Periodic transient beam loading effect in triple radio-frequency systems

Jincheng Xiao , Tianlong He *, Weimin Li†

* htlong@ustc.edu.cn

† lwm@ustc.edu

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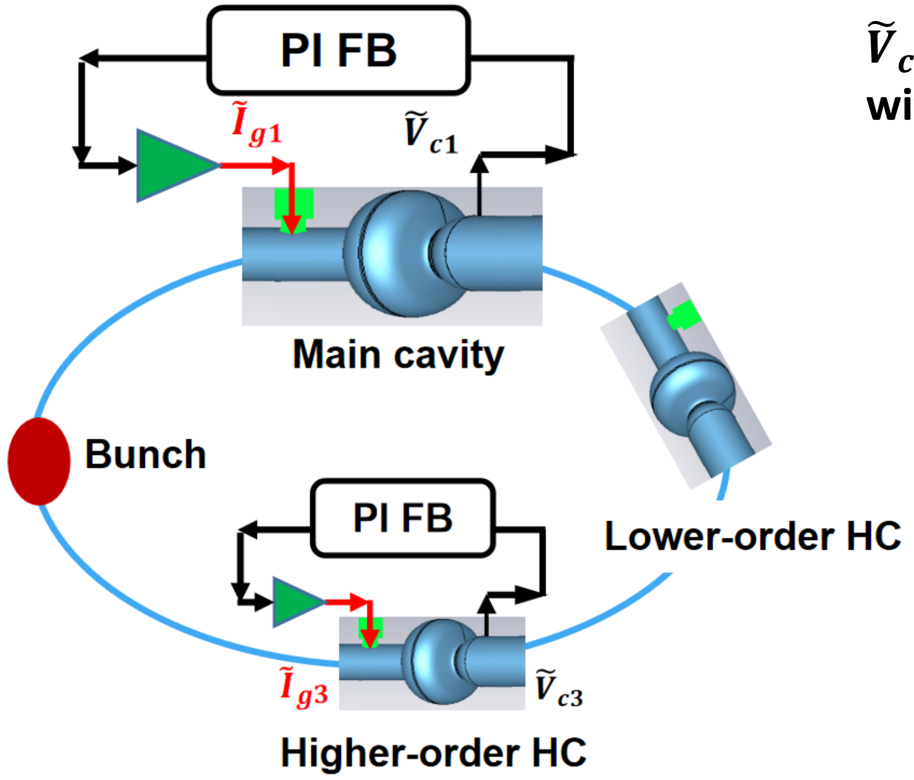
Outline

- Introduction to The Triple RF System
- Unexpected PTBL Effect
- PTBL Threshold Study for HALF

1. Introduction – Triple RF System



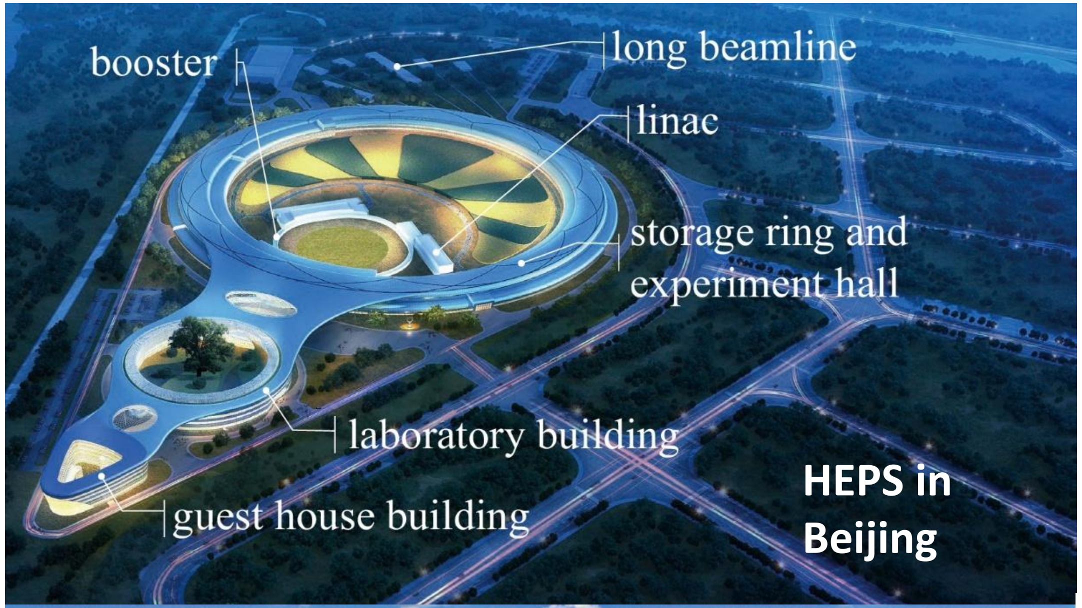
$\tilde{V}_{c1}$, typically an active cavity with negative detuning.

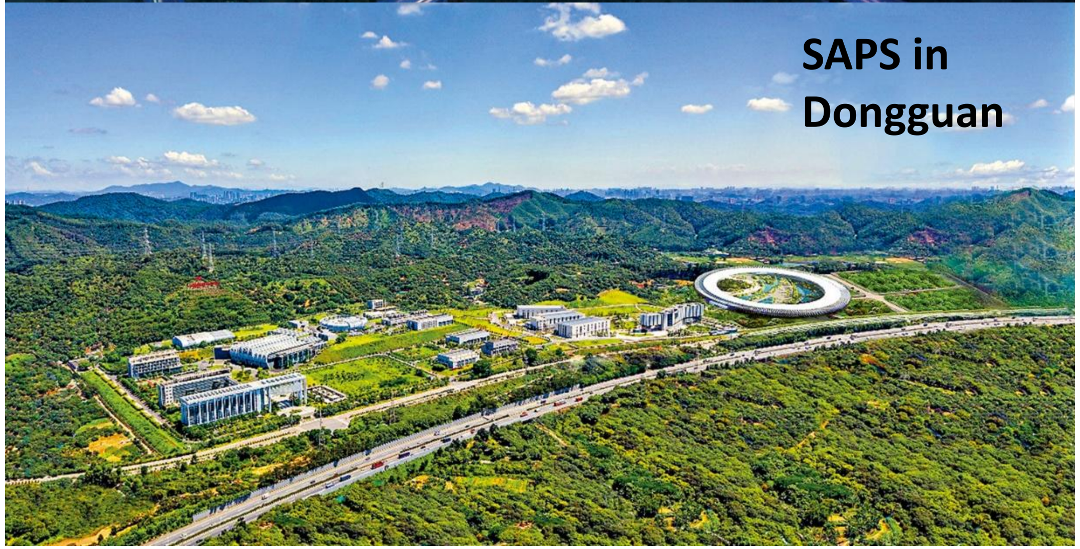
$\tilde{V}_{c2}$, both active and passive cavities are applicable, with positive detuning.

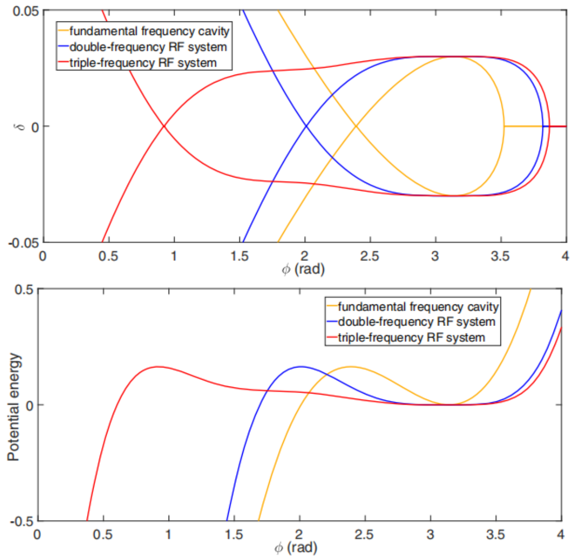
$$V_T = V_{mc}\sin(\omega_{rf}t + \phi_s) + k_1 V_{mc}\sin(n_1\omega_{rf}t + \phi_{hc1}) + k_2 V_{mc}\sin(n_2\omega_{rf}t + \phi_{hc2})$$

$\tilde{V}_{c3}$, typically an active cavity with negative detuning.

