Physics of Storage Rings
the very basics

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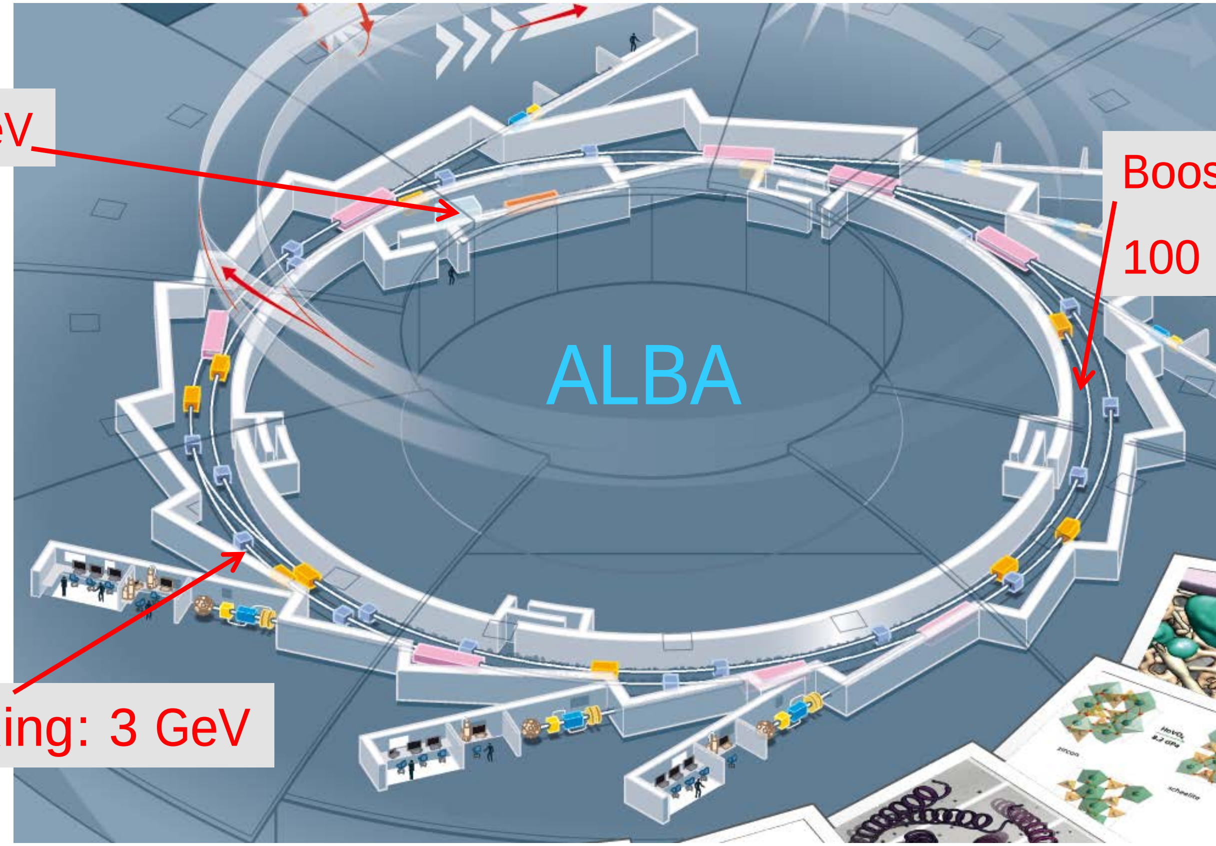
Seminars for the ALBA staff - Maxwell Auditorium - 27.10.2017

How to build a circular accelerator...



How to build a circular accelerator

How to **accelerate** charged particles?

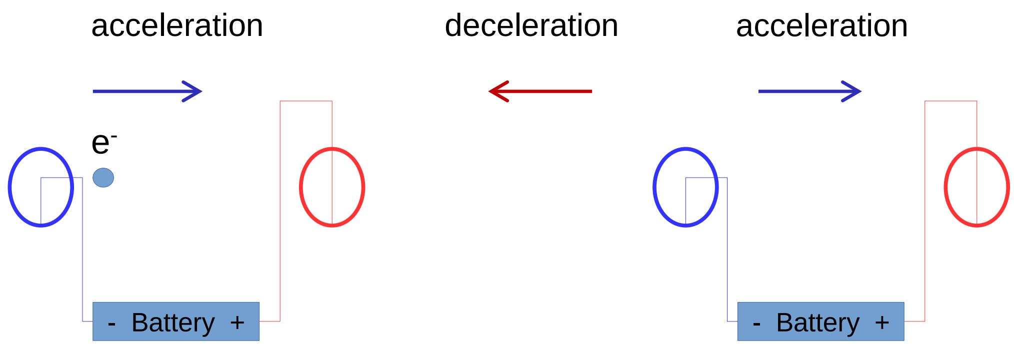
With **electric fields**

$$\boxed{\vec{F} = q\vec{E}}$$

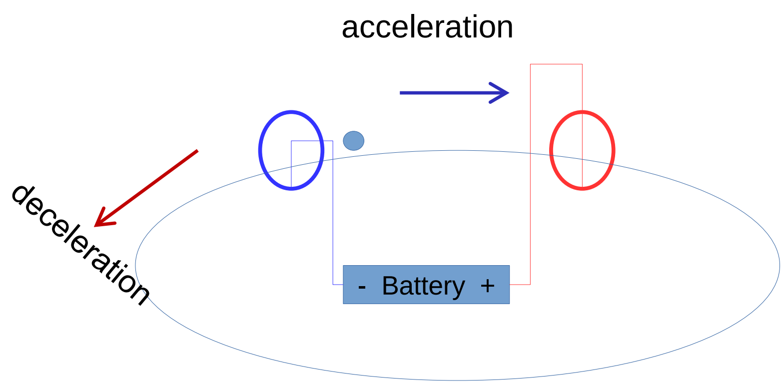
An electric field can work, but has a some limitations...

