = F_U y_D(x) + F_D y_D(x)$$

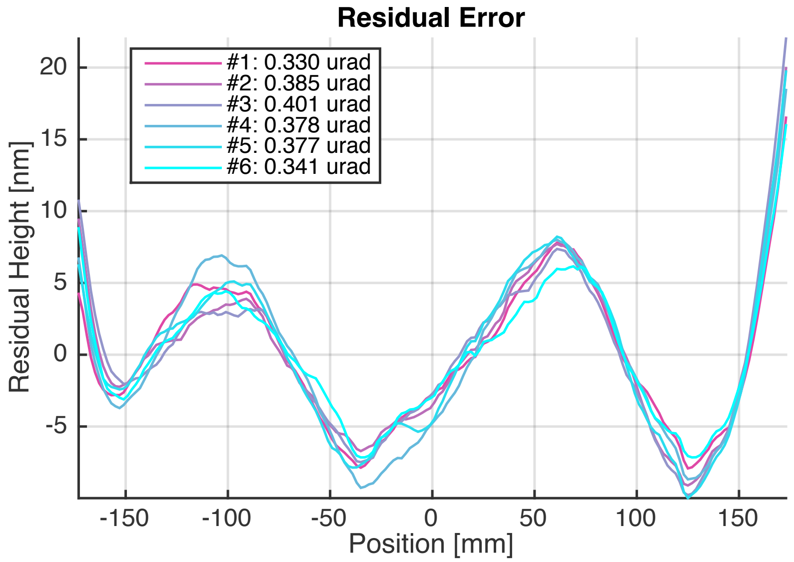
$$\begin{cases} \dfrac{Eb(x)h^3}{12a}\dfrac{d^2y_U}{dx^2}(x) = \dfrac{x}{L} - \dfrac{1}{2} \\ \dfrac{Eb(x)h^3}{12a}\dfrac{d^2y_D}{dx^2}(x) = -\dfrac{ax}{L} - \dfrac{1}{2} \end{cases}$$



Metrology of varied width mirror benders

After fitting data to the proposed base of functions, residual figure (polishing error) is equal for all bending positions



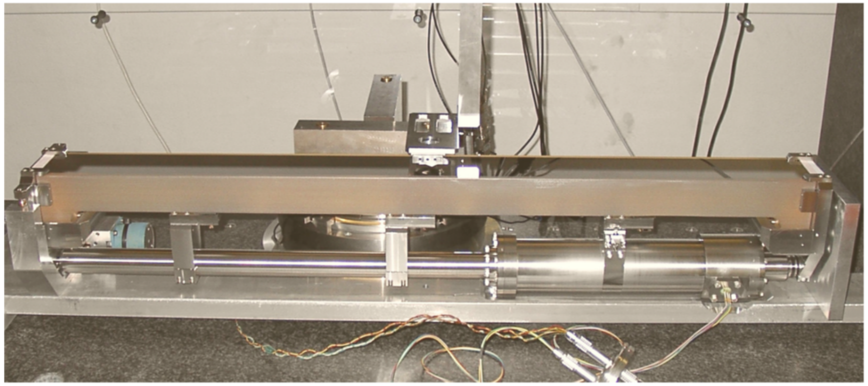


4. Mechanical design of mirror benders

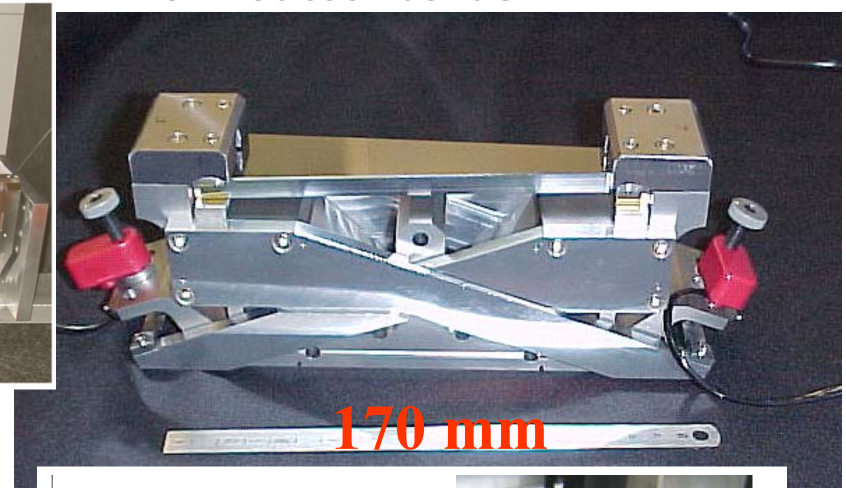
- *Functionalities of bender*
- *Parasitic forces*
- *Cyl benders, U-type, 4-point, scissor, bimorph, …*

A zoo of mechanical solutions

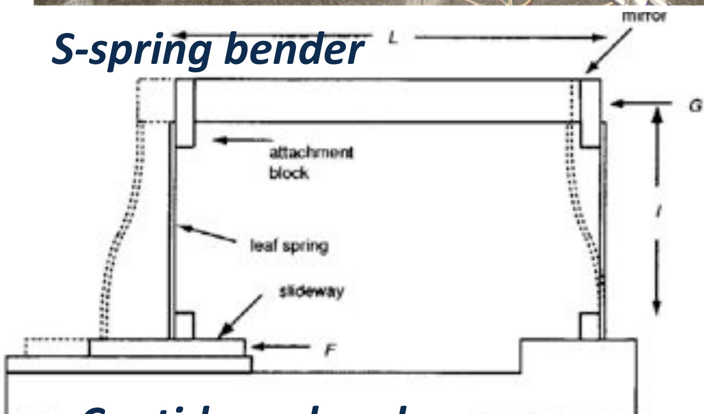
Cylindrical bender



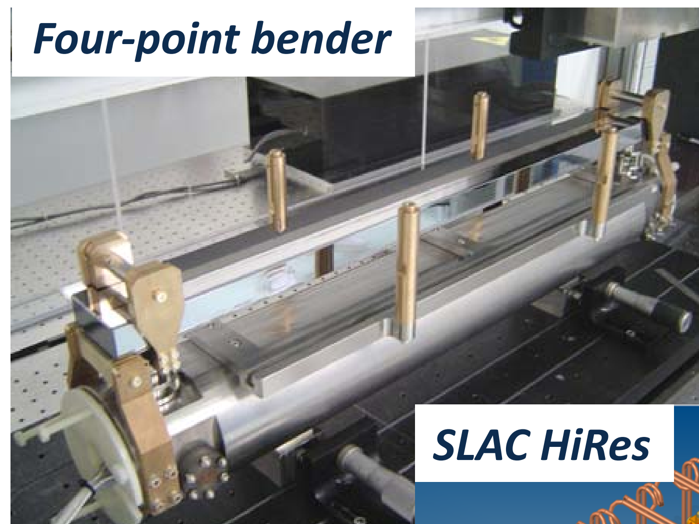
ESRF-Scissor bender



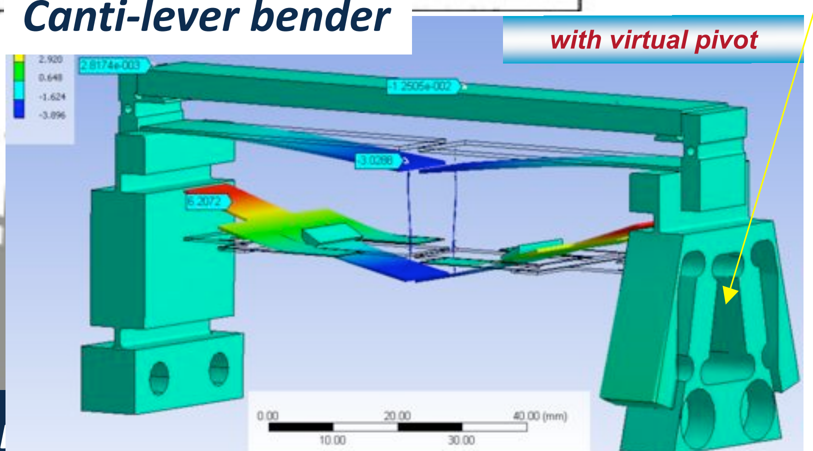
S-spring bender



Four-point bender



Canti-lever bender



SLAC HiRes

Mechanical design of benders

Functionalities of a bender

1. *To introduce the forces that deform the mirror to the required ellipse within tolerances.*

2. *To minimize parasitic forces that introduce undesired deformations*

3. *To introduce forces that deform the mirror to compensate gravity sag or other errors*

Mechanical design of benders

Functionalities of a bender

1. ***To introduce the forces that deform the mirror to the required ellipse within tolerances.***

