Abstract
Multibend Achromat (MBA) synchrotron designs are placing stringent mechanical tolerances on the magnet support systems. At the APS-U the magnet-to-magnet vibration tolerances are about 10 nm. [1]. Timelines, installation requirements, and budgets constrain the resources available for prototyping and physical testing. Reliance on FEA to predict dynamic response is paramount in insuring the tolerances are met. However, obtaining accurate results from a magnet support structure FEA is not as simple as analysing the CAD model of the structure.

The 16th century author Nostradamus published a collection of prophecies that since his time, have been held up as predictions of various world events. While it is attractive to think his collection of short poems can be used to foretell the future, in reality it is only the vagueness and absence of any dates that make them easy to apply in a posthoc basis. Arguably, a similar statement can be made about the use of FEA in predicting accelerator support response. In this paper the important contributors to FEA dynamic modelling will be discussed along with techniques that can be used to generate necessary data for models that can accurately predict response.

INTRODUCTION

Background
Numerous multi-bend achromat (MBA) synchrotrons are either under construction or planned [2, 3]. At the Advanced Photon Source (APS) a major upgrade is in the planning stages. The electron beam in this machine will have much smaller transverse dimensions, and consequently more stringent beam stability requirements than the current APS. In addition, the X-ray beams will be focused to ever smaller dimensions, placing more stringent stability requirements on the X-ray source point [4]. Both of these requirements result in magnet-to-magnet vibration tolerances on the order of 10 nm and magnet group-to-magnet group (girder) requirements of a few tens of nanometers.

Along with challenging physics requirements, facility, budget, and logistic constraints cause significant pressure on the design process. For example, the APS-Upgrade (APS-U) project spans multiple years, though the installation and commissioning phase is to take place over a single 12-month dark period. Critical engineering choices need to be made very early in the process. The combination of the physics and project planning constraints push engineering designs in new directions for which limited performance data are available. In the context of magnet support structures, predicting the modal and vibration response before construction are viewed by the APS-U engineering team as essential to completing the magnet support structure designs [5-7] and meeting the machine requirements in an economical and timely manner.

The goal is to have a validated process and set of models, by which the important magnet support system structural response characteristics such as natural frequency, vibration response, static deflection, and thermally-induced distortion can be predicted for each of the magnet support systems. Ideally, the testing of physical prototypes is reduced to an exercise of design verification rather than one of discovery, as shown in the workflow in Figure 1. This level of simulation is commonly referred to as virtual prototyping [8]. It is already being done in the automotive sector [9].

While the motivation for virtual prototyping in the automotive or aerospace sectors may be primarily one of reducing the cost of testing and of serial production products, there is certainly significant desire to produce products that have higher levels of out-of-the-box performance than ever before. This is the similarity to the synchrotron world—to engineer and build magnet support structures that have higher levels of mechanical stability than previous generations of synchrotrons.

Analytical and computational simulation have been used extensively in the design of synchrotron components, primarily in the area of absorbers and optics. Thermal finite element analysis (FEA) models for synchrotron radiation absorbers or optics depend on five important aspects: component geometry, boundary condition definition, material properties, thermal load profile and cooling fluid conditions/properties. These have all been characterized sufficiently such that performance can be well predicted from computational models. In fact, recent work has continued to refine the failure criteria through a better understanding of absorber failure mechanisms. The same level of a priori computational confidence is not currently possible in the arena of synchrotron structural dynamics.
The use of analytical and computational simulation in the structural dynamics area for synchrotron components differs from the thermal area. While in a typical optic or absorber thermal analysis the important area of the model does not extend beyond the boundaries of a single part, in a structural dynamics analysis, the model involves multiple parts and interconnections. Component geometry, mass, stiffness, and damping all play a role in the system behavior.

The Problem: Nostradamus We Are Not...