How to build a circular accelerator

Let's start with a very simple scheme: 2 batteries in a row



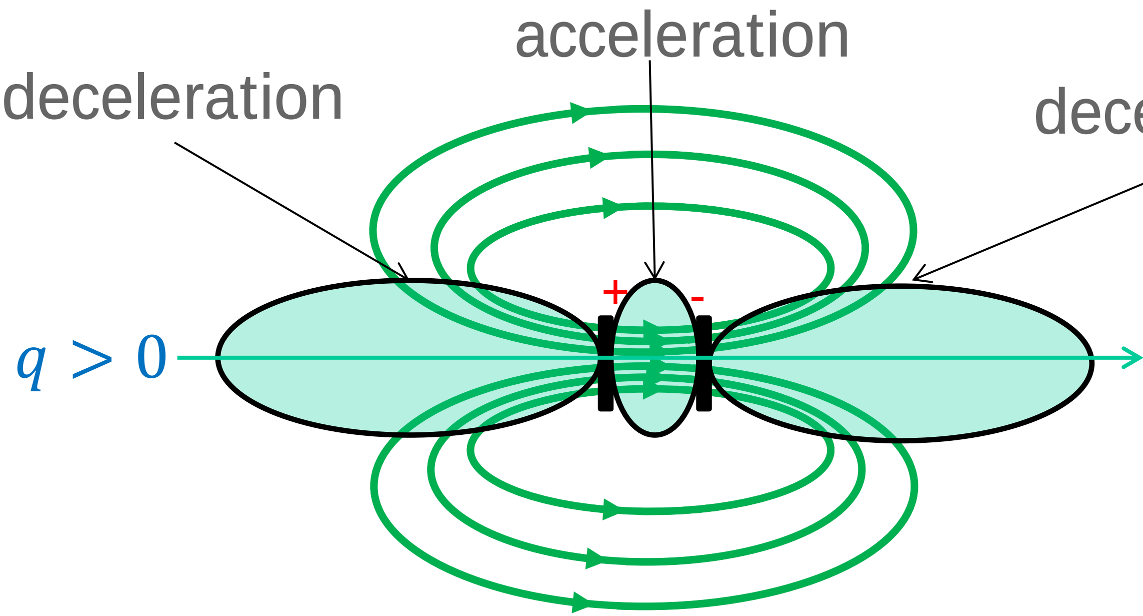
which is equivalent of making many turns with one battery:



How to build a circular accelerator

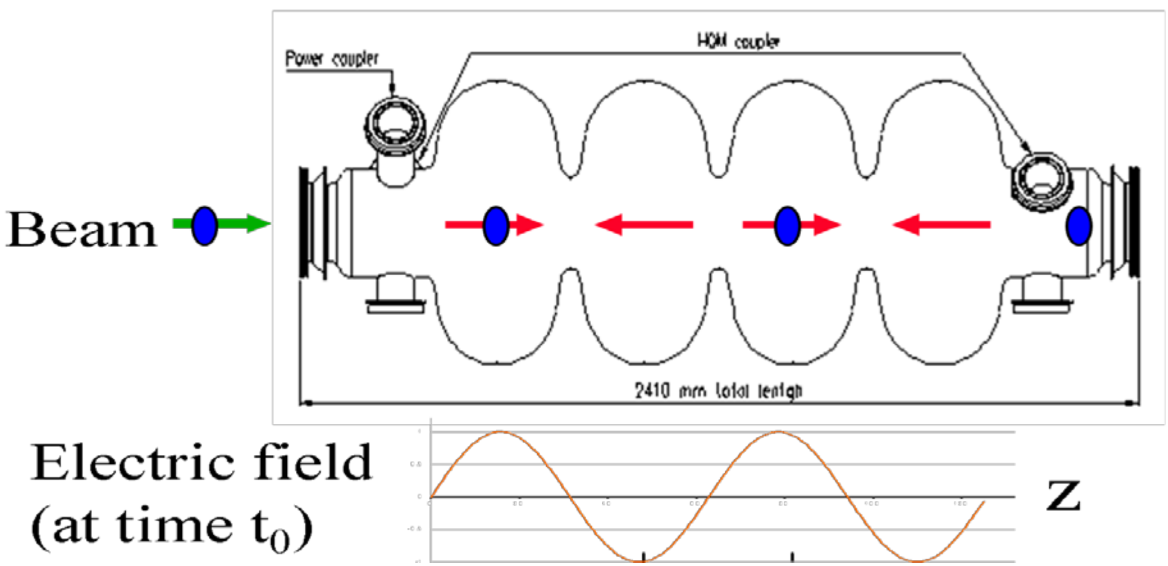
A **static** electric field accelerates only in a certain region, which is not good for multi-pass accelerators (circular acc.)...

$$W = \oint \vec{F}\,\overrightarrow{ds} = q \oint \vec{E}\,\overrightarrow{ds} = 0$$



How to build a circular accelerator

The most performing accelerator mechanism is a **time dependent electric** (which means **electro-magnetic**) field.