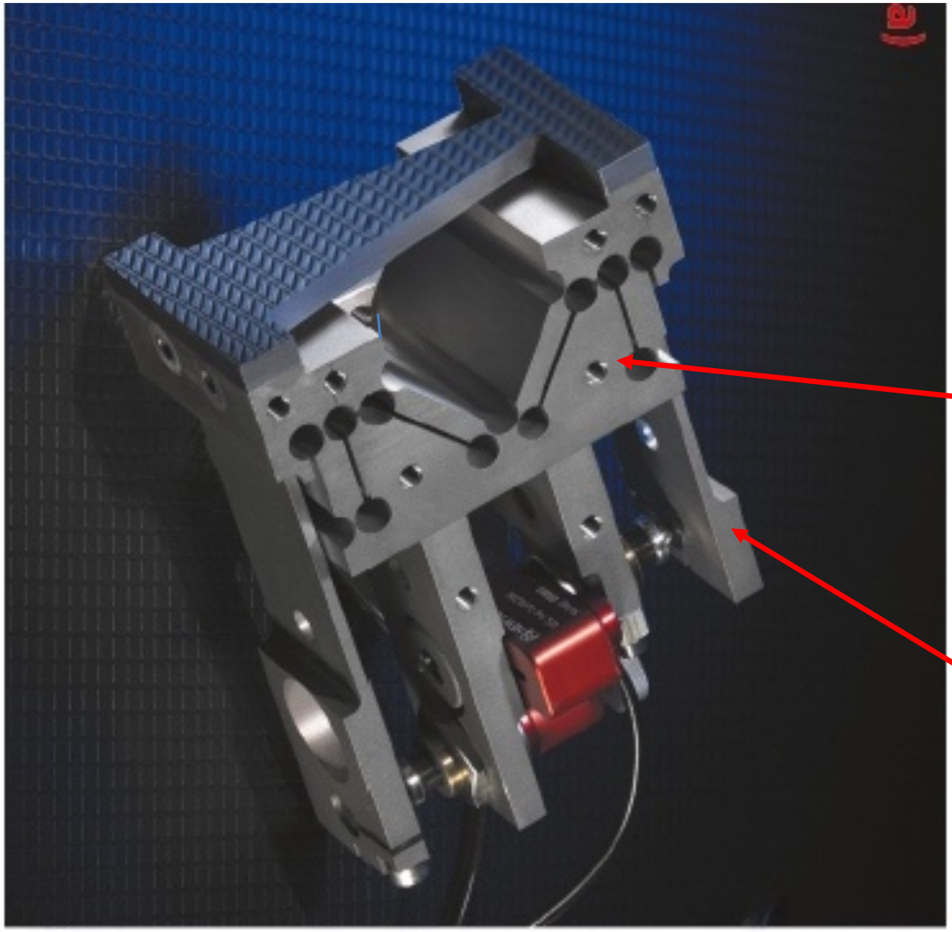
2. *To minimize parasitic forces that introduce undesired deformations*

3. *To introduce forces that deform the mirror to compensate gravity sag or other errors*

- ***Fix the application point and direction***

- ***Demagnify motor displacements to nanometer resolution deformation***

Apply controlled forces

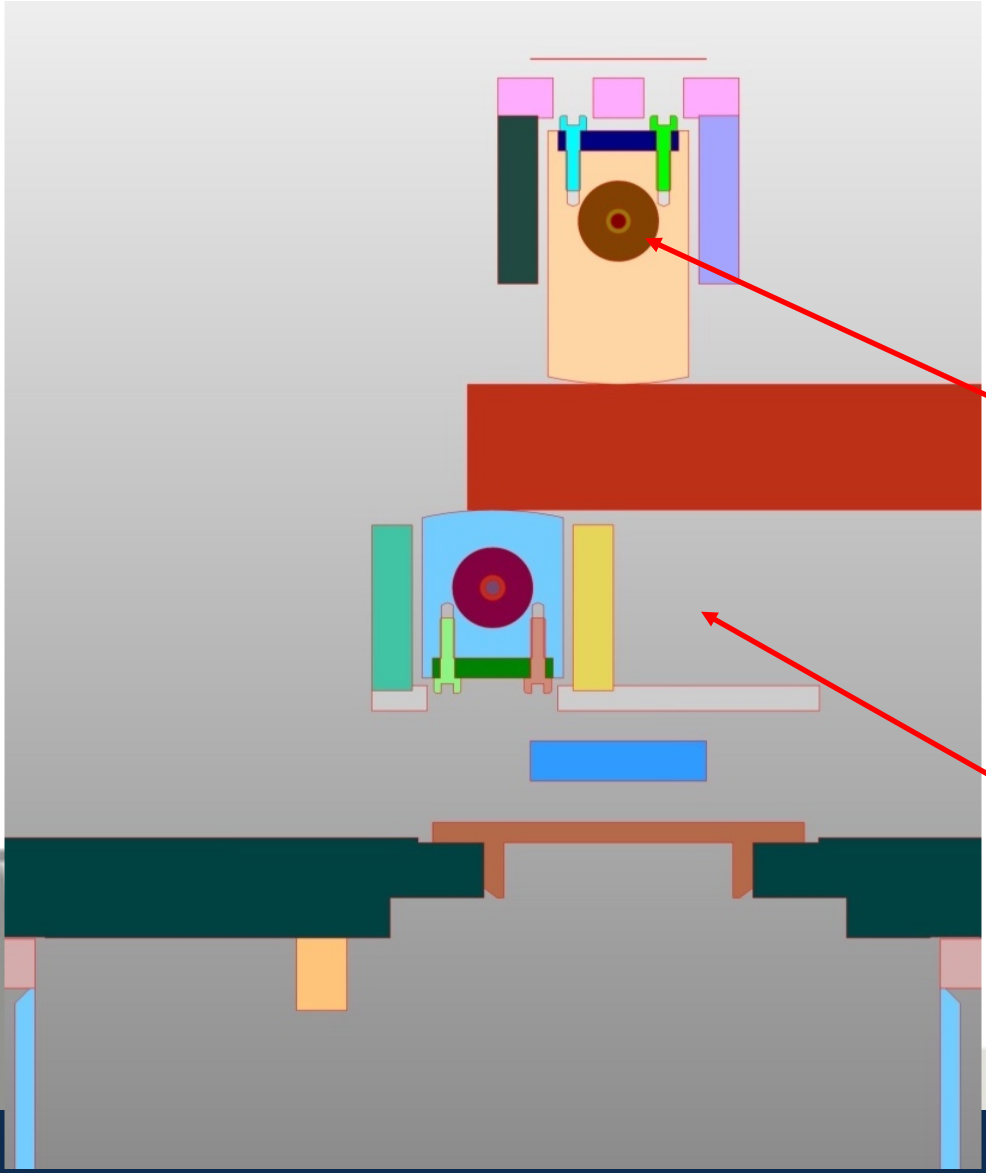


- ***Fix the application point and direction***
- ***Demagnify motor displacements to nanometer resolution deformation***

Flexure hinges point to the "rotation" point.
Mirror is glued (monolithic)

Lever (+bending ?) demagnifies motor linear motion to an angle constraint on the mirror ends.

Apply controlled forces

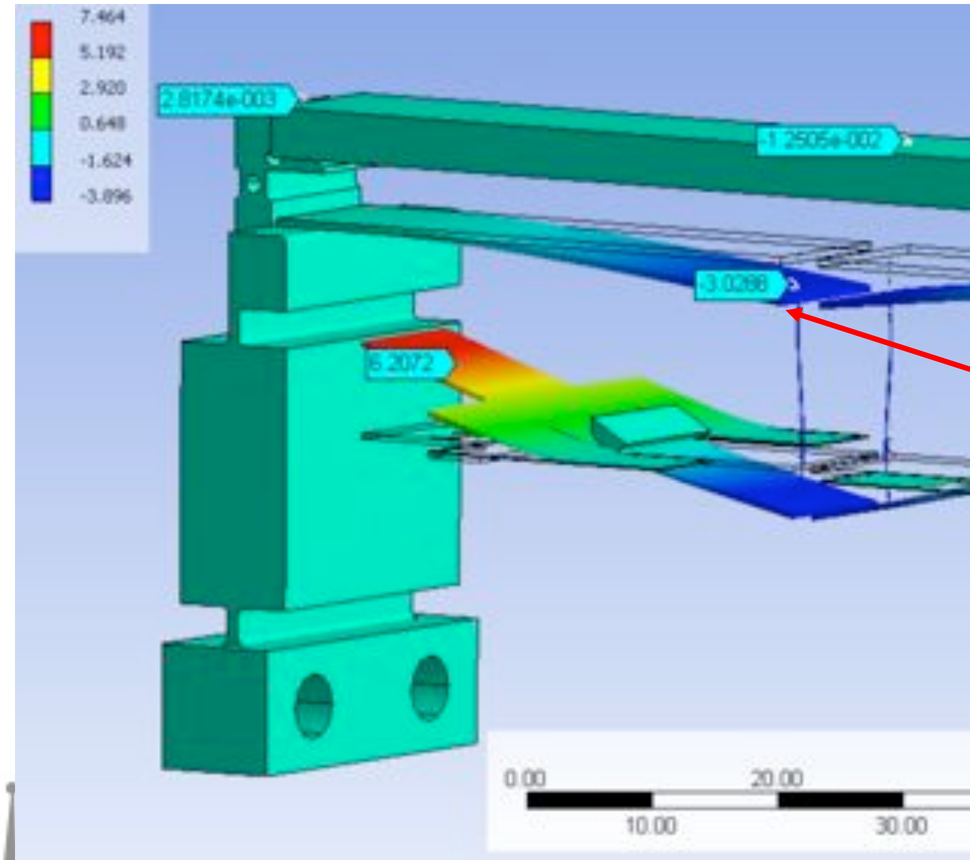


- ***Fix the application point and direction***
- ***Demagnify motor displacements to nanometer resolution deformation***

Pivot in center of contact surface ensures force is normal to the surface, and fixes application point