1. Introduction – Triple RF System – for Longitudinal Injection

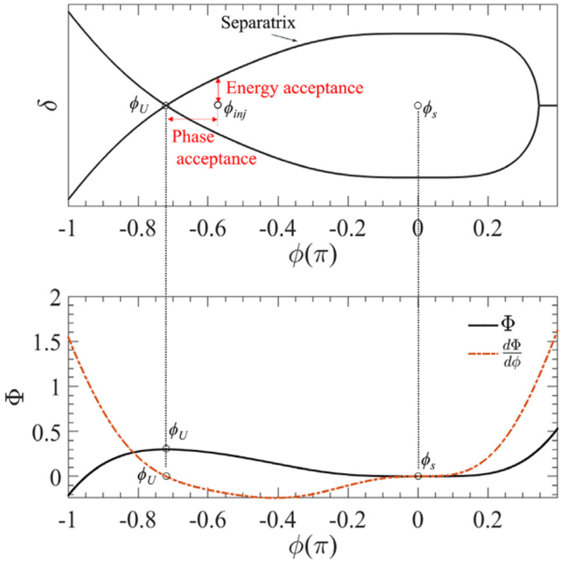






S. Jiang, G. Xu, PRAB, 2018, 21, 110701.

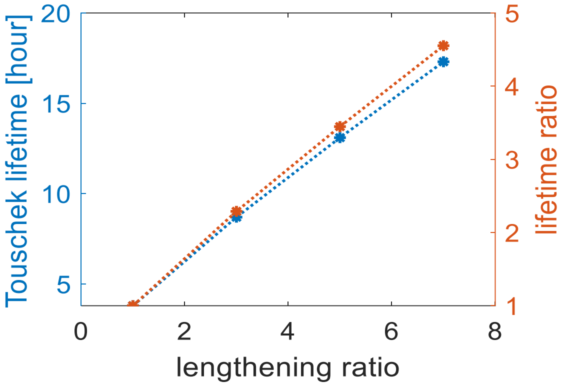
Longitudinal phase space & RF potential

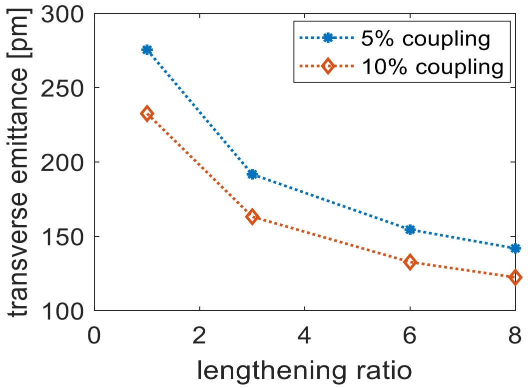


W. Liu, et al., NIMA, 2023, 1046, 167712.

1. Introduction – Triple RF System – for Longer Bunch Lengthening

Touschek lifetime and transverse emittance vs. lengthening ratio

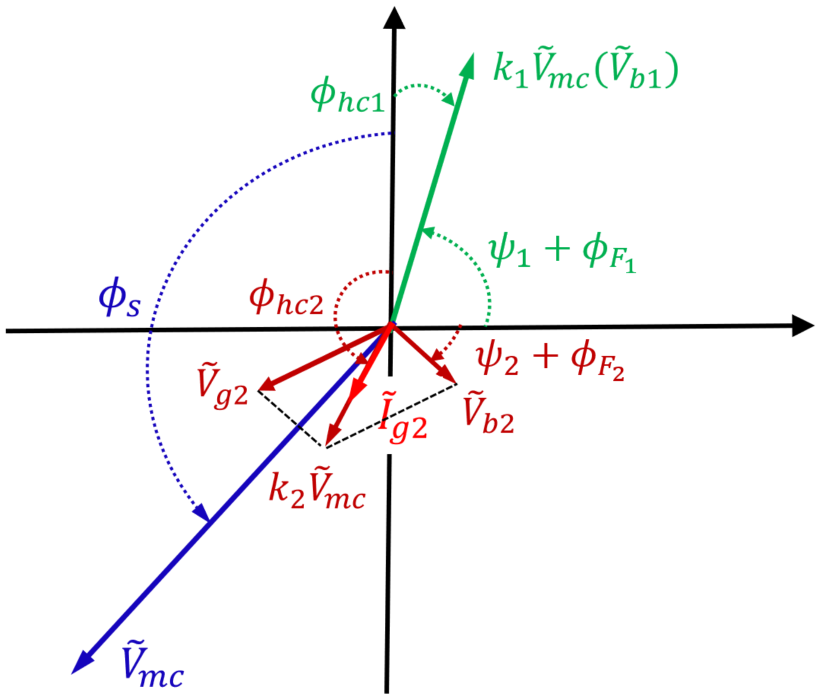




Optimum bunch lengthening conditions*

$$
\begin{cases}
\sin\phi_s = \dfrac{n_1^2 n_2^2}{a_{n_1 n_2}} \sin\phi_{s0} \\
k_1 = -\dfrac{1}{n_1}\sqrt{\dfrac{a_{n_1 n_2}\left(n_2^2-1\right) - n_1^2 n_2^4 \sin^2\phi_{s0}}{\left(n_1^2-1\right)\left(n_1^2-n_2^2\right)^2}} \\
k_2 = -\dfrac{1}{n_2}\sqrt{\dfrac{a_{n_1 n_2}\left(n_1^2-1\right) - n_2^2 n_1^4 \sin^2\phi_{s0}}{\left(n_2^2-1\right)\left(n_2^2-n_1^2\right)^2}} \\
\tan\phi_{hc1} = \dfrac{\operatorname{sgn}(n_1-n_2) n_1 n_2^2 \sin\phi_{s0}}{\sqrt{a_{n_1 n_2}^2 - n_1^4 n_2^4 \sin^2\phi_{s0}}} \\
\tan\phi_{hc2} = \dfrac{\operatorname{sgn}(n_1-n_2) n_1^2 n_2 \sin\phi_{s0}}{\sqrt{a_{n_1 n_2}^2 - n_1^4 n_2^4 \sin^2\phi_{s0}}}
\end{cases}
$$

Phasor diagram

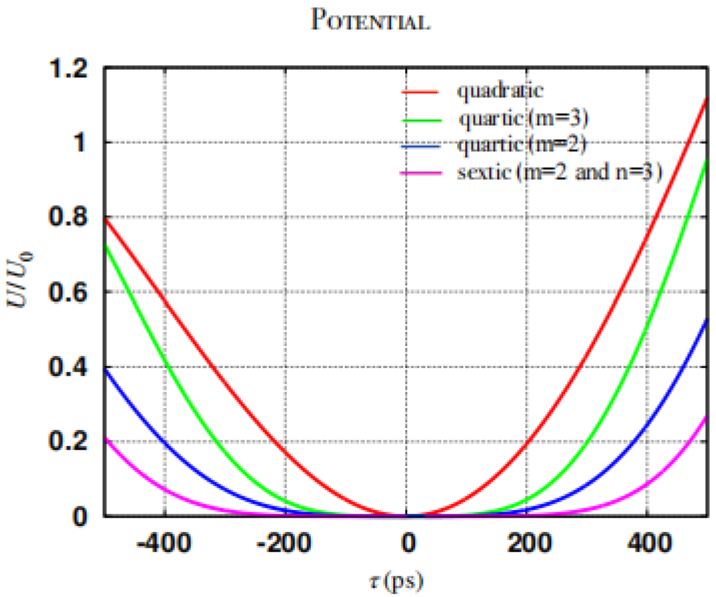


◆ **The more the bunch is lengthened, the better the performance.**

*G. Bassi, et al., in *IPAC'24*, 2949-2952, THBD2.

1. Introduction – Triple RF System – for Longer Bunch Lengthening

Total RF potential from superimposed HCs (NSLS-II U)



G. Bassi, et al., in *IPAC'24*, 2949-2952, THBD2.

The MAX-IV triple RF scheme

Åke Andersson, et al., at iFAST Low Emittance Rings workshop,2024.

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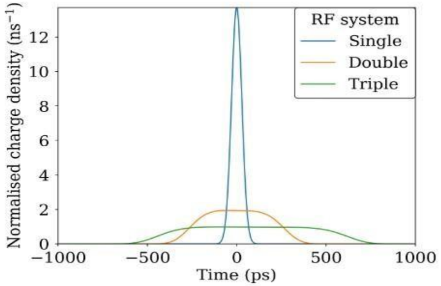


◆ Theoretically, the triple RF system can double the lengthening capability of the double RF system.

1. Introduction – Triple RF System – HALF

Table 1
Main parameters of HALF.

Parameter	Value
Energy	2.2 GeV
Circumference	479.86 m
Current	350 mA
Momentum compaction	9.4×10^{-5}
Harmonic number	800
Radiation energy loss per turn	400 keV
RMS energy spread	7.44×10^{-4}
Nominal bunch charge (80% filling)	0.875 nC
499.8 MHz MC R/Q	45 Ω
499.8 MHz MC loaded Q	1×10^5
3rd HC R/Q	39 Ω
3rd HC loaded Q	2×10^8
MC voltage amplitude	1200 kV
MC voltage phase (FP)	158.0 deg
HC voltage amplitude (FP)	374.2 kV
HC voltage phase (FP)	−7.68 deg

Current passive superconducting double RF system

VS.

TABLE I. Main parameters of HALF.