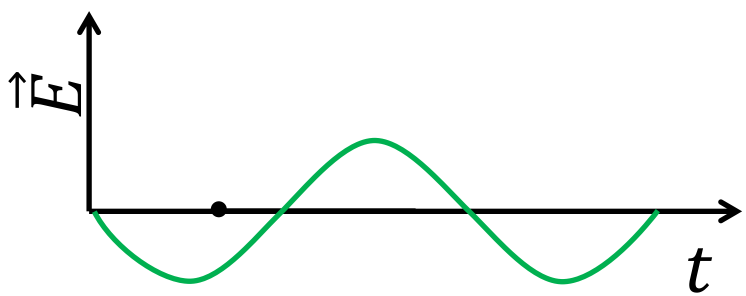


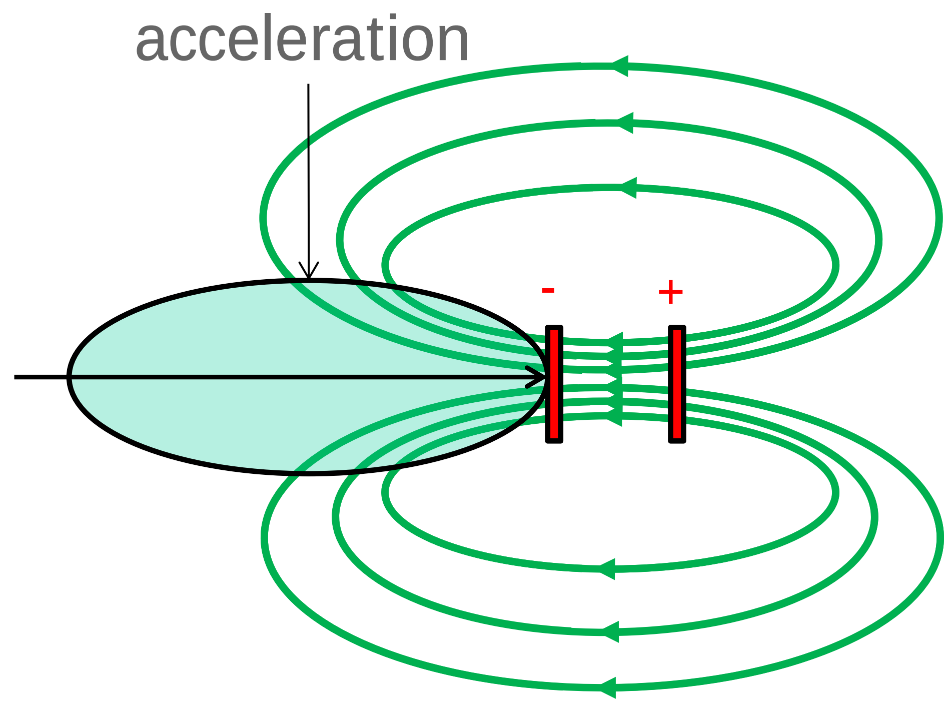
RF cavity

How to build a circular accelerator

A **time varying** electric field can easily accelerate:

$$\oint \vec{E}(t)\,\overrightarrow{ds} = -\iint \frac{\partial \vec{B}}{\partial t}\,\overrightarrow{da}$$





How to build a circular accelerator

A **time varying** electric field can easily accelerate:

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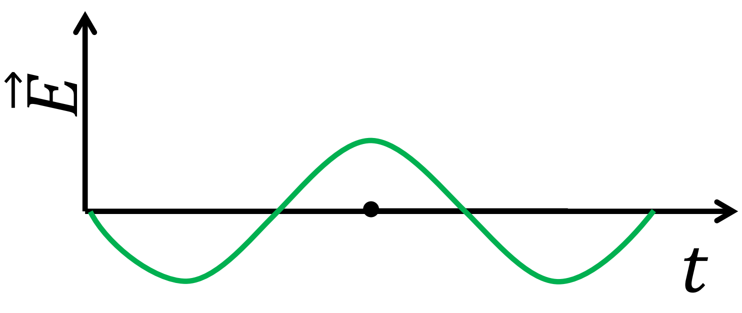

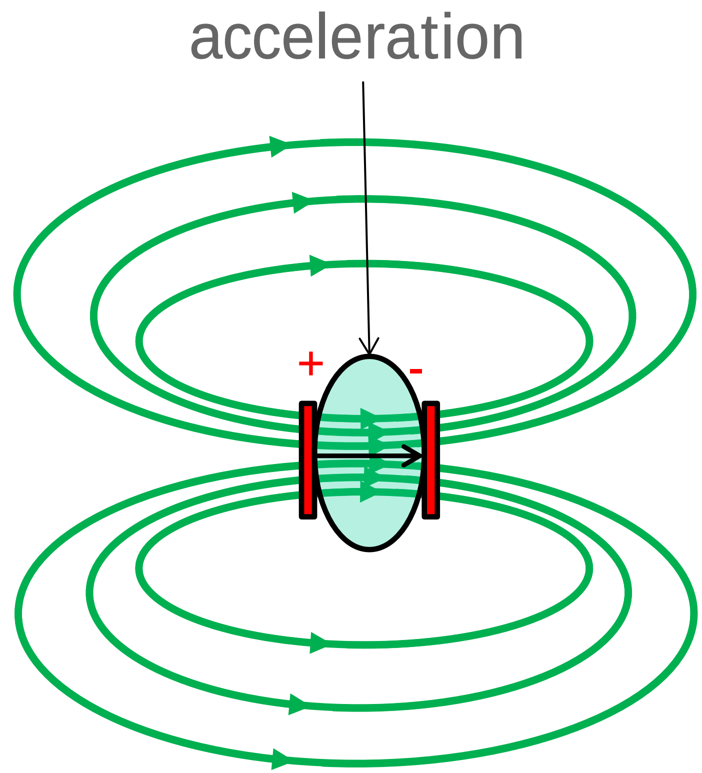

How to build a circular accelerator

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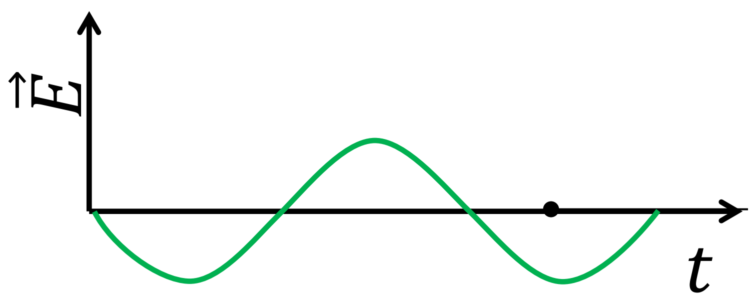
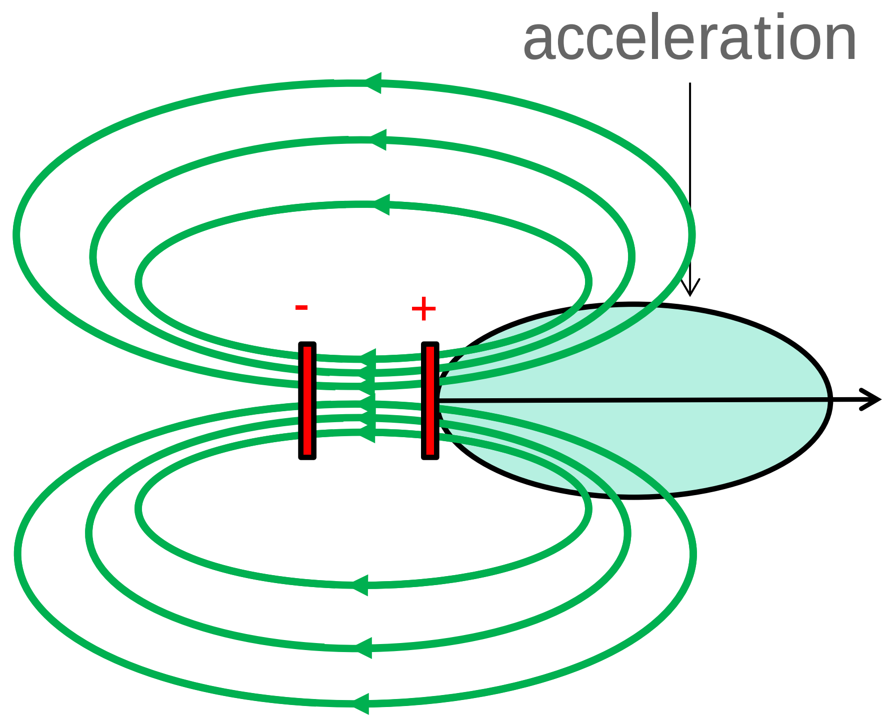