(Not visible) Force is exerted by a hellicoidal spring in compression

Apply controlled forces

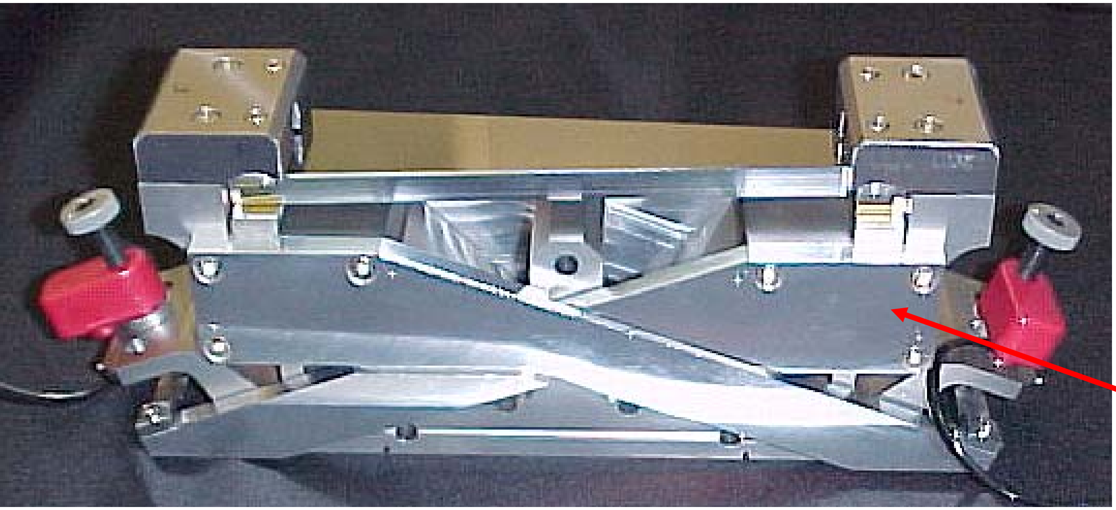


- ***Fix the application point and direction***
- ***Demagnify motor displacements to nanometer resolution deformation***

Demagnification of motor displacement by elastic bending of the cantilever

Apply controlled forces

- ***Fix the application point and direction***
- ***Demagnify motor displacements to nanometer resolution deformation***



Demagnification of motor displacement by a rigid long lever arm

Mechanical design of benders

Functionalities of a bender

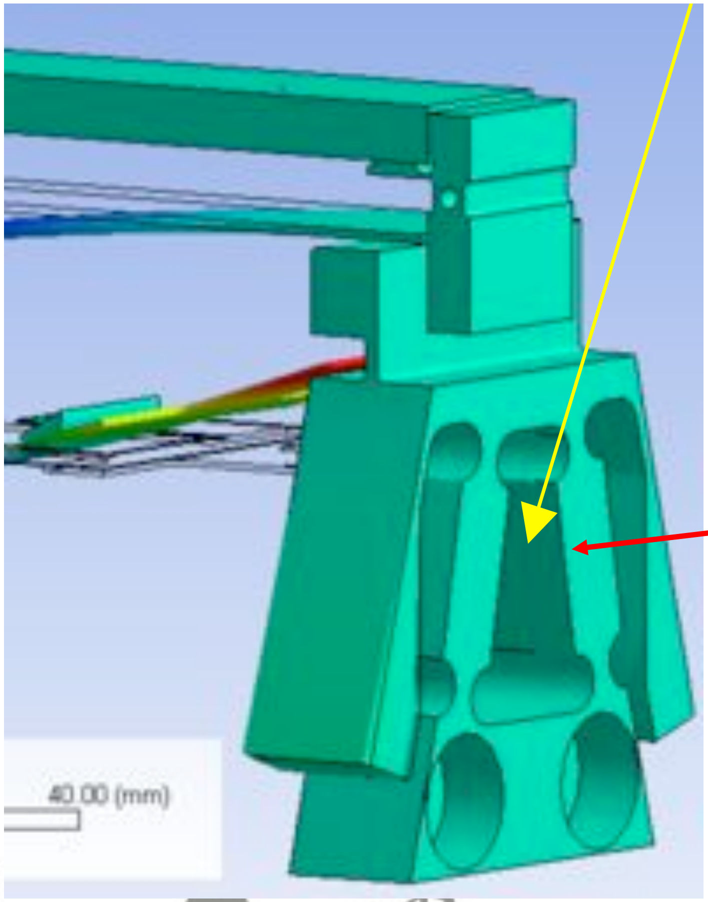
1. *To introduce the forces that deform the mirror to the required ellipse within tolerances.*

2. ***To minimize parasitic forces that introduce undesired deformations***

3. *To introduce forces that deform the mirror to compensate gravity sag or other errors*