Parameter	Symbol	Value
Ring circumference	C	480 m
Beam energy	E_0	2.2 GeV
Nominal beam current	I_0	350 mA
Longitudinal damping time	τ_z	14 ms
Momentum compaction	α_c	9.4×10^{-5}
Natural energy spread	σ_δ	7.4×10^{-4}
Harmonic number	h	800
Energy loss per turn	U_0	400 keV
Voltage of MC	V_{mc}	1.2 MeV
Natural rms bunch length	σ_τ	7 ps
Harmonic number of 3HC	n_1	3
Harmonic number of 5HC	n_2	5
Quality factor of 3HC	Q_1	1.68×10^5
Quality factor of 5HC	Q_2	1×10^5
R/Q of 3HC	R_1/Q_1	40 Ω
R/Q of 5HC	R_2/Q_2	40 Ω

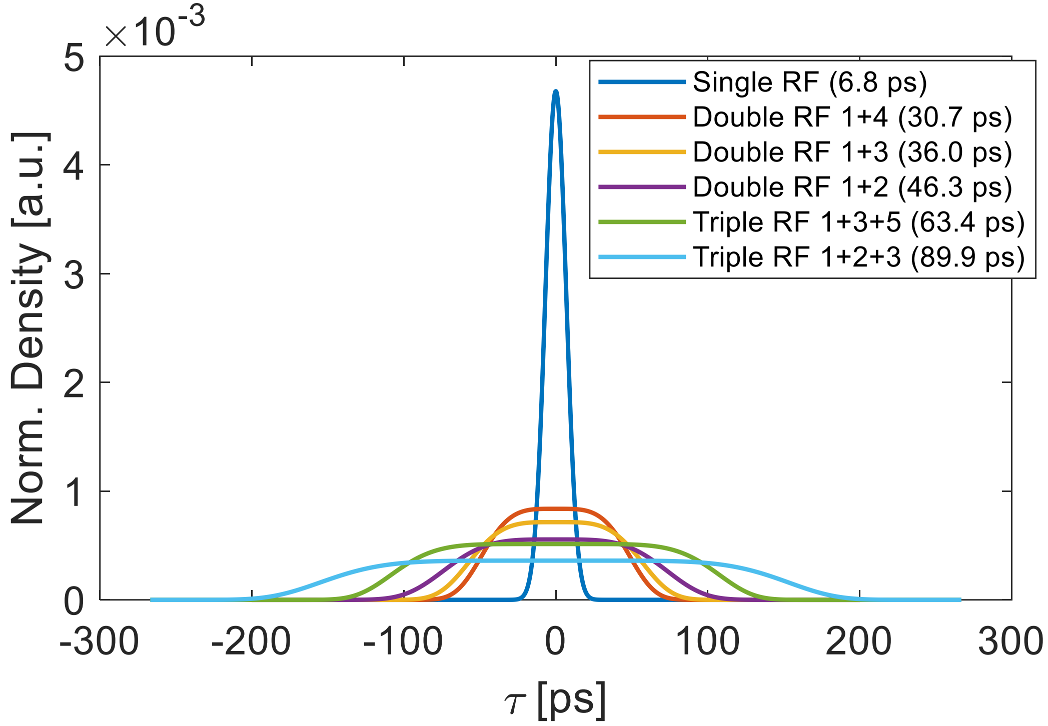
Pre-designed active superconducting triple RF System

1. Introduction – Triple RF System – HALF

Active HC

RF scheme	RMS length [ps]	Lengthening ratio
Single RF system 1	6.8	1
Double RF system 1,4	30.7	4.5
Double RF system 1,3	36.0	5.3
Double RF system 1,2	46.3	6.8
Triple RF system 1,3,5	63.4	9.3
Triple RF system 1,2,3	89.9	13.2

500 MHz + 1.5GHz + 2.5 GHz

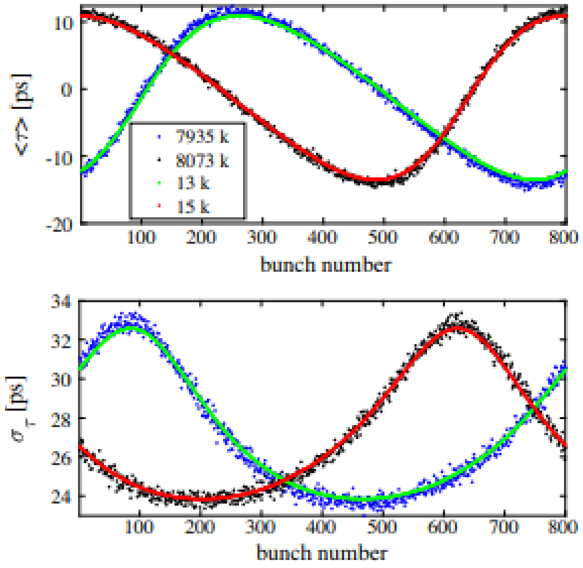


- Under optimum lengthening conditions, the theoretical RMS bunch length reaches 63.4 ps, which is 2.3 times longer than that achieved by the current passive double RF system.

2. Unexpected PTBL Effect – PTBL (Also Called Mode 1 Instability) in Double RF System

- **Characteristics: The bunch centroid position and length exhibit slow, periodic oscillations with a sinusoidal-like pattern that grow over time, typically requiring hundreds of thousands of turns to become observable.**

T.He,et al.,PRAB 25,024401(2022)



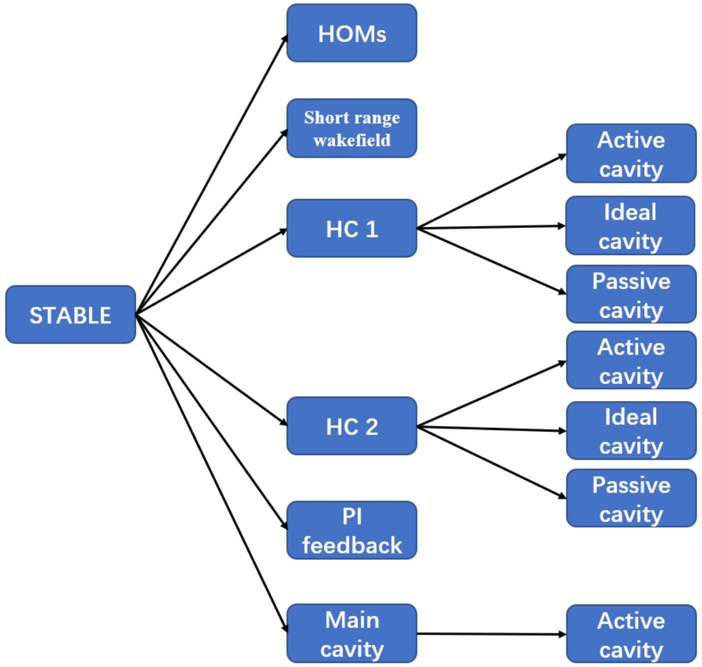
Overall bunch lengthening performance degradation

Date	Type	References
2018	Theory	M. Venturini, PRAB 21, 114404.
2022	Calculation	T. He, et al., PRAB 25, 024401.
2022	Calculation	T. Olsson, in *Proceedings of 3rd Workshop on Low Emittance Lattice Design.*
2022	Calculation	A. Gamelin, in *Proceedings of iFAST Workshop.*
2022	Calculation	F. J. Cullinan, et al., in *Proceedings of I. FAST Workshop.*
2024	Experiment	F. J. Cullinan, et al., PRAB 27, 044403.
2025	Calculation	A. Gamelin, et al., PRAB 28, 054401.
2025	Theory	M. B. Alves, et al., PRAB 28, 034401.

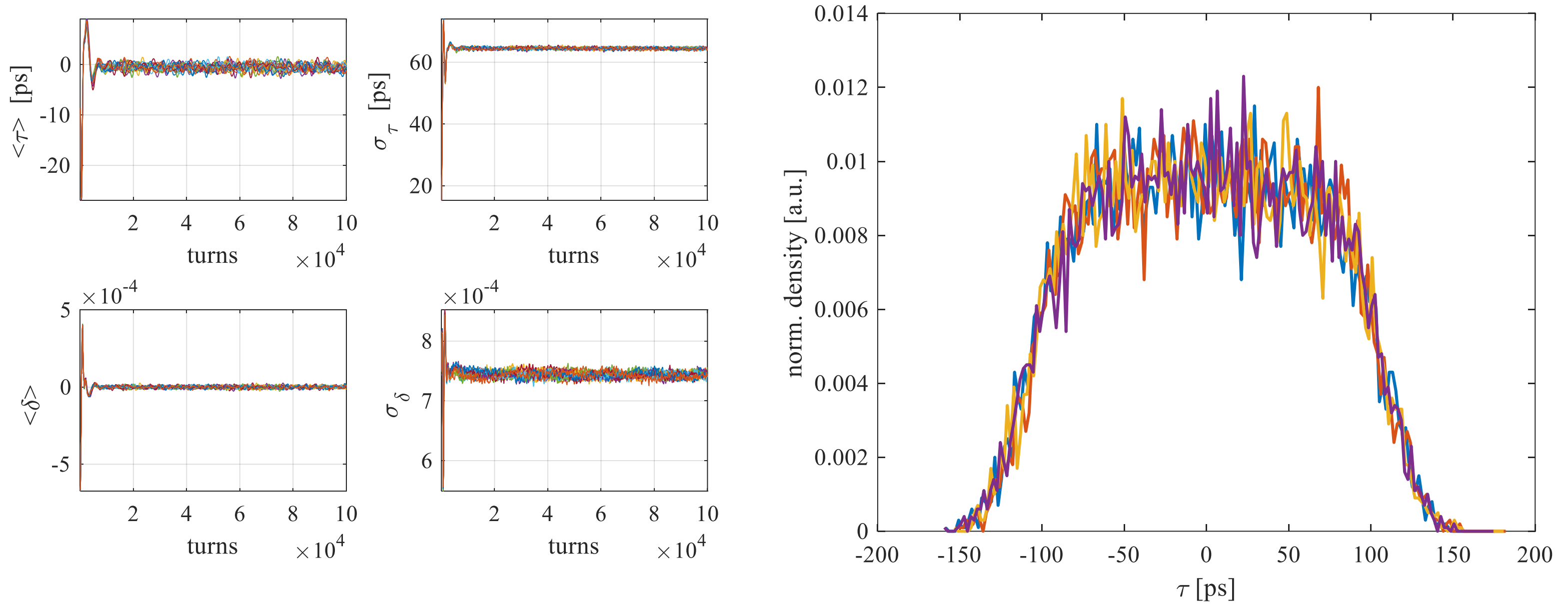
- **Although extensive research exists on the PTBL effect, its underlying mechanism and effective mitigation strategies remain incompletely understood, representing a significant area for further study.**