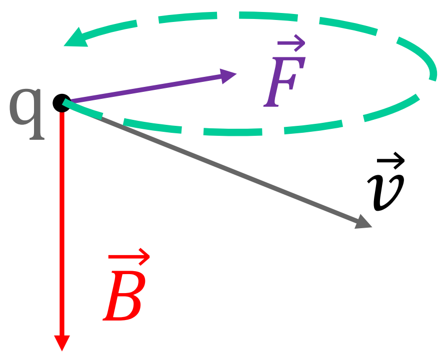


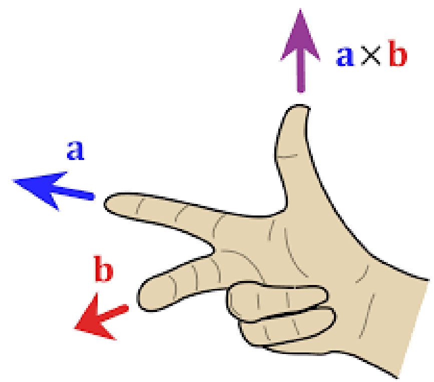
How to build a circular accelerator

... and how to **deflect** particle trajectories?

With **magnetic fields**

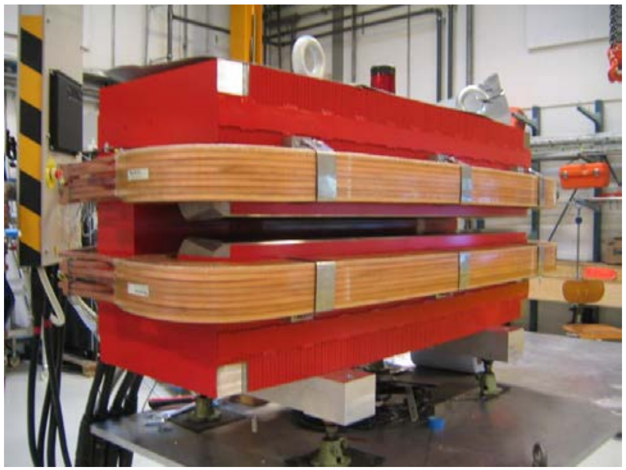
$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

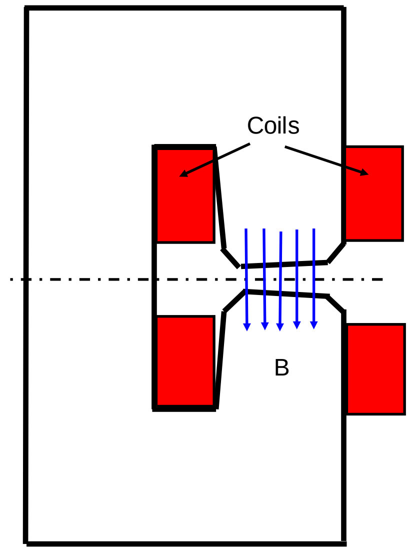


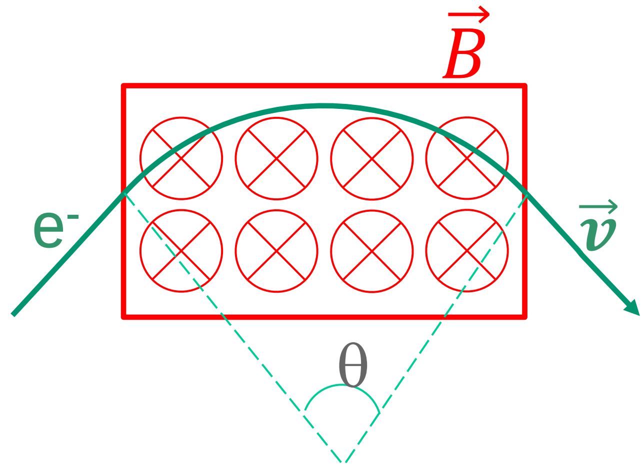


How to build a circular accelerator

Dipole magnet: constant magnetic field $\vec{B}$ perpendicular to $\vec{v}$







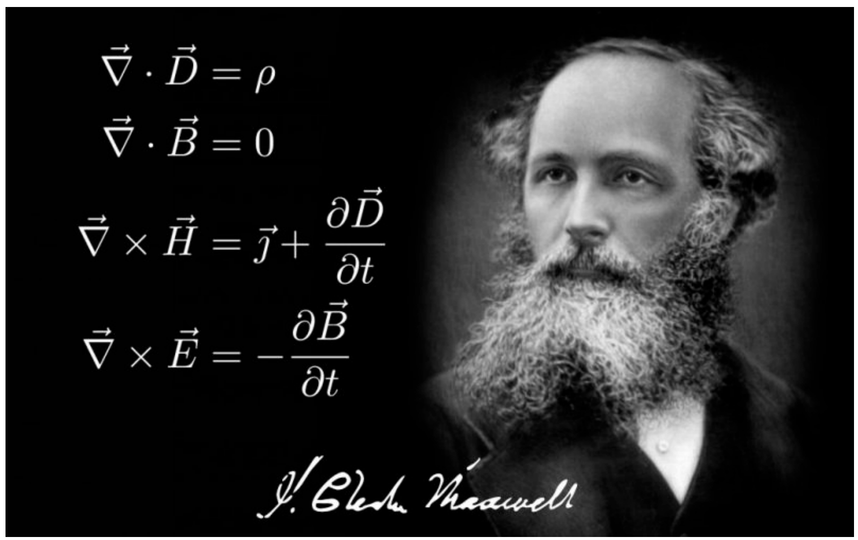
It deflects the particles on a circular trajectory.