- ***Avoid twist***
- ***Avoid forces along longitudinal axis***

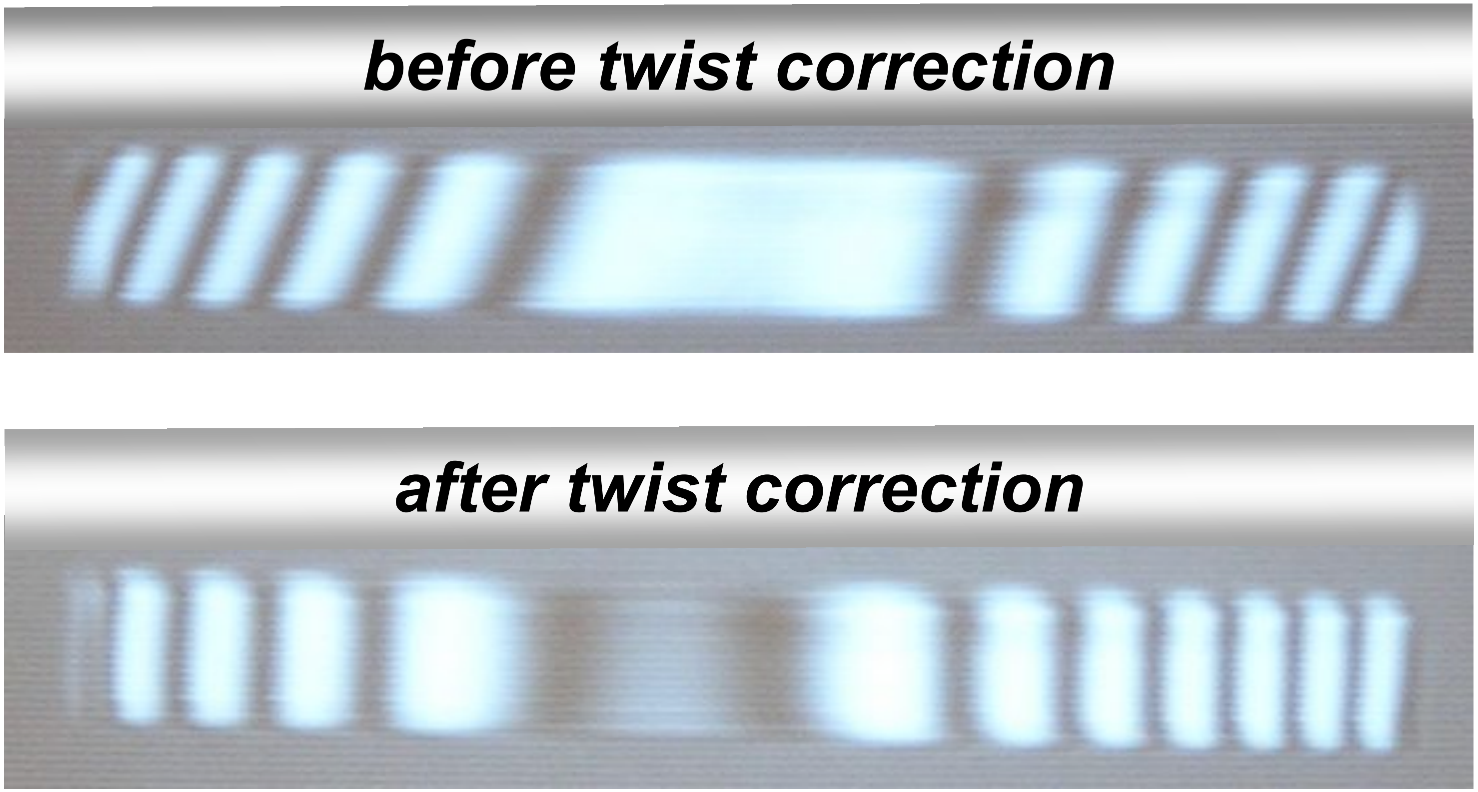
Control parasitic forces



- ***Avoid twist***
- ***Avoid forces along longitudinal axis***

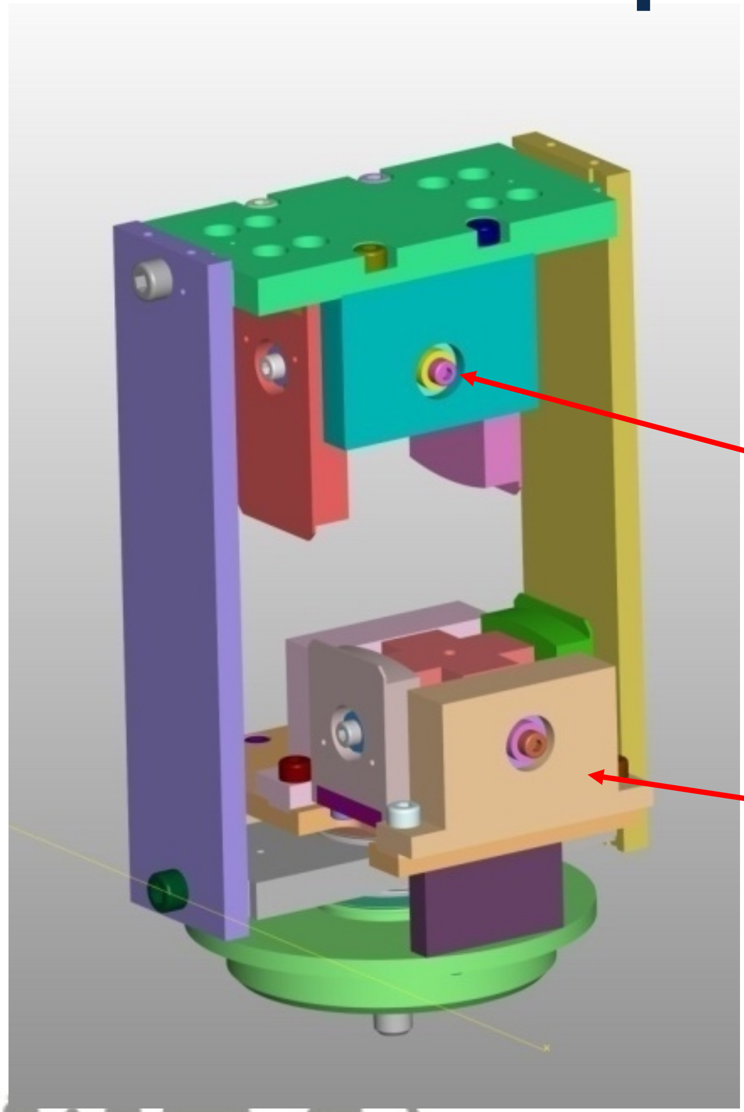
Virtual pivot to manually adjust the twist of one end of the mirror during metrology tests

Torsion (twist) adjustment



There are benders which are torsion free, and others which allow correcting it

Control parasitic forces



- ***Avoid twist***
- ***Avoid forces along longitudinal axis***

Bending force actuators allow pitch and roll to avoid introducing undesired deformations

Rolled articulation (on one side only) to allow the mirror settle its twist freely.

Mechanical design of benders

Functionalities of a bender

1. *To introduce the forces that deform the mirror to the required ellipse within tolerances.*

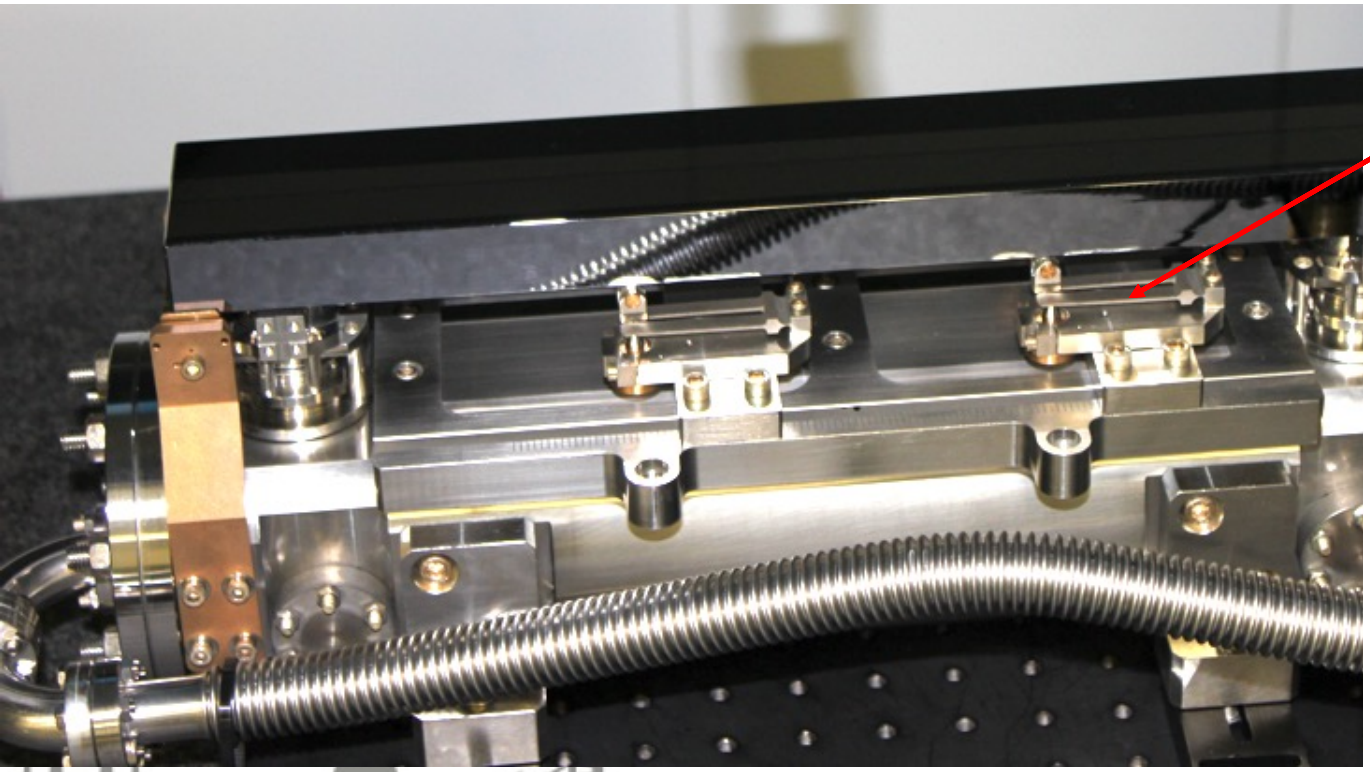
2. *To minimize parasitic forces that introduce undesired deformations*

3. ***To introduce forces that deform the mirror to compensate gravity sag or other errors***

- ***Gravity sag compensation***

Gravity sag compensation

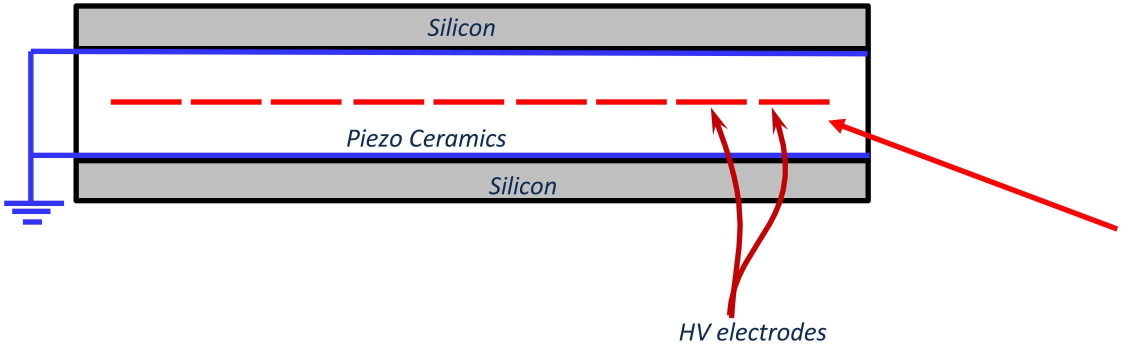
- ***Gravity sag compensation***



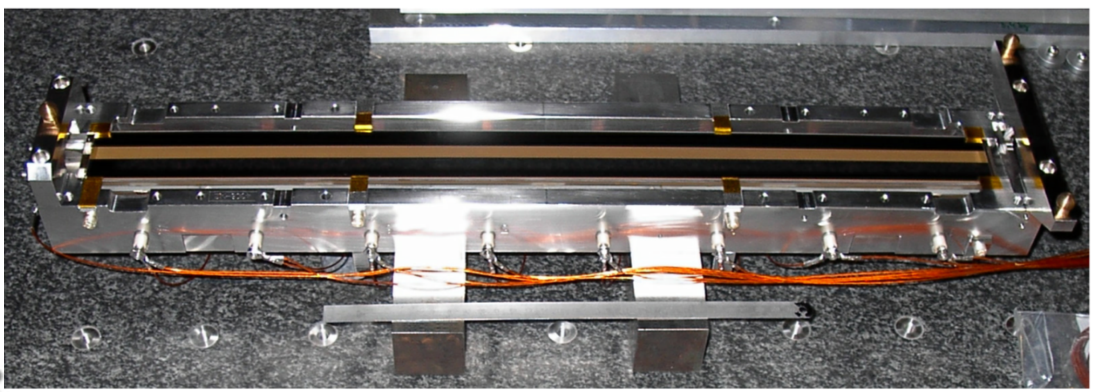
Manually adjustable springs, for gravity sag compensation