Simply using a CAD model of a complete magnet support structure for the starting point of either a modal or vibration response analysis will result in over predictions that are far from the measured result. For instance, a test article for the APS-U magnet support structures is shown in Figure 2. The geometry, mass, and material properties are well known for the girder, magnets and plinth. In fact, if a modal analysis were conducted using the CAD model for the girder and magnet assembly (not including the support and alignment mechanism), the results would be quite reasonable.

In contrast, if the same type of modal analysis were run for the whole test article, the results could over predict the natural frequencies by as much as 100%. A random vibration analysis would be even more erroneous. This is because the modal response of the assembly is highly dependent upon the stiffness of the support mechanism and the vibration response is highly dependent on both the stiffness and damping of the support mechanism. Using a CAD model of the support mechanism as the FE analysis model (just geometry, no accounting for interfaces) significantly over estimates the stiffness of the support and has no true relation to the damping present in the support. These properties are dependent not only on the component geometry but friction, loads, and all the minute interfaces.

A modal FEA model based on the CAD data can be tuned in a posthoc basis to match the experiment. One way this could be done is that the Young’s modulus of the support mechanisms might be altered such that simulation and experiment modal results can be more closely matched. Drawbacks include difficulty in determining the appropriate changes in modulus and material damping and difficulty in incorporating directional and load dependency. Most importantly, the results obtained with model tuning are obtained only after testing a complete structure, and are then only good for a specific case. Due to the posthoc nature of this method, it is difficult to generalize this process to other designs, much as it is difficult to generalize Nostradamus’ predictions to future events. Virtual design iteration as described in Figure 1 cannot be done.

Figure 1: An example of the design workflow. If the loop can be reliably closed between simulation and design (solid arrow), then reliance on feedback from testing to design (dashed arrow) can be reduced. The enabler for this to happen is strategic component testing and correct use of the identified properties in the analysis and simulation step.

Figure 2: APS-U magnet support test article. The arrow points to one of the supports. Below the support and alignment mechanism is the plinth and above the support and alignment mechanism are the girder and magnets. A thin epoxy grout layer couples the assembly to the floor.
any support mechanism can be quantified in a phenomenological manner and these data subsequently incorporated into analysis models. Model validation is also discussed.

METHODS OF PROPERTY ESTIMATION

In the most general sense the support system can be considered as in Figure 3. The support and alignment mechanism can be represented by stiffness and damping matrices.

![Figure 3: A generalized view of a magnet support mechanism. The support alignment mechanism is represented by the stiffness $K_J$ and the damping $C_J$. $F$ are the applied forces. $h$ is the height of the center of mass above the support and $d$ is the half-height of the support mechanism.](image)

The equations of motion describing the system are:

$$F = M \ddot{x} + C \dot{x} + K x.$$  \hspace{1cm} (1)

Where the coordinates are described by:

$$x = [X \ Y \ Z \ \theta_x \ \theta_y \ \theta_z].$$  \hspace{1cm} (2)

In the equations of motion, the mass matrix is diagonal (mass centered coordinate system). The stiffness and damping matrices are symmetric. The most general form of the support stiffness and damping matrices ($K_J$ and $C_J$) are of the form:

$$K_J = \begin{bmatrix} K_{XX} & K_{XY} & K_{XZ} & K_{X\theta_x} & K_{X\theta_y} & K_{X\theta_z} \\ K_{YX} & K_{YY} & K_{YZ} & K_{Y\theta_x} & K_{Y\theta_y} & K_{Y\theta_z} \\ K_{ZX} & K_{ZY} & K_{ZZ} & K_{Z\theta_x} & K_{Z\theta_y} & K_{Z\theta_z} \\ K_{\theta_xX} & K_{\theta_yX} & K_{\theta_zX} & K_{\theta_x\theta_x} & K_{\theta_y\theta_x} & K_{\theta_z\theta_x} \\ K_{\theta_yX} & K_{\theta_yY} & K_{\theta_yZ} & K_{\theta_y\theta_x} & K_{\theta_y\theta_y} & K_{\theta_y\theta_z} \\ K_{\theta_zX} & K_{\theta_zY} & K_{\theta_zZ} & K_{\theta_z\theta_x} & K_{\theta_z\theta_y} & K_{\theta_z\theta_z} \end{bmatrix}_{sym}$$  \hspace{1cm} (3)