How to build a circular accelerator

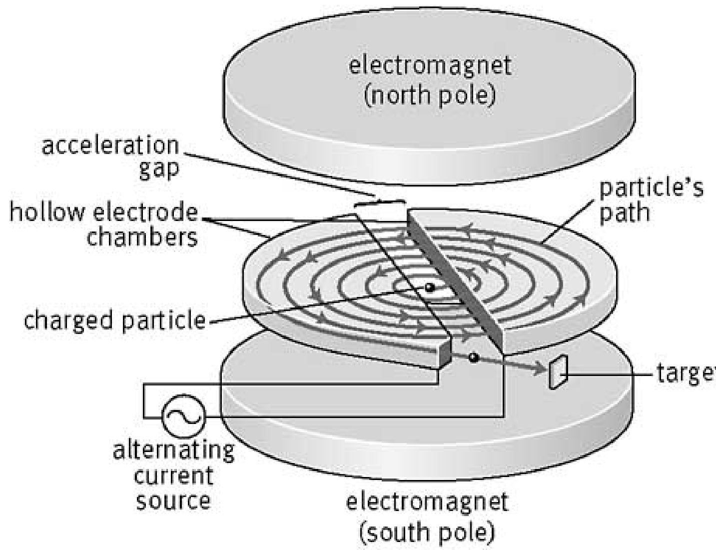
In summary:

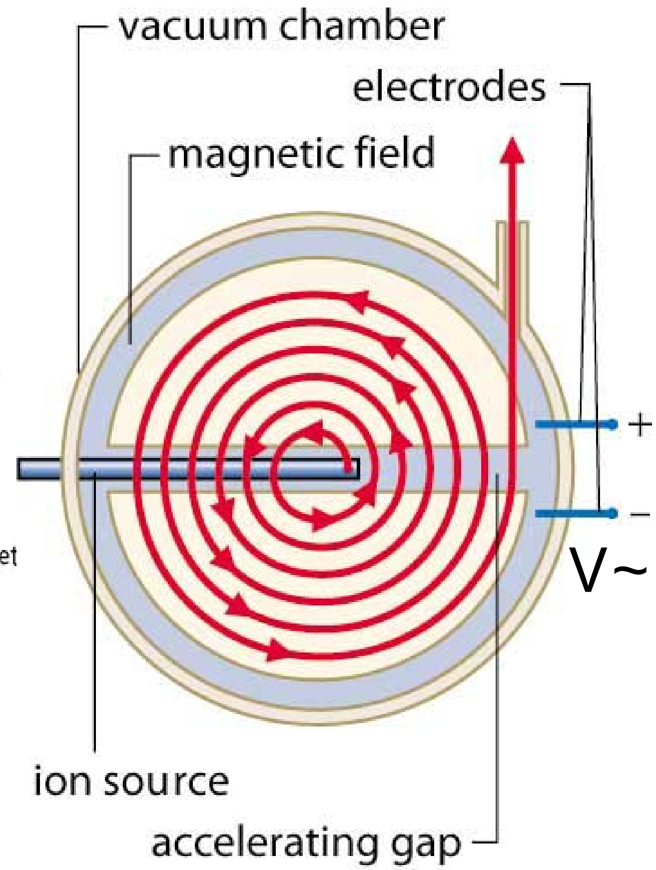
1. Electric field: can produce higher energy gains if it varies with time.
2. Magnetic field: works for bending

So far, basically everything we do with accelerators is explained by **Maxwell's equations** (1861)