Adaptive optics

- ***Gravity sag compensation***
- ***Adaptive optics***

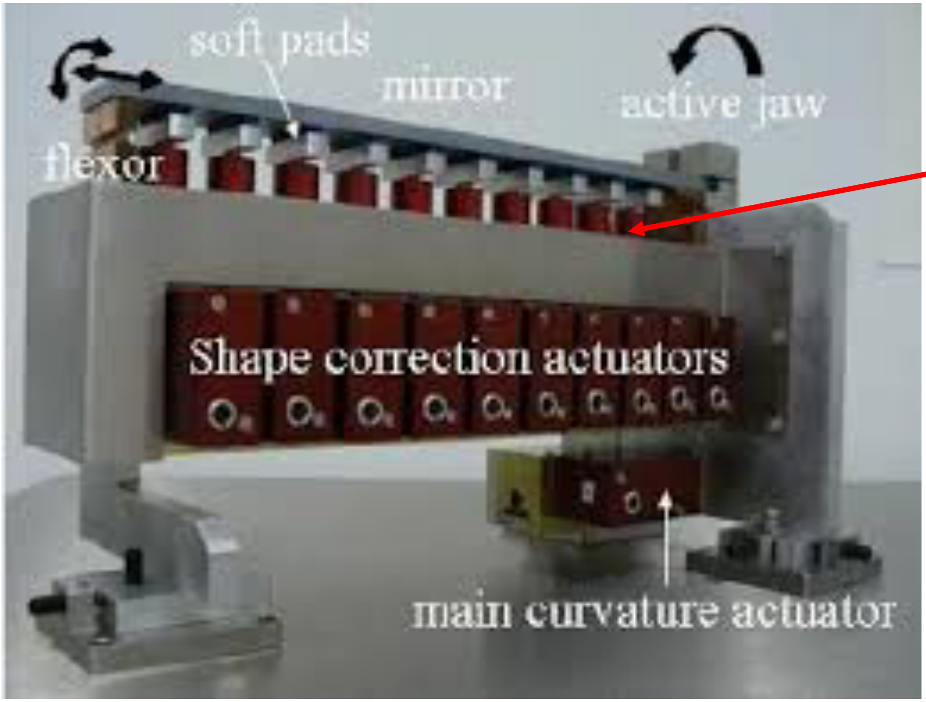


Piezo – electric corrctors embedded in the mirror bulk



Adaptive optics

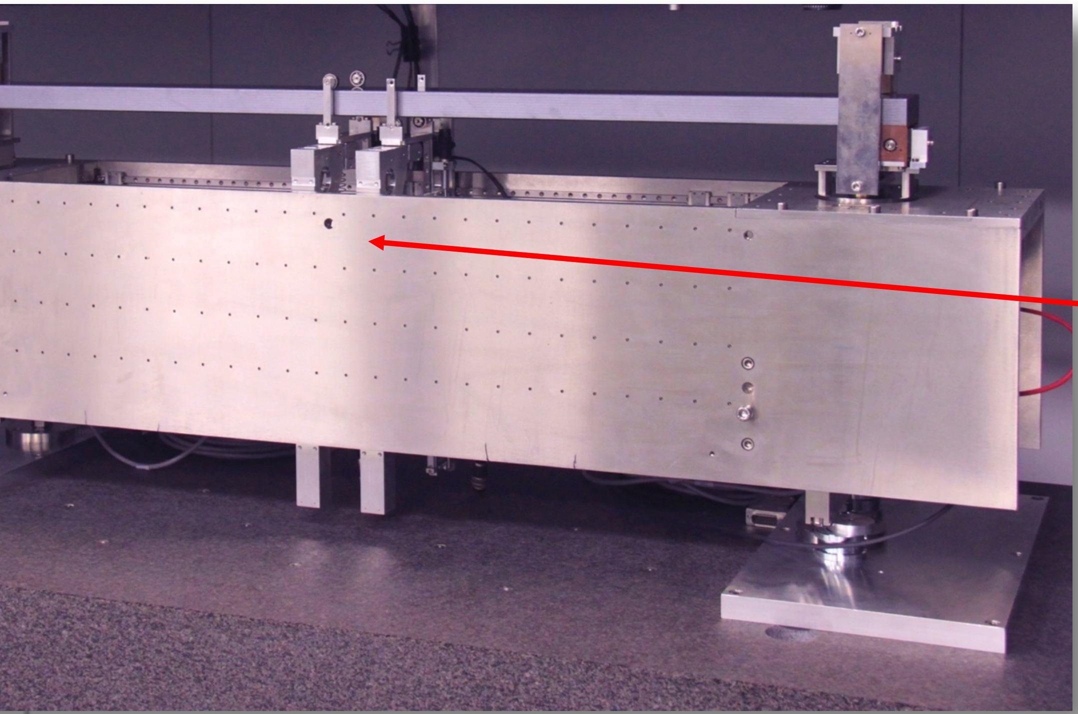
- ***Gravity sag compensation***
- ***Adaptive optics***



Nanometer resolution –stiff- pushing/pulling motors

Adaptive optics

- ***Gravity sag compensation***
- ***Adaptive optics***



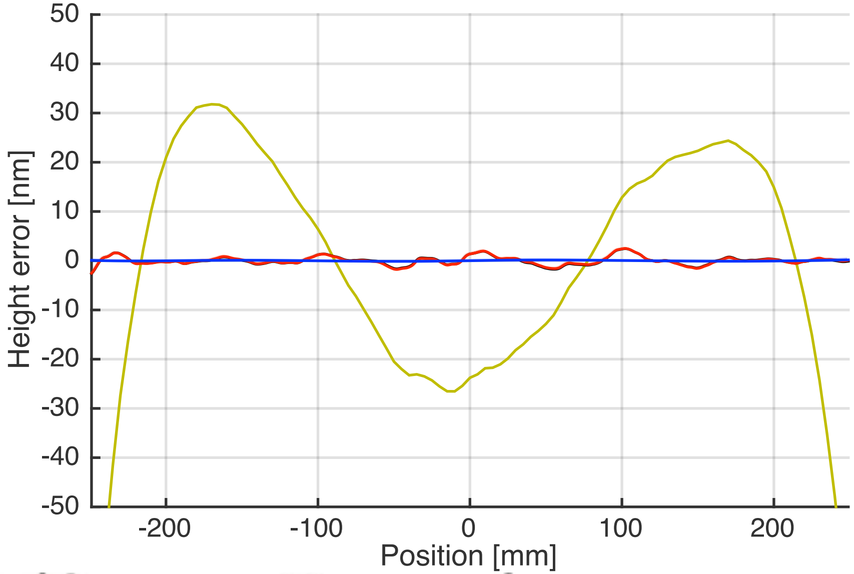
Force-stabilized – motorized correctors.

Adaptive optics

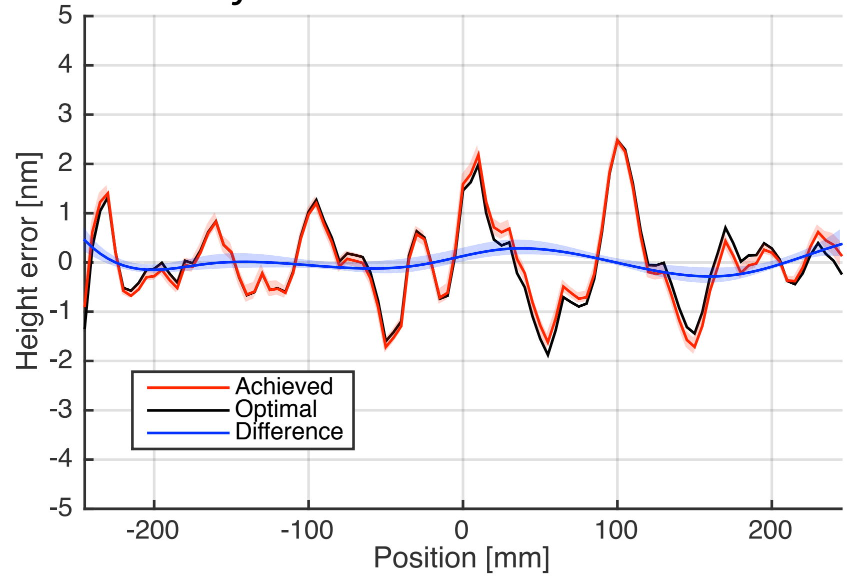
Results of the ALBA-SENER nanobender

Using 4 actuators, 500 mm

Initial slope error 0.87 µrad rms
Initial surface Error: 23.2 nm rms



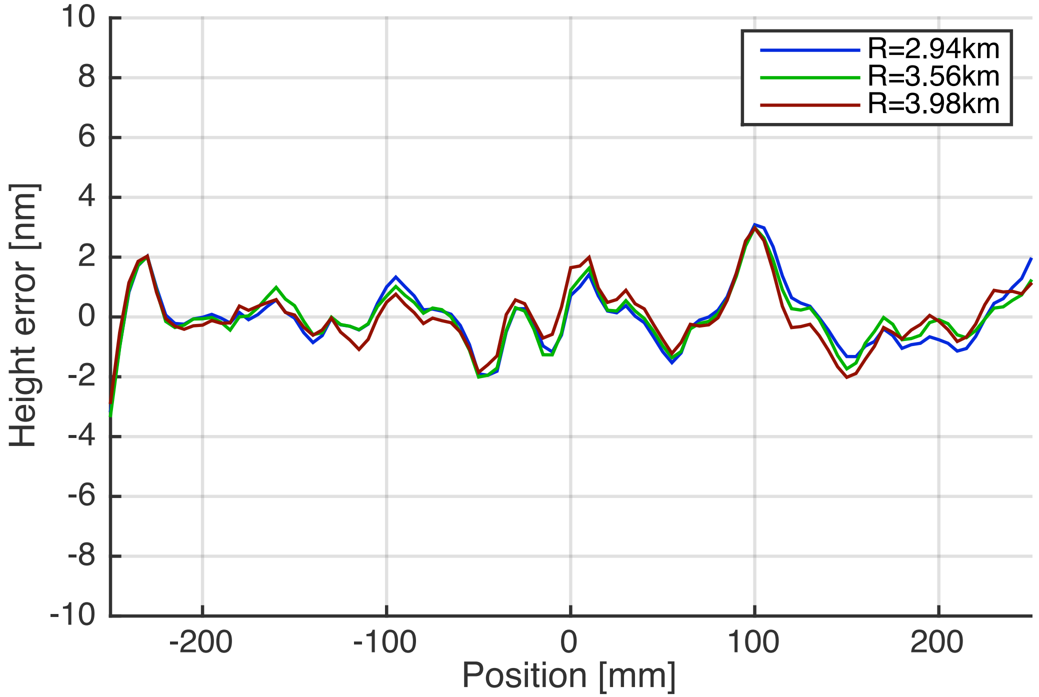
Slope Error: 0.115 µrad rms
Surface Error: 0.858 nm rms



Difference to model-base optimum of 0.08 nm

Adaptive optics

The correction is preserved within the nanometer with variations of the radius of curvature of 35% .