2. Unexpected PTBL Effect – Tracking Simulation

Tracking Simulation Code-STABLE (MATLAB)



STABLE module design



Result @ $I_0 = 45$ mA, 800 bunches, 10,000 particles per bunch, tracking 100,000 turns takes 748 s.

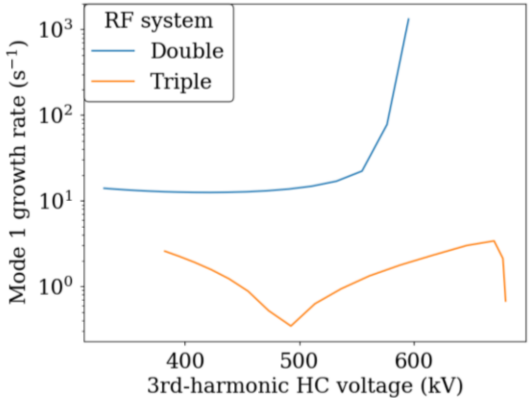
- STABLE utilizes GPU acceleration to enable significantly faster tracking of particle longitudinal motion in multi-RF systems.
- Developed with a modular architecture and added new modules for higher order HC, ideal cavities, and PI feedback control.

2. Unexpected PTBL Effect – More Severe Instability

- Contrary to the expectation that a 5HC would suppress the 3HC-driven PTBL.
- Manifested as larger transient bunch distributions in the saturated state and a higher growth rate.

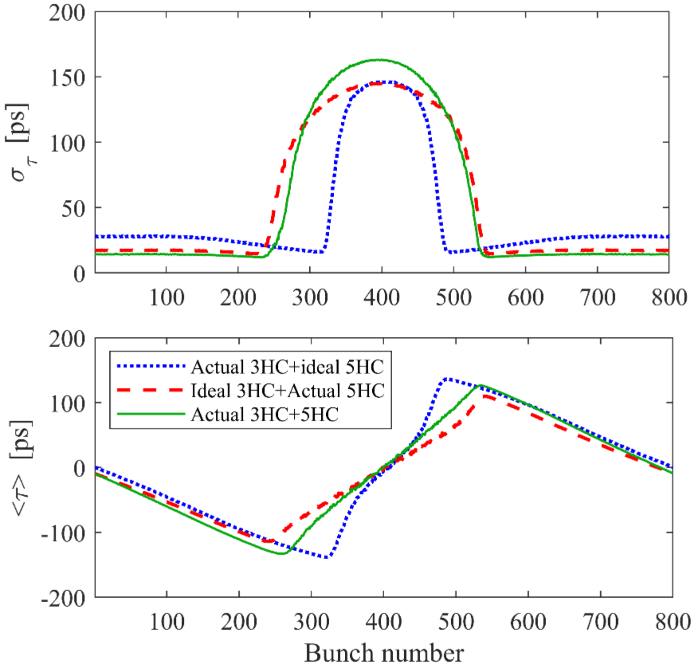
Mode 1 stability

- Impedance of 5th-harmonic cavity damps mode 1

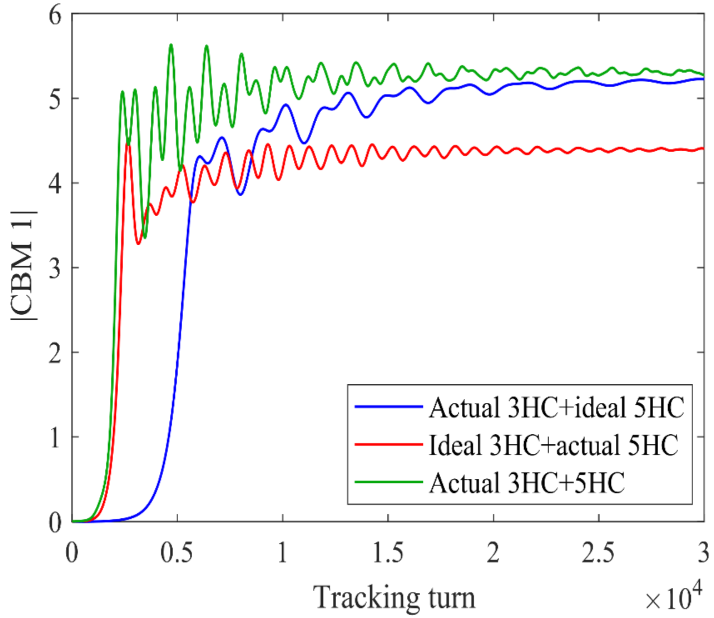


F. J. Cullinan, HarmonLIP, MAX IV, Lund, Sweden, October 2022

5HC damps mode 1 expectedly



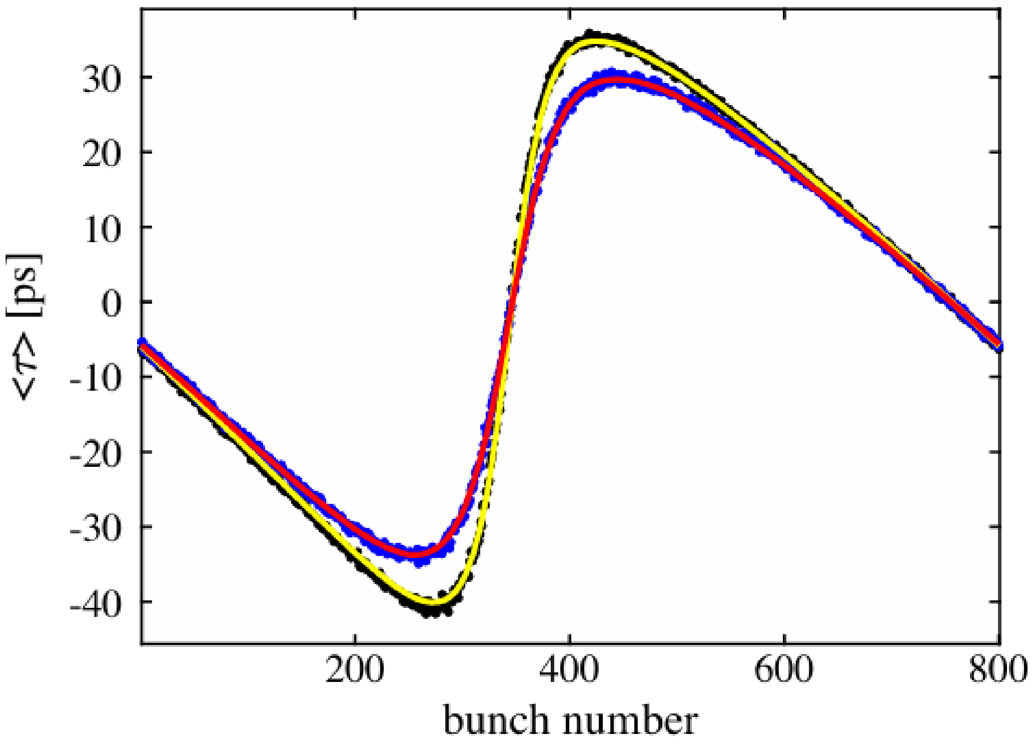
Tracking simulation results



Growth rate modulus analysis

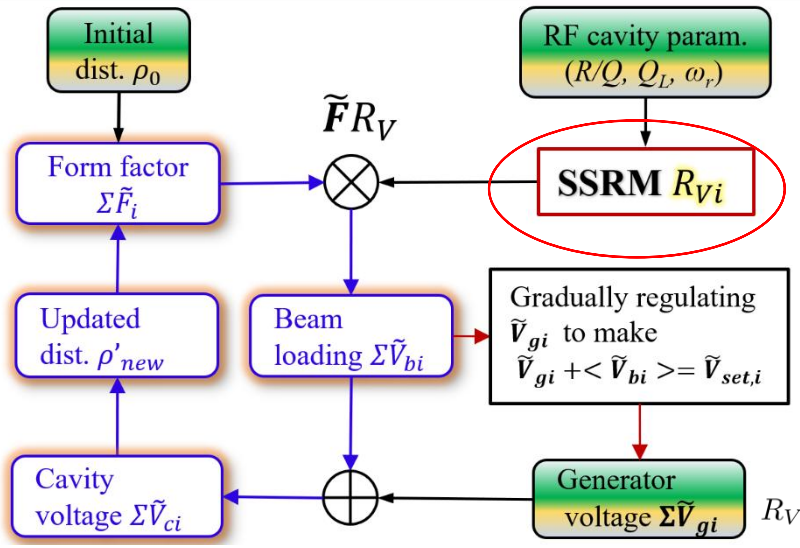
2. Unexpected PTBL Effect – Semi-Analytical Algorithm