Let’s start building a Cyclotron



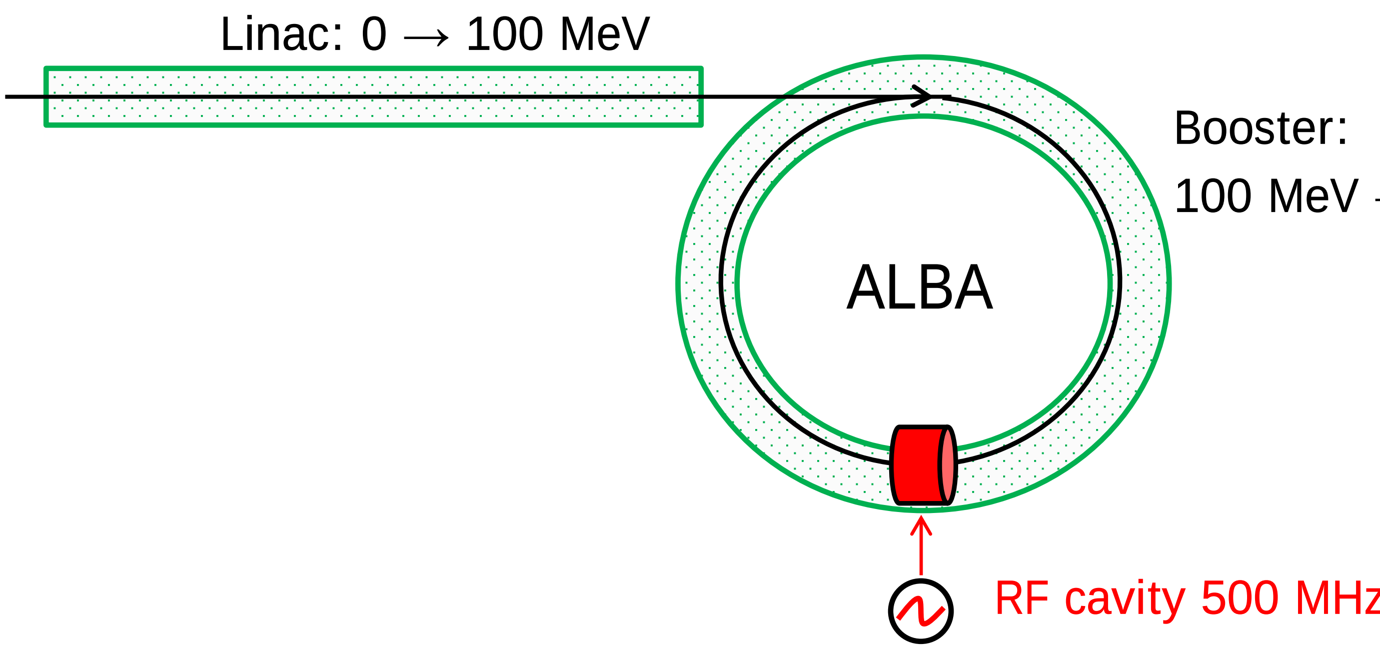


As the beam energy grows, if we want to perform a circle instead of a spiral path, the magnetic field B should be “ramped”.

And so we do in the ALBA booster!

Linac + Booster

There are around 90 synchrotron radiation facilities in the world using this configuration:



In the ALBA booster, will the revolution time of the accelerated electrons be shorter turn after turn?

A bit of special relativity

As Albert Einstein showed to the world in 1905:

- Particles have a maximum velocity: the speed of light (c)
- Particles have a minimum energy ("rest energy"): $E_0=mc^2$
- Measured elapsed times and distances are different in different reference frames (time dilation and length contraction)

A bit of relativity

A particle moving at a relative speed v experiences different kinematics according to the relativistic factor γ:

$$\gamma = \frac{E}{E_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

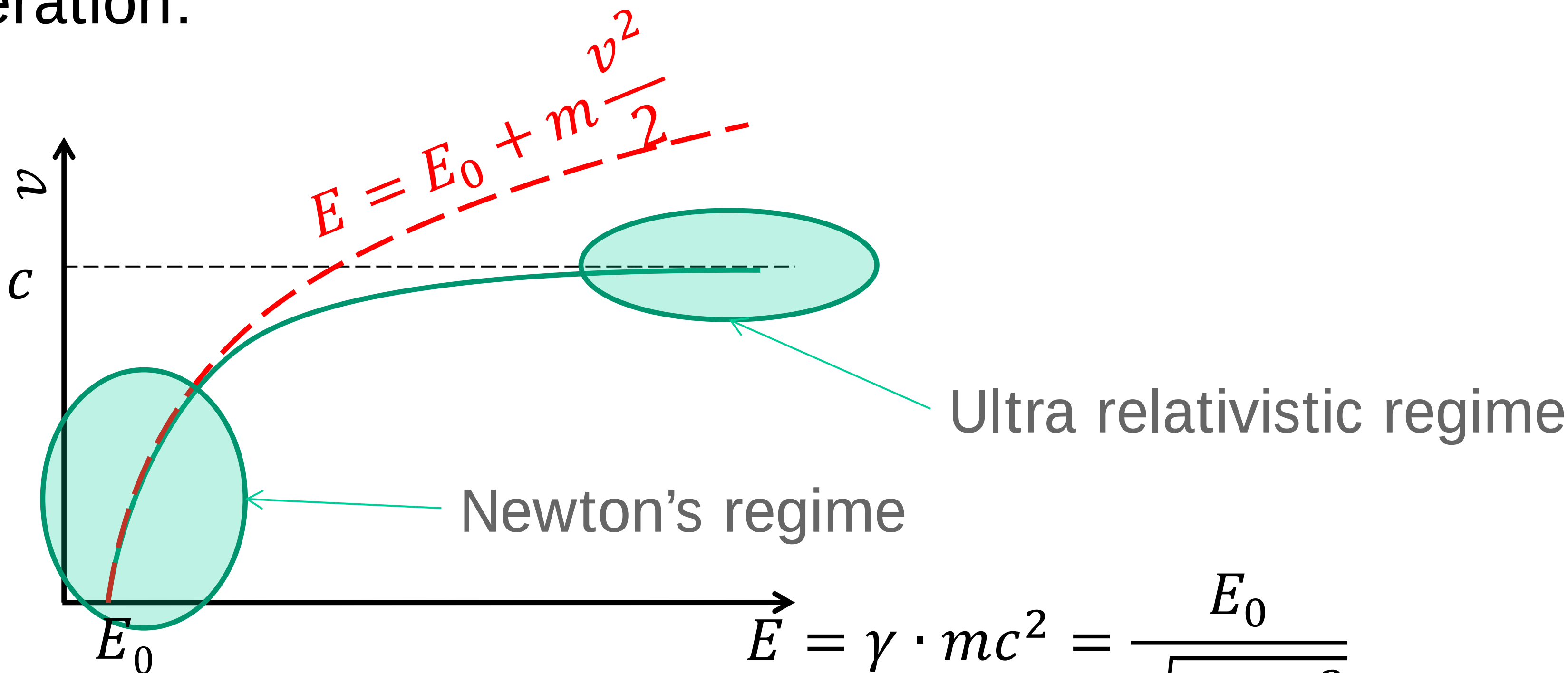
The particle's momentum *p* and the kinetic energy *E* are expressed as follows:

$$E = \gamma m c^2 \quad \left(\frac{1}{2} m v^2\right)$$

$$p = \gamma m v \quad (mv)$$

A bit of relativity

If the truth be told, nowadays, many accelerators do almost no acceleration:



$$E = \gamma \cdot mc^2 = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

... as a result, in our booster, despite the energy is growing, the electrons take the same time to complete a turn.