Notice the off-diagonal coupling terms, relating translation-translation, rotation-rotation, and translation-rotation. It is reasonable in the case of the magnet support system to neglect the coupling terms, resulting in a diagonal stiffness matrix. In the case of a diagonal $K_J$, the stiffness matrix $K$ becomes:

$$K = \begin{bmatrix} K_{xx} & 0 & 0 & 0 & 0 & (h + d)K_{zz} \\ 0 & K_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & - (h + d)K_{zz} & K_{xx} & 0 \\ 0 & 0 & 0 & 0 & - (h + d)K_{zz} & K_{yy} \\ (h + d)K_{zz} & 0 & 0 & 0 & 0 & K_{yy} \\ (h + d)K_{zz} & 0 & 0 & 0 & 0 & K_{zz} + (h + d)^2 K_{zz} \end{bmatrix}$$  \hspace{1cm} (4)

Three methods were considered for determining the stiffness and damping of the support mechanism, that is $K_J$ and $C_J$. The methods, merits, and results for each are discussed below.

“Constitutive” FE Modeling

Virtual single-axis testing can be used in FE modeling to estimate the support stiffness. This sub-modeling approach is “constitutive” in the sense that all parts and interfaces are modeled. Allows all of the interfaces to be captured, including the threads and the spherical washer geometry. An example of a support post similar to those used in the MAX IV design [10] and one of designs considered for the APS-U is shown in Figure 4.

![Figure 4: Views showing a) a cutaway of the MAX IV support post, and b) the FEM of the same part. The parts are: 1-post, 2-adjusting nut, 3-spherical washer set, 4-spacer, and 5-sliding plate. The dark areas in view b) are the interfaces and have high mesh density.](image)

![Figure 5: Chart showing the results from three different virtual stiffness tests compared the stiffness value determined from an experiment.](image)
displacement probed. Results for the lateral stiffness are shown in Figure 5. The experimental measurement is considered to be the target stiffness. It can be seen that the solid model, one where the interfaces are not modelled, over predicts the stiffness (steeper slope). Whereas it can be seen that in the frictional model the stiffness is dependent on the number of frictional interfaces. Not shown here are the dependence of the stiffness upon element size for the solid model and upon friction coefficient and preload for the frictional model. In short, this result confirms the over-prediction of a solid model (regardless of element size). The constitutive model presents the opposite problem of under prediction. Other than a negative correlation to the number of frictional interfaces and a positive correlation to friction level, this under prediction is not readily understood. The conclusion is that while convenient to implement, it would not be possible to use this method to estimate support stiffness without prior measurement for calibration.

**Static Testing**

Conventional static stiffness testing was investigated. Figure 6. Fixturing was designed to allow axial and lateral stiffness measurements. For each type of measurement, preload equal to the weight force from the magnet assembly was applied. Then, an Intron machine was used to apply the load and record the displacements.

![Figure 6: A post-type support installed in an Intron testing machine for lateral stiffness testing.](image)

This type of testing is typically limited to linear measurements. It was not possible to measure torsional stiffnesses in this manner. The vertical and transverse stiffness ($K_{xx}$, $K_{yy}$, and $K_{zz}$) obtained appeared reasonable in form. However, when used in a FE model and the resulting system dynamics did not match the experiment.

**Dynamic Testing**

The dynamic test method involves constructing a test setup similar to that shown in Figure 3. The practical realization of this is shown in Figure 7. In the dynamic testing a single support is used and a test mass is installed on top of the support. Equation (1) describes the system and the stiffness matrix is as in equation (4). Both large (~2900 kg) and small (~120 kg) test masses were used. The test mass is symmetric and the support is known to be centered under the mass because it was balanced. A hammer with integral force transducer is used to impact the assembly and the response is measured at multiple points with a set of triaxial accelerometers.