- **For PTBL effect in double RF system, semi-analytical algorithm yields results consistent with tracking simulation.**



T.He,et al.,PRAB 25,024401(2022)

Schematic of the improved semi-analytical algorithm



Steady-state response matrix of RF cavity voltage

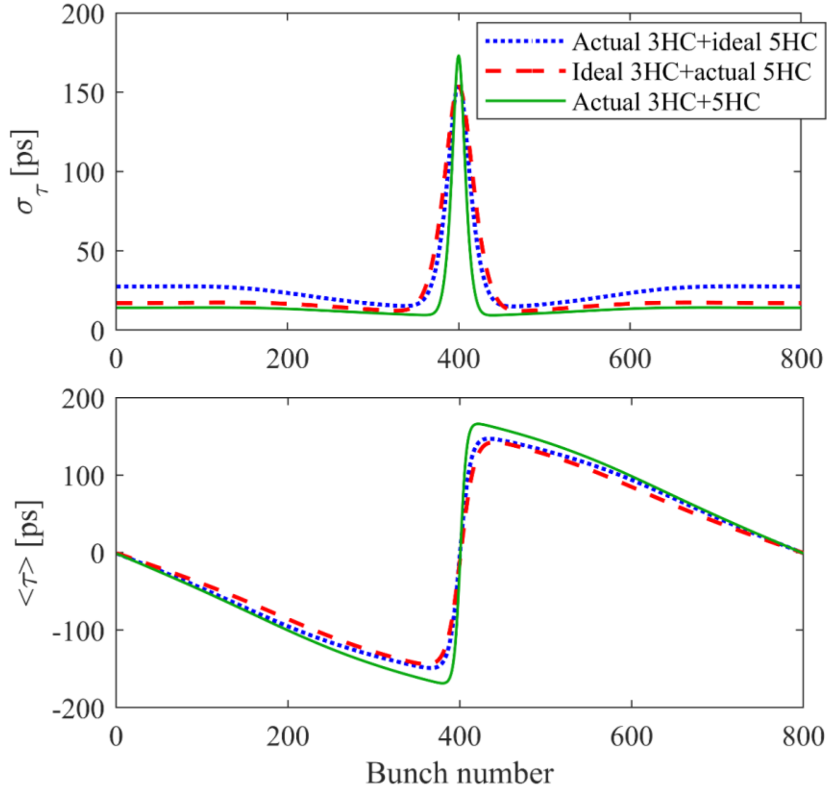
$$R_V = \begin{bmatrix} V_0 e^{h\phi_b} & V_0 e^{\phi_b} & \cdots & V_0 e^{(h-1)\phi_b} \\ V_0 e^{(h-1)\phi_b} & V_0 e^{h\phi_b} & \cdots & V_0 e^{(h-2)\phi_b} \\ \vdots & \vdots & \ddots & \vdots \\ V_0 e^{\phi_b} & V_0 e^{2\phi_b} & \cdots & V_0 e^{h\phi_b} \end{bmatrix}$$

$$\tilde{V}_{b,i}|_l = -2k\tilde{F}_{n,l} q_l e^{j\delta_{i,l} - \tau_{i,l}} \sum_{N=0}^{\infty} e^{(j\omega_r - \frac{\omega_r}{2Q_L})NT_0} = \frac{-2k\tilde{F}_{n,l} q_l}{1 - e^{(j\omega_r - \frac{\omega_r}{2Q_L})T_0}} \times e^{(j\omega_r - \frac{\omega_r}{2Q_L})\Delta t_{i,l}}$$

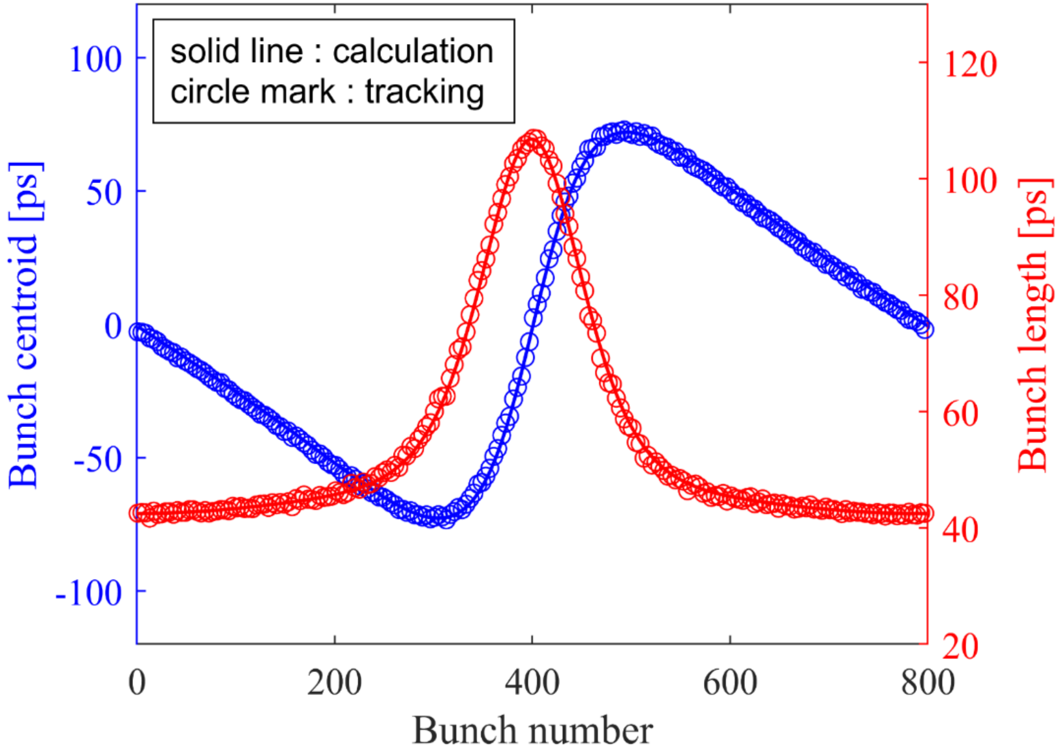
$$\boldsymbol{V_b} = \tilde{\boldsymbol{F}} R_V, \quad \text{where} \quad \tilde{\boldsymbol{F}} = (q_1 \tilde{F}_1, \ldots q_h \tilde{F}_h)$$