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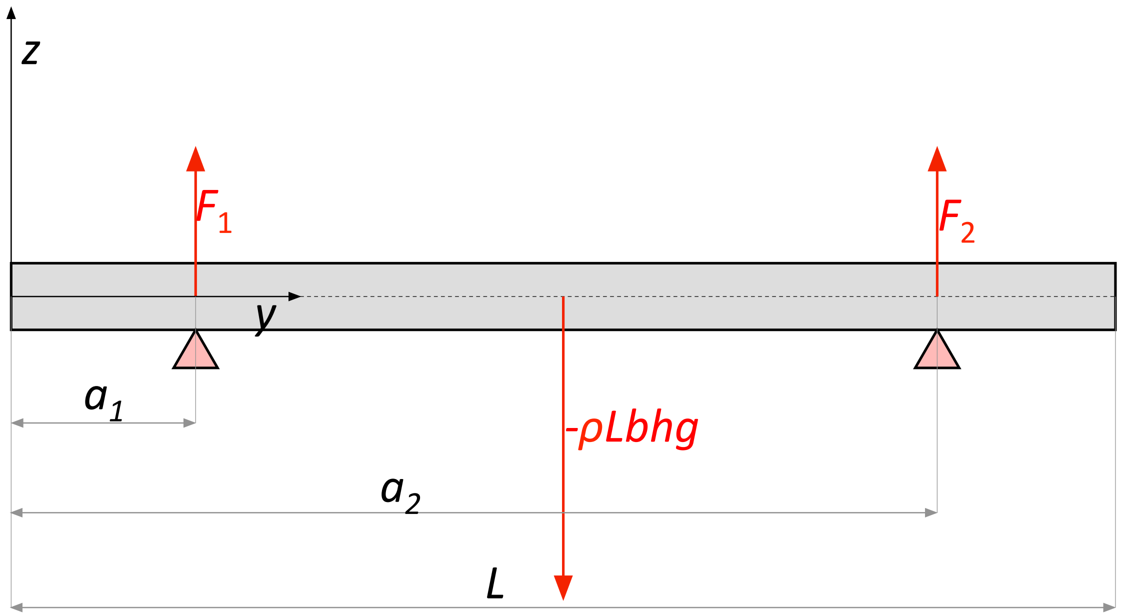
Ray Barrett (ESRF)

SENER
IRELEC

Backup slides – gravity sag correction

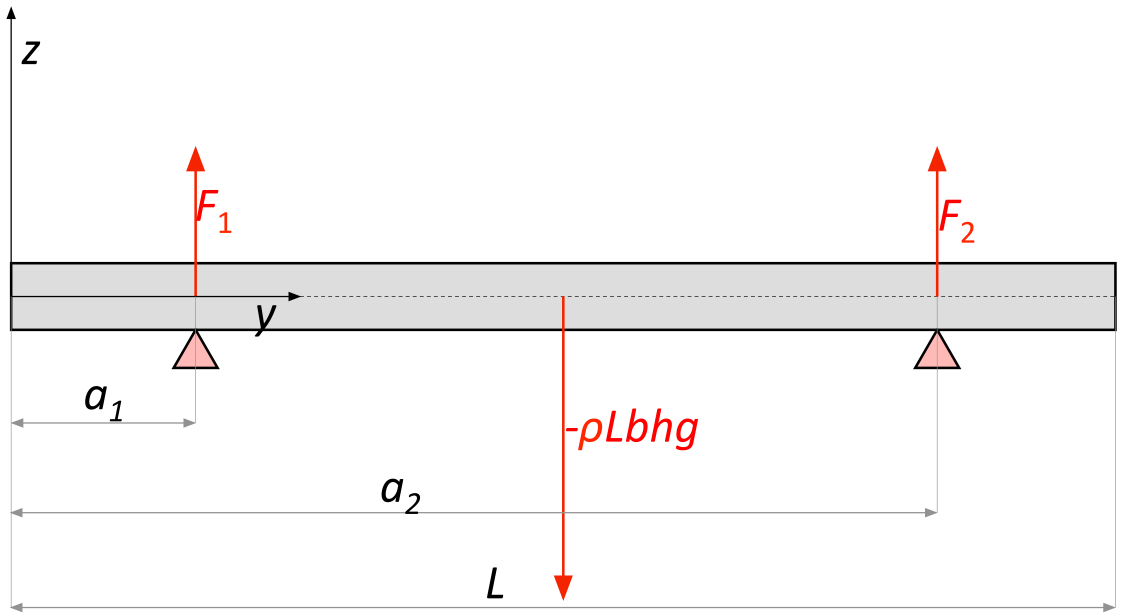
- *Rectangular footprint case*
- *Custom footprint case*

Gravity sag calculation



1. ***Equilibrium of forces and torques:*** *to determine all the forces.*
2. ***Function of momenta:*** *to determine the RHS of the E-B equation.*
3. ***Integration.***
4. ***Boundary conditions:*** *to determine the integration constants.*

Equilibrium of forces



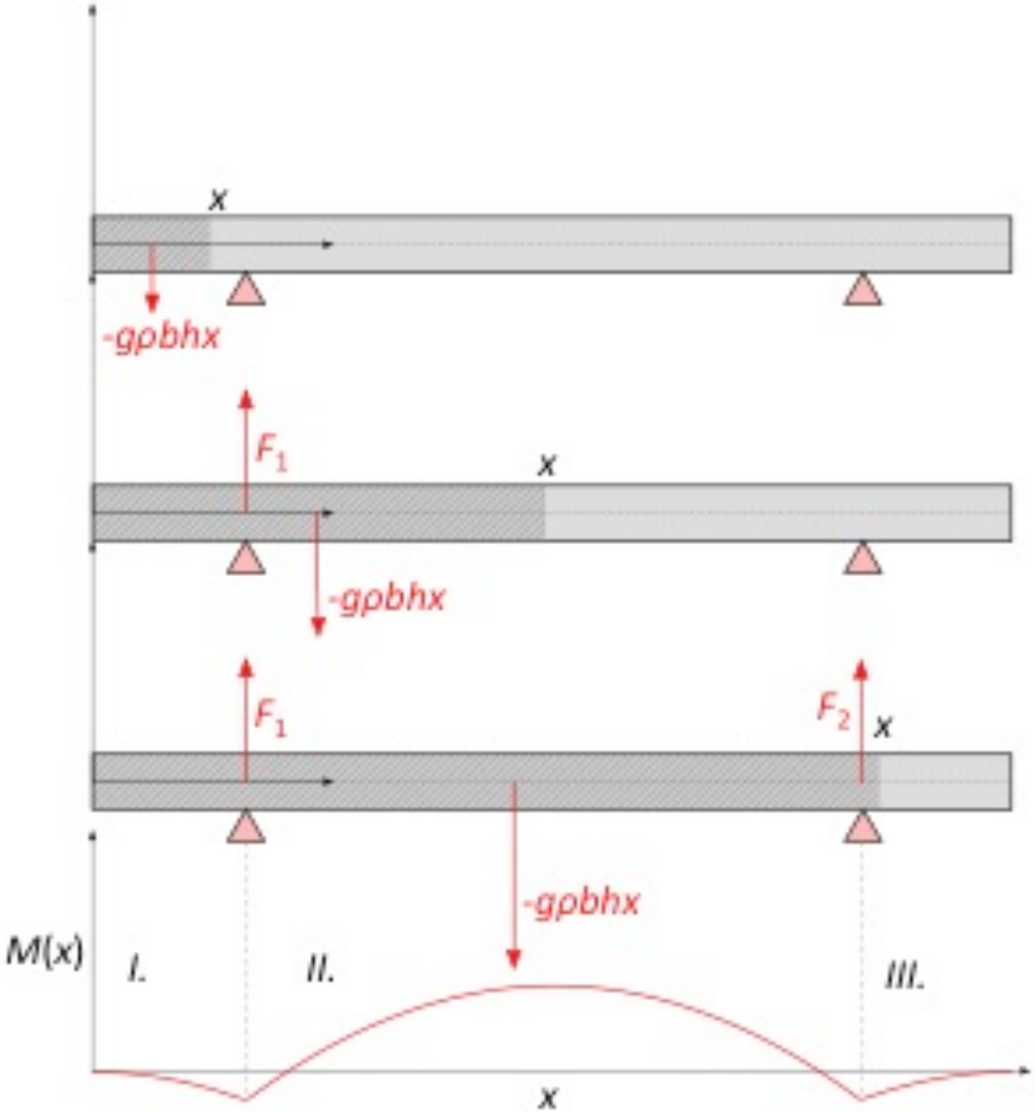
- *All forces and torques must add to zero, otherwise the mirror would be accelerating linearly or angularly.*
- *Torques are calculated from the origin of coordinates (not the center of the mirror)*

$$\begin{cases} F_1 + F_2 - g\rho L b h = 0 \\ a_1 F_1 + a_2 F_2 - \dfrac{g\rho L^2 b h}{2} = 0 \end{cases}$$

$$F_1 = gm\frac{a_2 - L/2}{a_2 - a_1}$$

$$F_2 = gm\frac{L/2 - a_1}{a_2 - a_1}$$

Function of moments



Region I. (only weight)