Dynamic testing has a few advantages over the constitutive FE and static testing approaches. First, the whole stiffness matrix can be determined if desired. Whereas the FE and static approaches require distinct test for each stiffness value. The dynamic approach can use one set of measurements to extract all of the stiffnesses. There are two basic ways in doing dynamic testing, a modal approach and a frequency response function-based (FRF) approach [11].

![Figure 7: Dynamic testing setup for wedge jack-type support (Unisorb RK-IV is shown). 1-jack and base, 2-accelerometers (hammer on table), and 3-test mass.](image)

The modal approach was used for this work. It is important to choose the test mass such that it is rigid in the frequency range of interest. Another important consideration is that the base of the isolator is “fixed”. Alternatively, this test could be performed with free boundary conditions. In the modal approach an experimental modal analysis (EMA) was conducted and the six rigid body modes were identified. The natural frequencies, mode shapes, and damping were then used along with the equations of motions (1), with diagonal stiffness matrix (4), in a Matlab script to solve for the support stiffnesses. As a check for self-consistency, a FE model of the test was built using the identified stiffness data and a modal analysis was run. The model was found to be in agreement with the experiment.

**APPLICATION OF PROPERTY DATA**

**Property Usage**

Regardless of how the support properties are obtained these data need to be incorporated into models of the complete support system to be useful. This overall model could be analytical, or as in this paper, a FE model. In the case of the FE model, each type of FE software typically has discrete stiffness elements. Through the use of discrete stiffness elements, the stiffness matrix can be entered directly. In this manner, a simple testing method can be used to generate data that can then be used in a variety of magnet support structures to investigate and predict performance. It is important to note that this same analysis mode could not
**Model Validation**

The method and data were validated by comparing the results from a free boundary condition, experimental modal analysis conducted on a prototype magnet support system, shown in Figure 8. Free boundary conditions are the easiest to create.

![Figure 8: Setup for the free boundary condition, EMA conducted on the ~10000 kg APS-U magnet support prototype. The whole assembly was hung from an overhead crane. The support dynamics were sufficiently lower in frequency than the structural dynamics such that they did not confound the identification of these dynamics.](#)

The test was conducted in a manner similar to the single-support test. A large instrumented hammer was used to excite the structure and the response was measured in multiple locations on the structure. Commercial modal analysis software was used to identify and visualize the natural frequencies and mode shapes.

A FE model of the test setup was also constructed in ANSYS. Discrete stiffness elements were used to incorporate the support stiffness identified in the single-support test. A modal analysis was run with free boundary conditions. The resulting natural frequencies and modes were then compared to the natural frequencies and modes from the EMA as shown in Table 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>EMA</th>
<th>FEA</th>
<th>% diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64 Hz</td>
<td>67 Hz</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>91 Hz</td>
<td>90 Hz</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>130 Hz</td>
<td>122 Hz</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>137 Hz</td>
<td>137 Hz</td>
<td>0</td>
</tr>
</tbody>
</table>

It can be seen that the results from the experiment and finite element model compare well. The jack stiffness can also be incorporated in other support structure models for which tests have not been done or are not planned.

**CONCLUSION**

We have developed a testing and modeling process that “closes the loop” on the design-analysis-testing workflow as shown in Figure 1. Establishing this process provides confidence in simulation results and enables the exploration of many different designs using the same components.

**ACKNOWLEDGMENT**

The authors would like to acknowledge the assistance of Josh Abraham and the Argonne riggers and Bruce Hoster and the APS machine shop. In addition, we thank Dr. Sheldon Mustovsky at the Illinois Institute of Technology for his assistance with static testing.

**REFERENCES**