T. Olsson, et al.,PRAB, 2018, 21(12): 120701.

2. Unexpected PTBL Effect – Semi-Analytical Algorithm



Results @$I_0 = 350$ mA

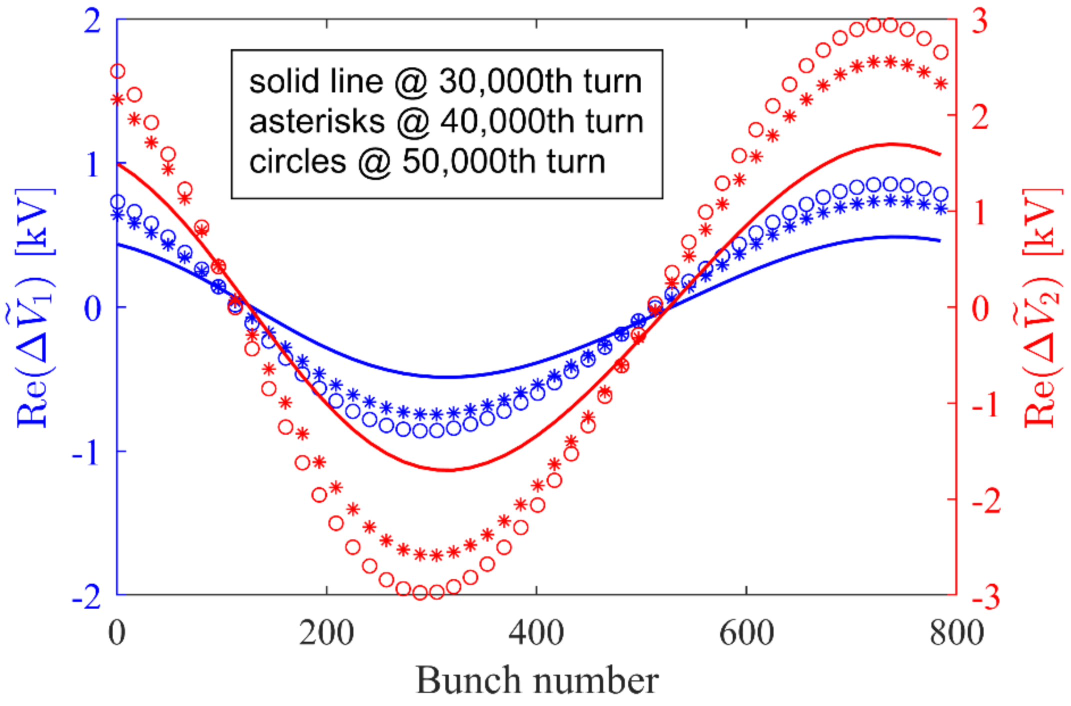


Benchmark test @$I_0 = 55$mA

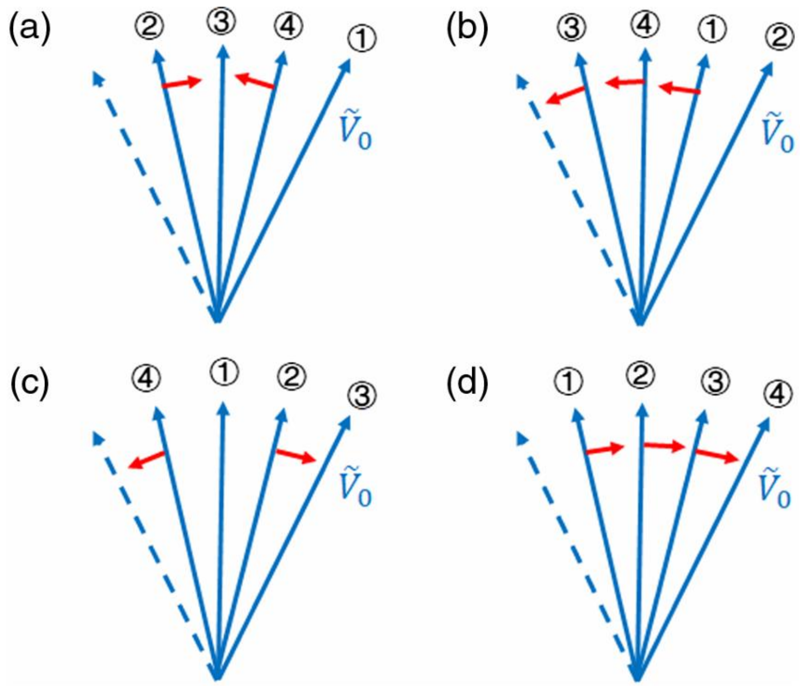
- It is reasonable to infer that when a bunch splits rapidly into two sub-bunches due to the strong PTBL effect, the particles within these sub-bunches may not easily merge in subsequent tracking turns.
- Both methods show that PTBL threshold current is only 50 mA!

2. Unexpected PTBL Effect – Theoretical Analysis

◆ Tracking simulations reveal that the real-part perturbations of the cavity voltages in the two HCs remain inphase throughout.



Real-part perturbations at different tracking turns



T.He, PRAB 25, 094402 (2022)

Analysis of the most-distorted bunches in centroid perturbation

◆ An analysis using this method and considering only centroid perturbations would suggest that the real-part cavity voltage perturbations are antiphase.

2. Unexpected PTBL Effect – Theoretical Analysis

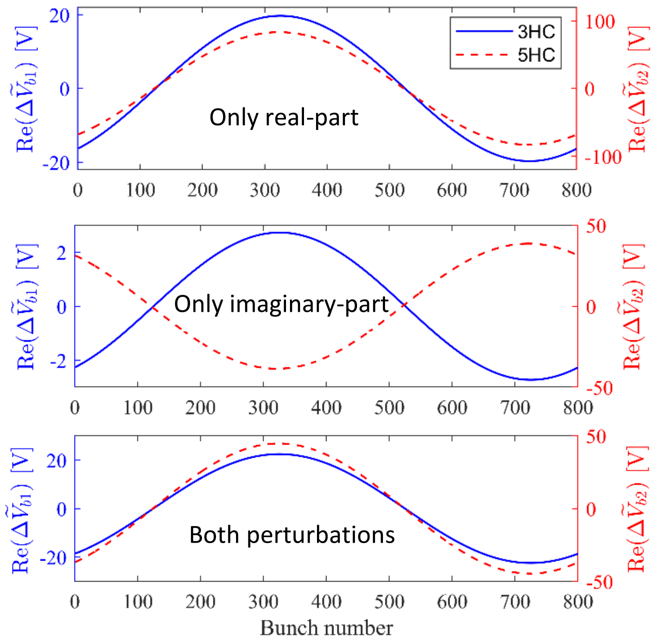
◆ **Both centroid and bunch length perturbations are considered[1,2,3].**

For a symmetric Gaussian bunch

$$\begin{cases} \tilde{F} \cong (1 - j\omega_r\langle\tau\rangle)e^{-\frac{\omega_r^2\sigma_\tau^2}{2}} \\ \dfrac{\partial \mathrm{Re}(F)}{\partial\sigma_\tau} \cong -\omega_r^2\sigma_\tau e^{-\frac{\omega_r^2\sigma_\tau^2}{2}}, \dfrac{\partial \mathrm{Re}(F)}{\partial\langle\tau\rangle} \cong 0 \\ \dfrac{\partial \mathrm{Im}(F)}{\partial\langle\tau\rangle} \cong -\omega_r e^{-\frac{\omega_r^2\sigma_\tau^2}{2}}, \dfrac{\partial \mathrm{Im}(F)}{\partial\sigma_\tau} \cong 0 \end{cases}$$

SSRM helps to analyse the influence of perturbations on cavity voltage

$$\mathrm{Re}(\Delta V_{hci}) = \mathrm{Re}(\Delta\tilde{F})\mathrm{Re}(R_{Vi}) - \mathrm{Im}(\Delta\tilde{F})\mathrm{Im}(R_{Vi})$$



Impact by the real-part and imaginary-part perturbations of bunch form factor

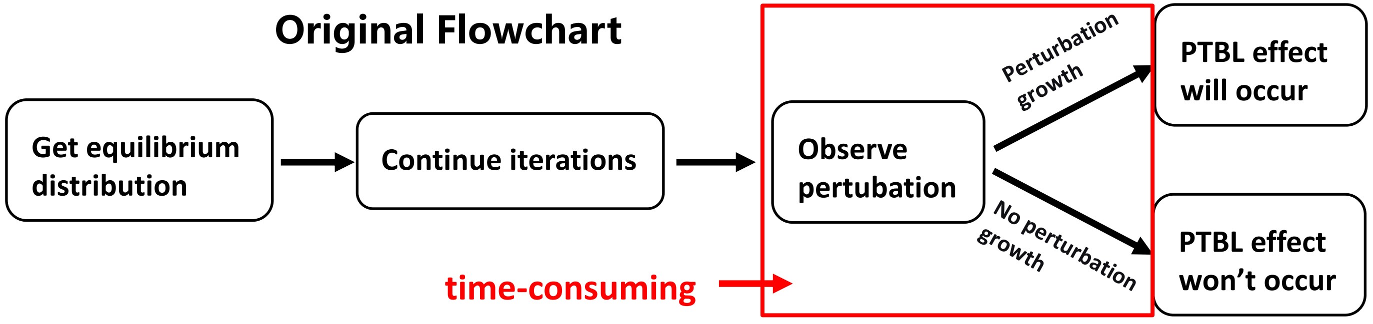
◆ **Analysis using the SSRM reveals that bunch length perturbations play a major role in the inphase relationship.**

[1] A. Gamelin, et al., PRAB, 2025, 28, 054401.
[2] M. B. Alves, PRAB, 2025, 28, 034401.
[3] F. J. Cullinan, et al., PRAB, 2024, 27, 044403

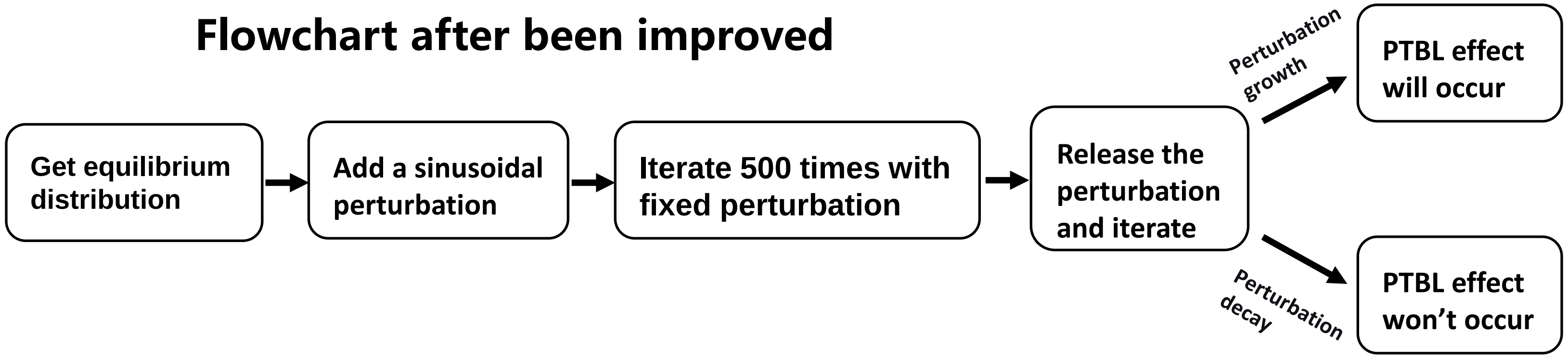
3. Threshold Study – Efficient Determination of PTBL Threshold