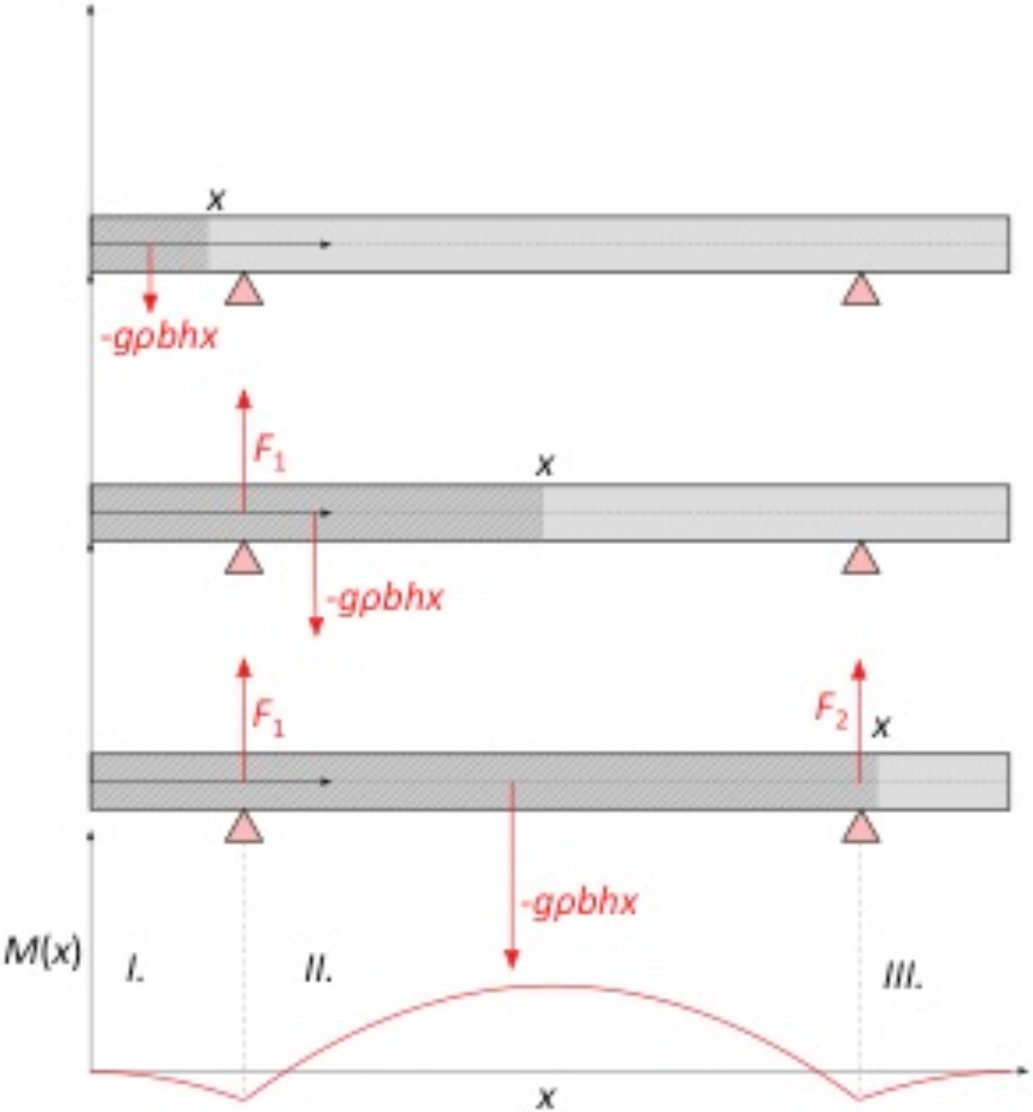
$$F_g(x) = -g\rho bhx$$

$$a_g(x) = \frac{x}{2}$$

$$M_I(x) = F_g(x)\left(x - a_g(x)\right)$$

$$= -\frac{1}{2}g\rho bhx^2$$

Function of moments

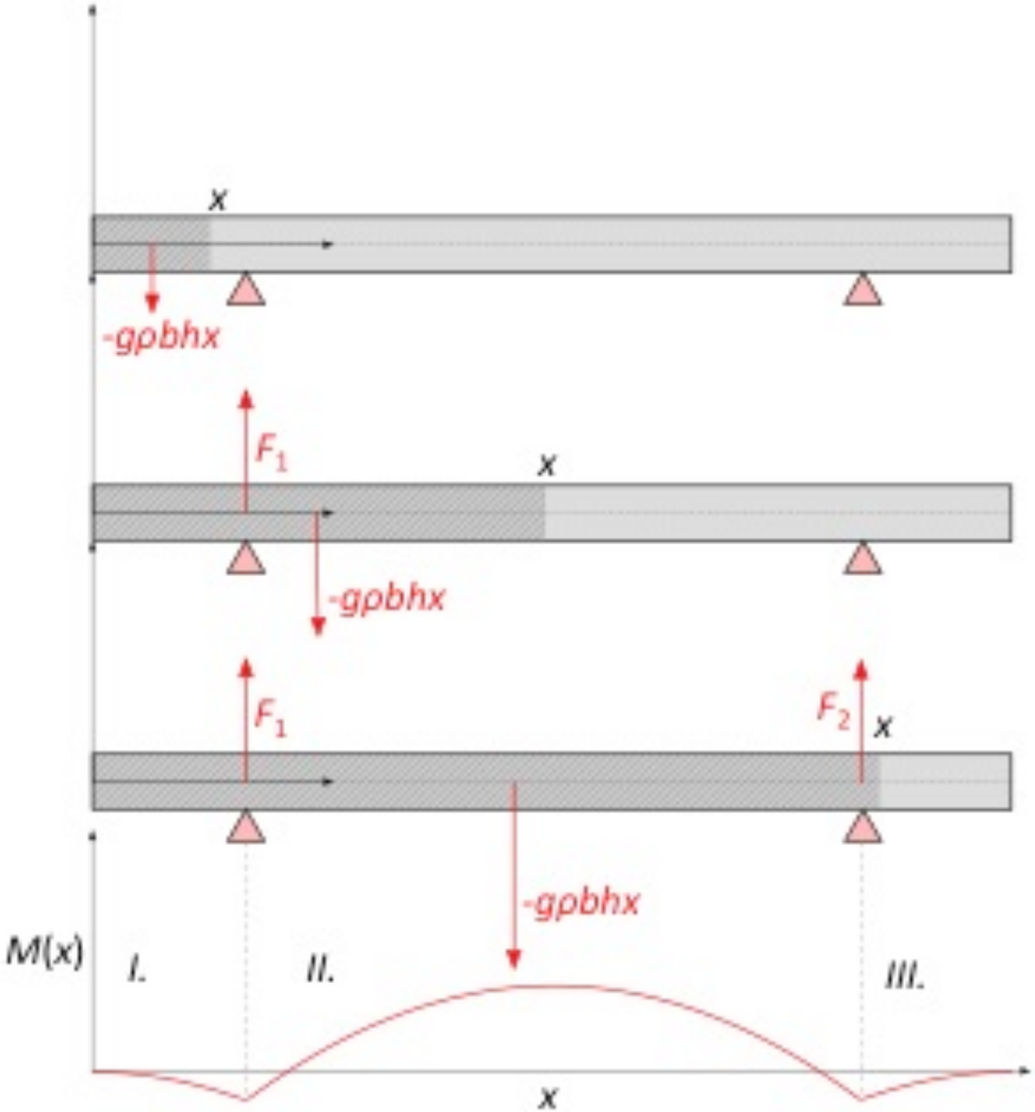


Region I. (only weight)

$$F_g(x) = -g\rho b h x$$

$$a_g(x) = \frac{x}{2}$$

Function of moments



Region II. (weight, F_1)

$$M_{II}(x) = -\frac{1}{2} g\rho bhx^2 + F_1(x - a_1)$$

Region III. (weight, F_1, F_2)

$$M_{III}(x) = -\frac{1}{2} g\rho bhx^2 + F_1(x - a_1) + F_2(x - a_2)$$

Integration (1)

- *Each region has to be integrated separately, because the function is not continuous.*
- *In all the cases the function of momenta is a polynomial of order 2. Which when integrated twice becomes a polynomial of order four.*

$$z_G(x) = z_0 + z_1 x + z_2 x^2 + z_3 x^3 + z_4 x^4$$

$$\frac{dz_G}{dx}(x) = z_1 + 2z_2 x + 3z_3 x^2 + 4z_4 x^3$$

$$\frac{d^2 z_G}{dx^2}(x) = 2z_2 + 6z_3 x + 12z_4 x^2$$

By defining :

$$k = \frac{Ebh^3}{12}$$

We have:

$$\frac{Ebh^3}{12}\frac{d^2 z_G}{dx^2}(x) = 2kz_2 + 6kz_3 x + 12kz_4 x^2$$

Integration (2)