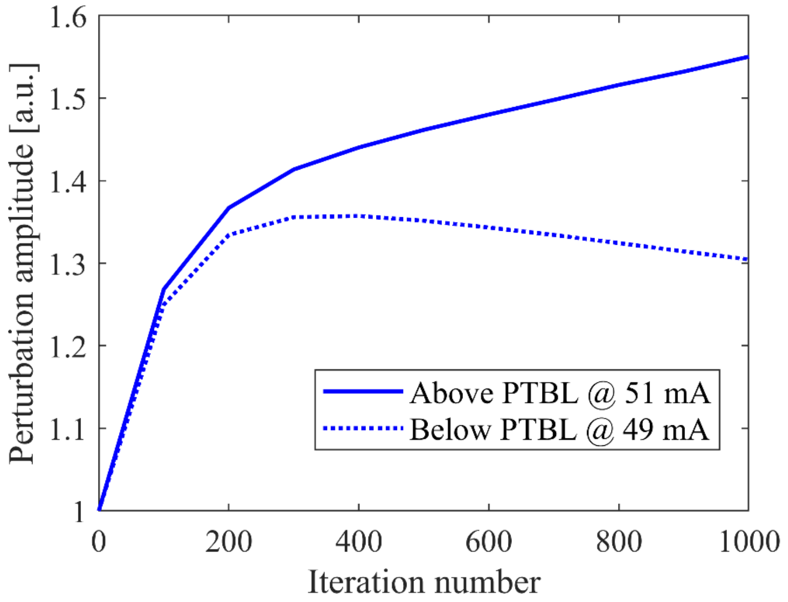
◆ Core concept: artificially introducing larger sinusoidal perturbations

Original Flowchart



Flowchart after been improved





Consistent with the results of previous two methods.

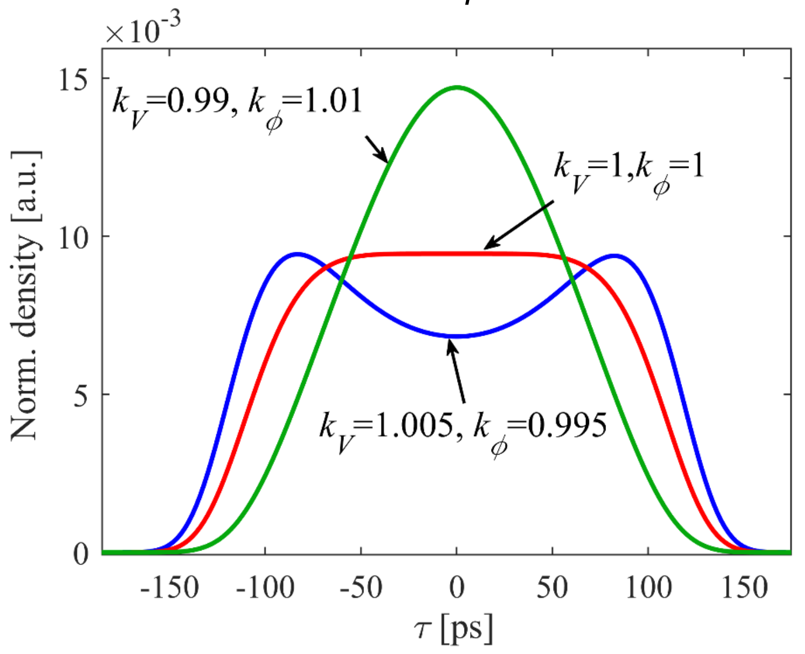
◆ Taking HALF as an example, when the tested beam current approaches the actual threshold, the improved method requires approximately 25 seconds on a conventional computer (for 1000 iterations), whereas observing the natural growth of perturbations using the original semi-analytical algorithm requires at least 5000 iterations, taking about 125 seconds.

3. Threshold Study – Symmetric Bunch Distributions with Different Lengths

Two methods

$$V_T = V_{mc}\sin(\omega_{rf}t + \phi_s) + k_V k_1 V_{mc}\sin(n_1\omega_{rf}t + k_\phi \phi_{hc1}) + k_2 V_{mc}\sin(n_2\omega_{rf}t + \phi_{hc2})$$

$$k_V + k_\phi = 2$$



Solve the following system of equations by fixing k_1 at a relatively small value

$$\sin\phi_s + k_1\sin\phi_{hc1} + k_2\sin\phi_{hc2} = U_0/eV_{mc}$$

$$\cos\phi_s + k_1 n_1\cos\phi_{hc1} + k_2 n_2\cos\phi_{hc2} = 0$$

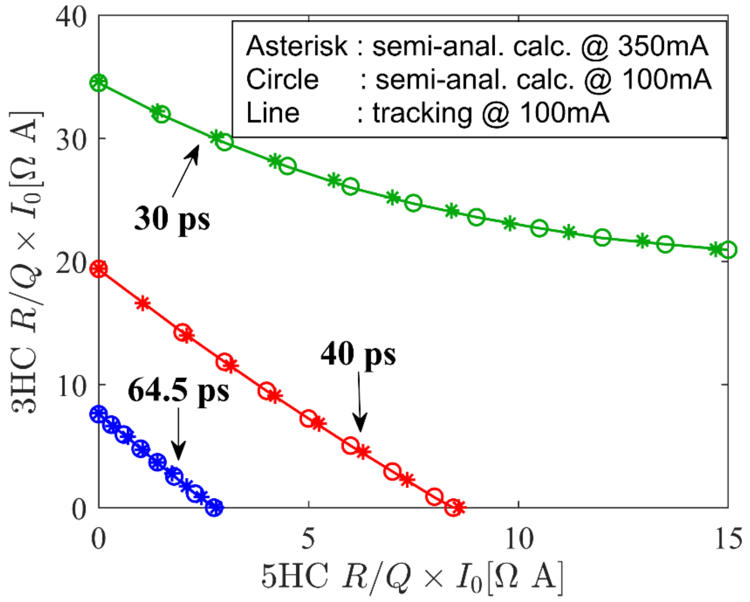
$$\sin\phi_s + k_1 n_1^2\sin\phi_{hc1} + k_2 n_2^2\sin\phi_{hc2} = 0$$

$$\cos\phi_s + k_1 n_1^3\cos\phi_{hc1} + k_2 n_2^3\cos\phi_{hc2} = 0$$

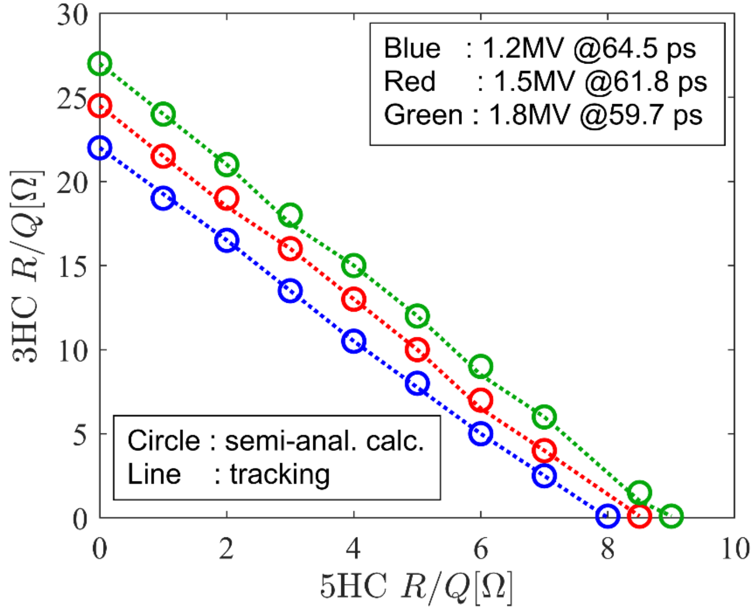
- Only shorter bunch distributions can be generated.
- HCs require lower cavity voltages at the same bunch length.

3. Threshold Study – Threshold Study for HALF

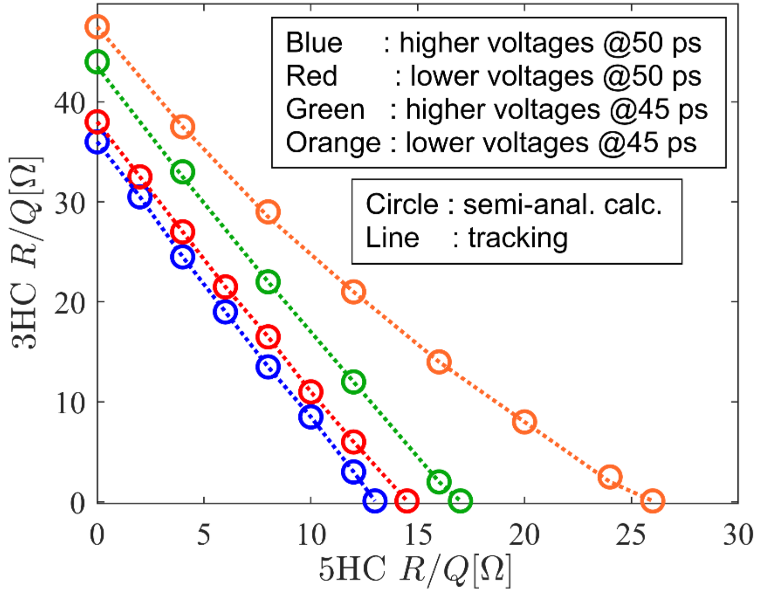
R/Q threshold @ different bunch length and beam current



R/Q threshold @different MC voltage



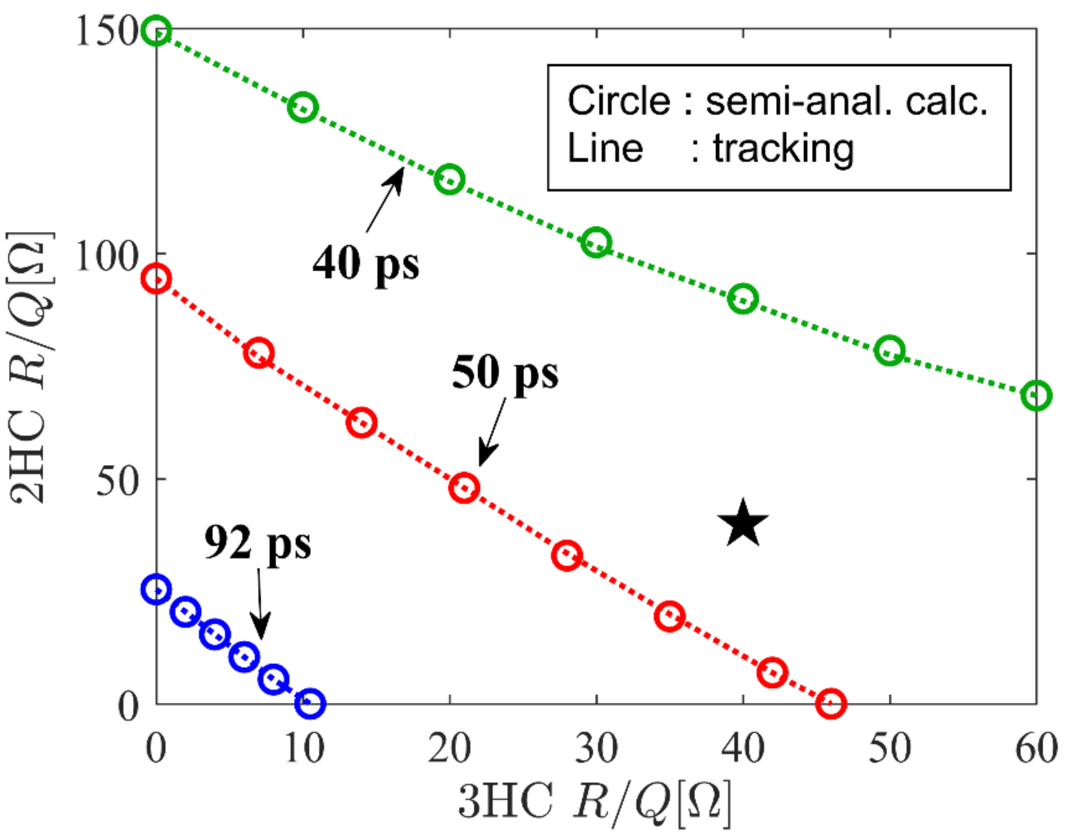
R/Q threshold @ different HC voltage



- $R/Q \times I_0$ determines PTBL threshold.
- The R/Q threshold of the lower order HC is inversely proportional to that of the higher order HC, further demonstrating two HCs jointly enhance the PTBL effect.
- A series of parametric design analyses demonstrate that, while maintaining the target beam current and bunch lengthening, adjusting other parameters doesn’t yield particularly effective improvement.

3. Threshold Study – Triple RF System @ 2HC + 3HC

R/Q threshold @ 2HC + 3HC and different bunch length



- Needed higher HC voltages.
- Longer bunch lengthening compared with 3HC + 5HC
- Higher R/Q threshold at identical bunch length compared with 3HC + 5HC
- Under current parameters (Black star), the achievable stable bunch length remains significantly short compared to the optimum lengthening case.

3. Summary

- Extended tracking simulation code STABLE to enable longitudinal beam dynamics studies in triple RF systems.
- Building on the SSRM, improved the semi-analytical algorithm and developed a new method for the efficient determination of PTBL thresholds。
- Severe PTBL effect imposes stringent requirements on the R/Q values of both HCs in the triple RF system.

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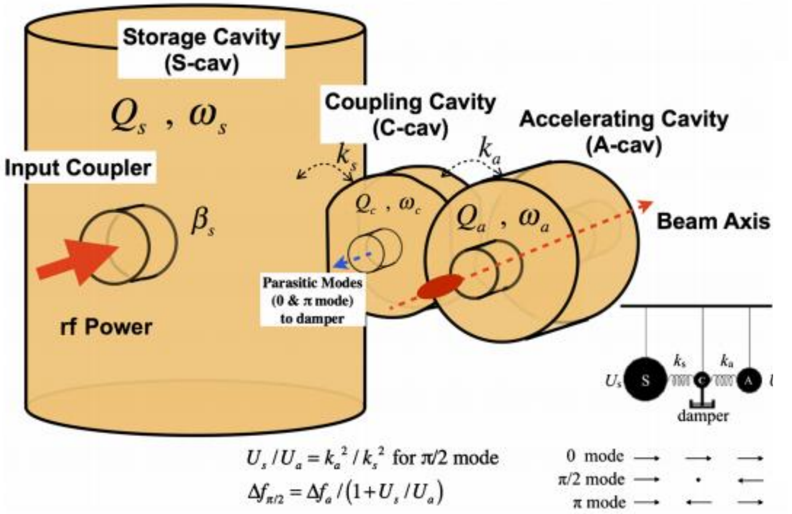
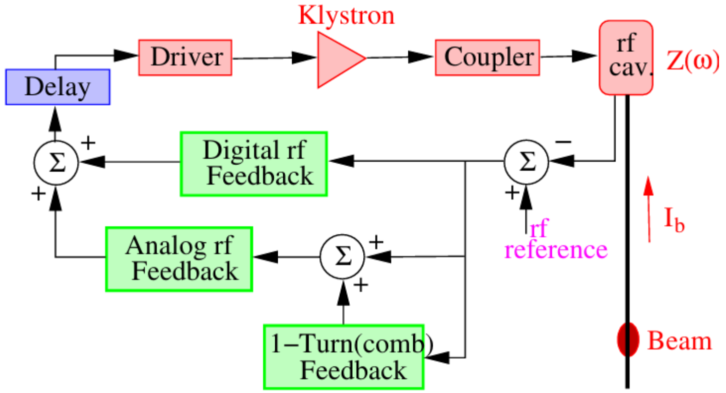
4. Future Work

ARES cavity



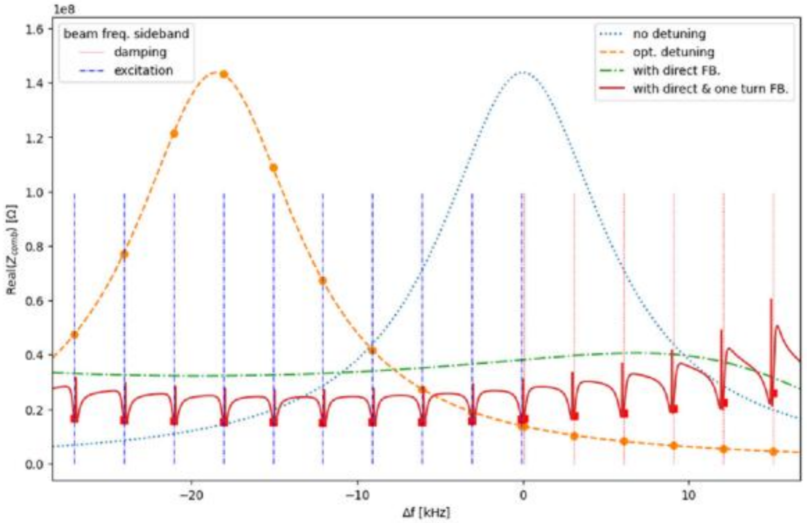
T. Kobayashi, et al., PRAB, 19, 062001 (2016)

Feedback block diagram



P. Baudrenghien, T. Mastoridis
PRAB 20, 011004 (2017)

Open-loop vs. closed-loop cavity impedance



H. Zhu, et al., Radiation Detection Technology and Methods (2023) 7:210–219

- **Research on the PTBL effect in triple RF system using room-temperature cavity scheme.**
- **Feasibility study of low-R/Q RF cavities.**
- **Exploring PTBL instability suppression via feedforward and feedback control.**

Thank you！