Notation change !!!

$$z_i \rightarrow A_i, B_i, C_i$$

$$2kz_2 + 6kz_3x + 12kz_4x^2 = M(x)$$

Region I.

$$M_I(x) = -\frac{1}{2}g\rho bhx^2$$

Implies

$$A_2 = 0$$

$$A_3 = 0$$

$$A_4 = -\frac{g\rho bh}{24k}$$

Unknown

$$A_0, A_1$$

Region II.

$$M_{II}(x) = -a_1F_1 + F_1x - \frac{1}{2}bghx^2\rho$$

Implies

$$B_2 = -\frac{a_1F_1}{2k}$$

$$B_3 = \frac{F_1}{6k}$$

$$B_4 = -\frac{bgh\rho}{24k}$$

Unknown

$$B_0, B_1$$

Region III.

$$M_{III}(x) = (-a_1F_1 - a_2F_2) + (F_1 + F_2)x - \frac{1}{2}bghx^2\rho$$

Implies

$$C_2 = -\frac{a_1F_1 + a_2F_2}{2k}$$

$$C_3 = \frac{F_1 + F_2}{6k}$$

$$C_4 = -\frac{bgh\rho}{24k}$$

Unknown

$$C_0, C_1$$

Boundary conditions

1. Continuity between regions + at tbese points deformation is zero

$$z_I(a_1) = 0$$
$$z_{II}(a_1) = 0$$
$$z_{II}(a_2) = 0$$
$$z_{III}(a_2) = 0$$

2. Derivability (continuity of 1st derivative) between regions

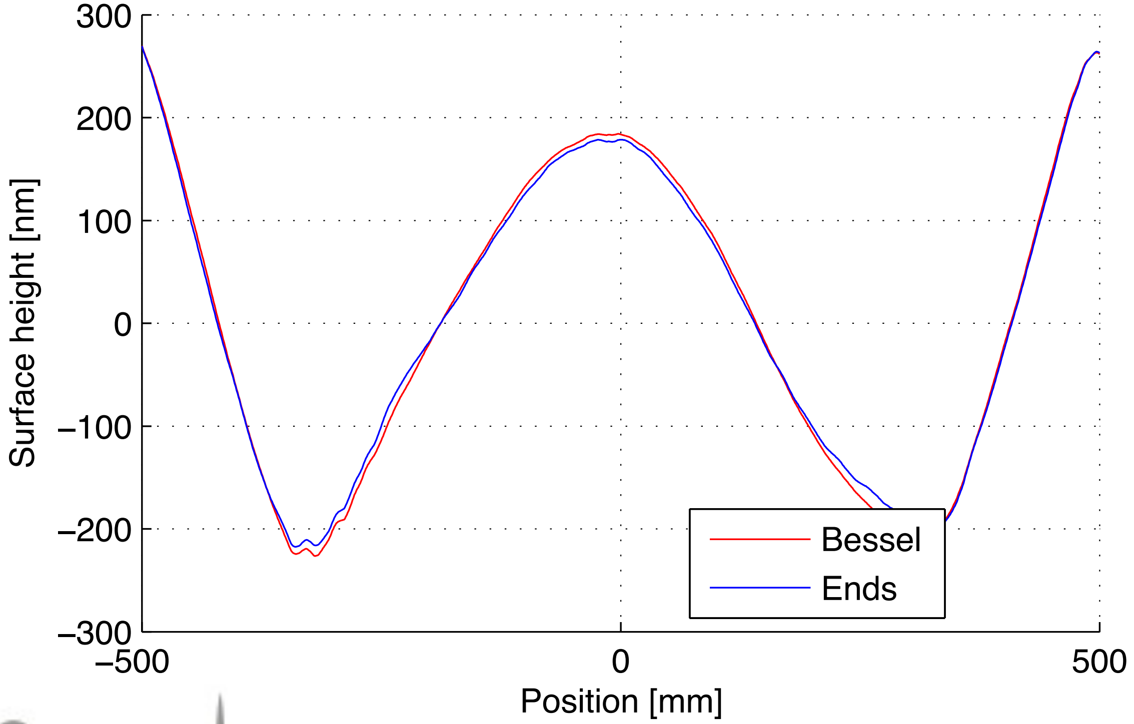
$$z_I(a_1) = z_{II}(a_1)$$
$$z_{II}(a_2) = z_{III}(a_2)$$

The system is linear on A_0, A_1, B_0, B_1, C_0, C_1, and can be solved with unique solution (using Mathematica).

➔ *See Mathematica Notebook:* **GravitySagCalculation.nb**

Example of gravity sag correction

A mirror measured with supports at the Bessel points and at the ends. Gravity sag is removed in both cases.



Gravity sag of arbitrary profile mirror

The width of the mirror is not constant

$$\frac{Eb(x)h^3}{12}\frac{d^2 z_G}{dx^2}(x) = M_G(x) + M_1(x) + M_2(x)$$

M_G: gravity, M_1: support 1, M_2: support 2

Calculating M_G

- *Mass at the left of x:* $$m_L(x) = \rho h\left(\int_{-L/2}^{x} b(x')dx'\right) \qquad I_0(x)$$
- *Mass center:* $$x_L(x) = \int_{-L/2}^{x} x'b(x')dx' \Big/ \int_{-L/2}^{x} b(x')dx' \qquad \frac{I_1(x)}{I_0(x)}$$

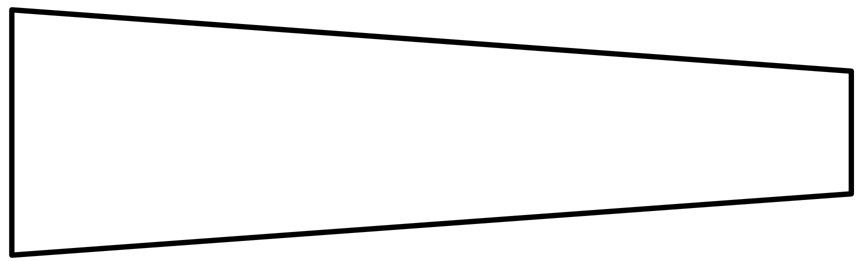
M1, M2

$$M_i(x) = [x - x_i]F_i\theta(x - x_i)$$

$\theta(x)$ *Heavyside Step function*

Gravity sag of arbitrary profile mirror

- *The width of the mirror is not constant.*



- *The equation is no-longer the integral of a polynomial.*

$$\frac{d^2 z_G}{dx^2}(x) = 12\frac{M_G(x) + M_1(x) + M_2(x)}{Eb(x)h^3}$$

M_G: gravity, M_1: support 1, M_2: support 2

- *It can be integrated numerically.*
- *All the functions of x become vectors (linear arrays)*

$$f(x) \rightarrow f(x_n) \rightarrow f(n\Delta x) \rightarrow f_n$$

Moment of inertia of the weight

Calculating $M_G(x)$

- *Mass at the left of x:* $$m_L(x) = \rho h\left(\int_{-L/2}^{x} b(x')dx'\right)$$

- *Mass center:* $$x_L(x) = \int_{-L/2}^{x} x'b(x')dx' \Big/ \int_{-L/2}^{x} b(x')dx'$$

These become numerical integrals

$$(m_L)_n = \rho h \Delta x \sum_{m=1}^{n} b_m \qquad (x_L)_n = \sum_{m=1}^{n} x_m b_m \Big/ \sum_{m=1}^{n} b_m$$

If you like you can apply more sophisticated integration methods, Simpson's rule, Spline,…

And the final expression of $M_G(x)$ is also a vector

$$M_G(x) = -g m_L(x)[x - x_L(x)] \qquad (M_G)_n = -g(m_L)_n[x_n - (x_L)_n]$$

Moments of inertia of the supports

Calculating $M_1(x)$, $M_2(x)$

- *Need to know the actual values of F_1 and F_2 (not just its expression as a function of mirror dimensions).*

$$F_1 = gm_0 \frac{x_2 - x_0}{x_2 - x_1}$$

$$F_2 = gm_0 \frac{-x_1 + x_0}{x_2 - x_1}$$

m_0: total mass
x_0: mass center of the mirror

- *They can be written in close expression using the Heaviside function step function*

$$M_i(x) = (x - x_i)F_i\theta(x - x_i)$$

$$(M_i)_n = (x_n - x_i)F_i\theta(x_n - x_i)$$

Integration

- *Once all the functions of x are evaluated, they can be numerically integrated*
- *Note that, since we use close expressions, there are no boundary conditions between zones.*

$$z(x) = c_o + c_1 x + \int_{-L/2}^{x} \int_{-L/2}^{x''} 12 \frac{M_G(x') + M_1(x') + M_2(x')}{E h^3 b(x')} dx'\, dx''$$

- *Which becomes*

$$z_n = c_o + c_1 x_n + \frac{12}{E h^3} \sum_{m=1}^{n} \sum_{p=1}^{m} \frac{(M_G)_p + (M_1)_p + (M_2)_p}{b_p}$$

- *Average height and average pitch are undetermined. Often, these are simply set to average zero as they do not carry information about the mirror figure, but about its position.*