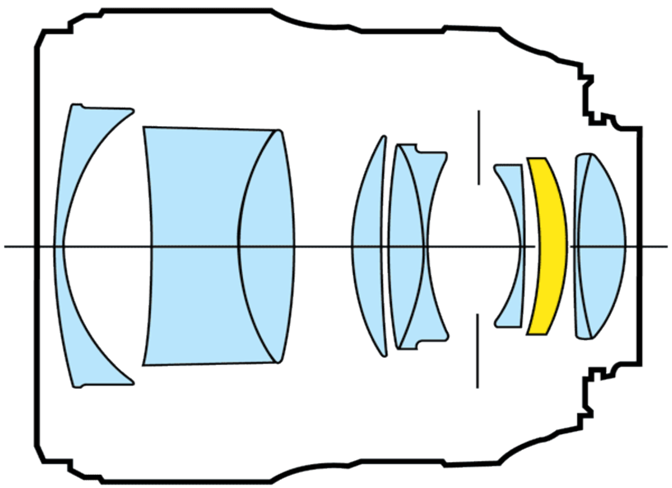
Divergence and spread out of particle beams

Particle beams (as light) always diverge and **need some focusing system** if you want them not to spread out!

Beam optics: analogy with a camera

Beside dipoles, we need other dedicated magnets which act as lenses.

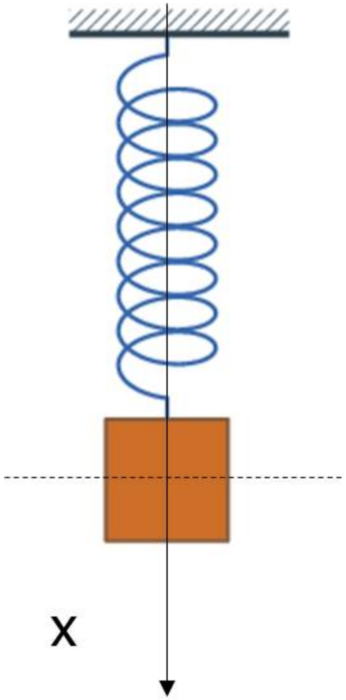
The arrangement of magnets (also called "lattice") must be somehow similar to a camera focusing system.



The question is: what a magnet must be like to focus particle trajectories?

Magnetic focusing: analogy with the spring

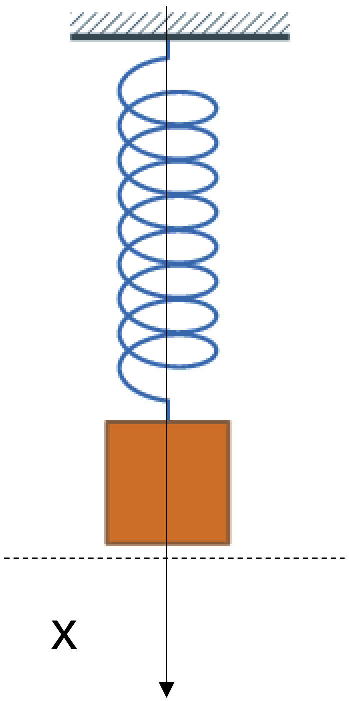
Spring:



If the spring is in its equilibrium position $x = 0$ there is no oscillation

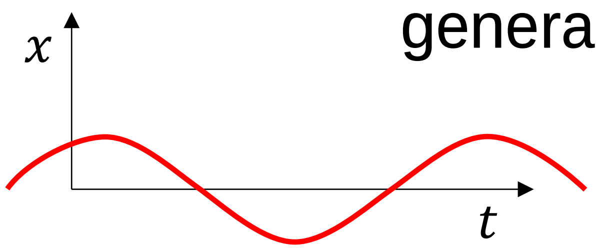
Magnetic focusing: analogy with the spring

Spring:



When we move it away, the spring has a restoring force (opposite to the displacement) proportional to the elongation x

$$F = m \cdot \frac{d^2x}{dt^2} = -kx$$



general solution: free harmonic oscillation

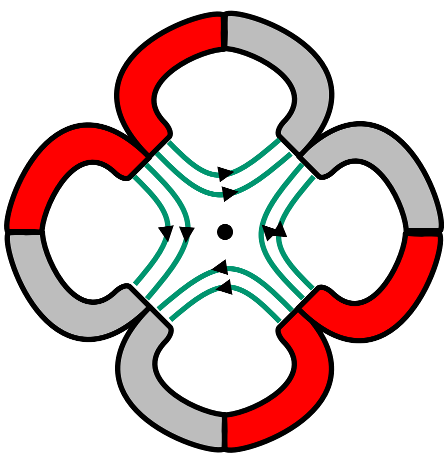
$$x(t) = A \cdot cos(\omega t + \varphi)$$

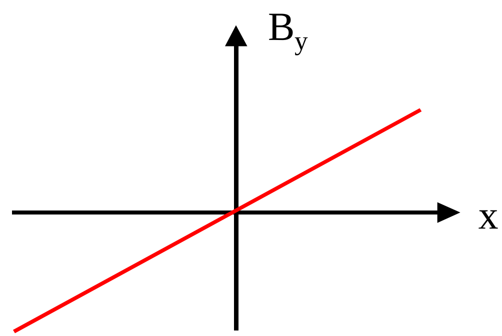
Focusing: quadrupole magnet

So, coming back to our storage ring, we need a force that rises as a function of the displacement from the design orbit...

$$F(x) = qvB(x) \rightarrow B(x) = g \cdot x$$

4 horseshoes or... 4 alternated iron poles + coils:



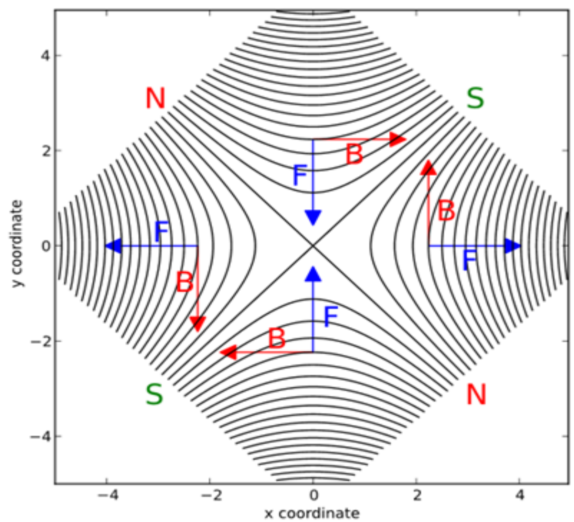


$$B(x) = g \cdot x$$

g = gradient

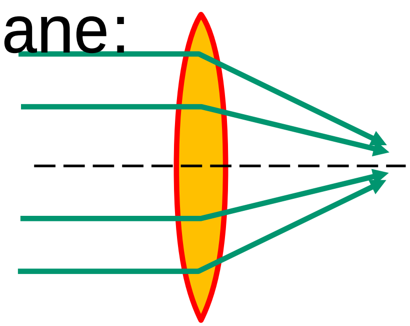
Focusing: quadrupole magnet

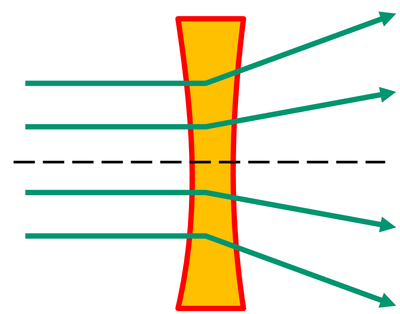
A quadrupole magnet has a linear magnetic field:



$$B_x = -gy$$
$$B_y = gx$$

A quadrupole focuses in one plane BUT defocuses in the other plane:


horizontal plane

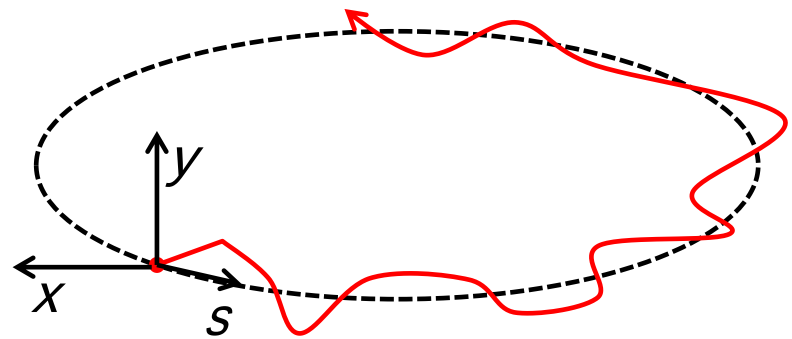

vertical plane

Betatron oscillations

Due to focusing and defocusing quadrupoles, the displacement with respect to the design orbit **oscillates in the transverse plane (x, y)**:

Hill's equations

$$\boxed{\begin{aligned} \frac{d^2x}{ds^2} + k(s)x = 0 \\ \frac{d^2y}{ds^2} - k(s)y = 0 \end{aligned}}$$



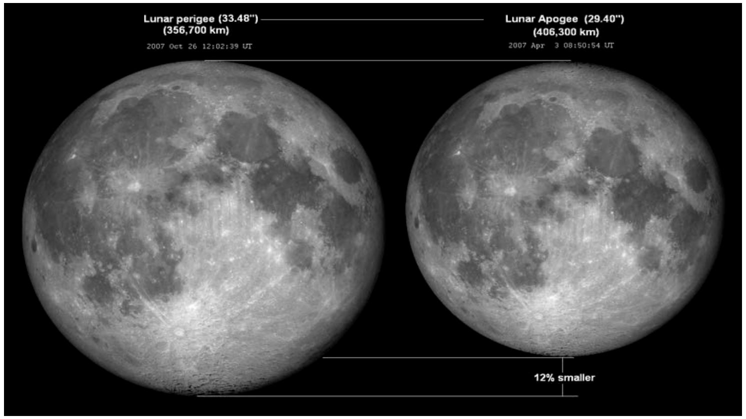
with: $$k = \frac{qg}{E/c}$$

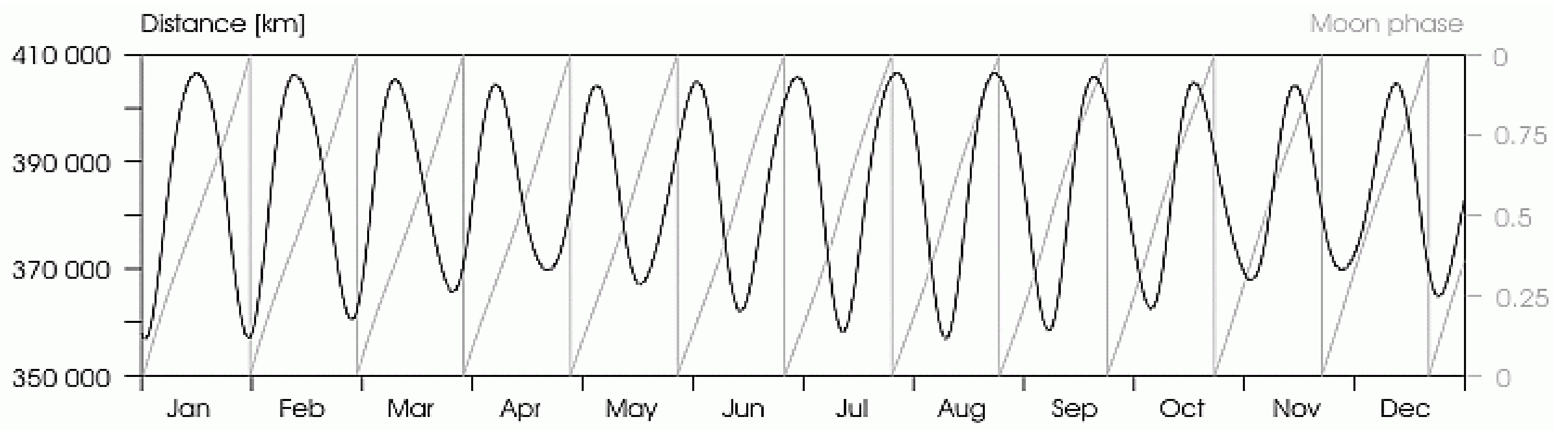
and since it is a circular accelerator, *k(s)* is periodic:

$$k(s+L) = k(s)$$

Betatron oscillations

Such type of equation was already solved in 1886 by **G.W. Hill** when studying the motion of the lunar perigee.





Betatron oscillations

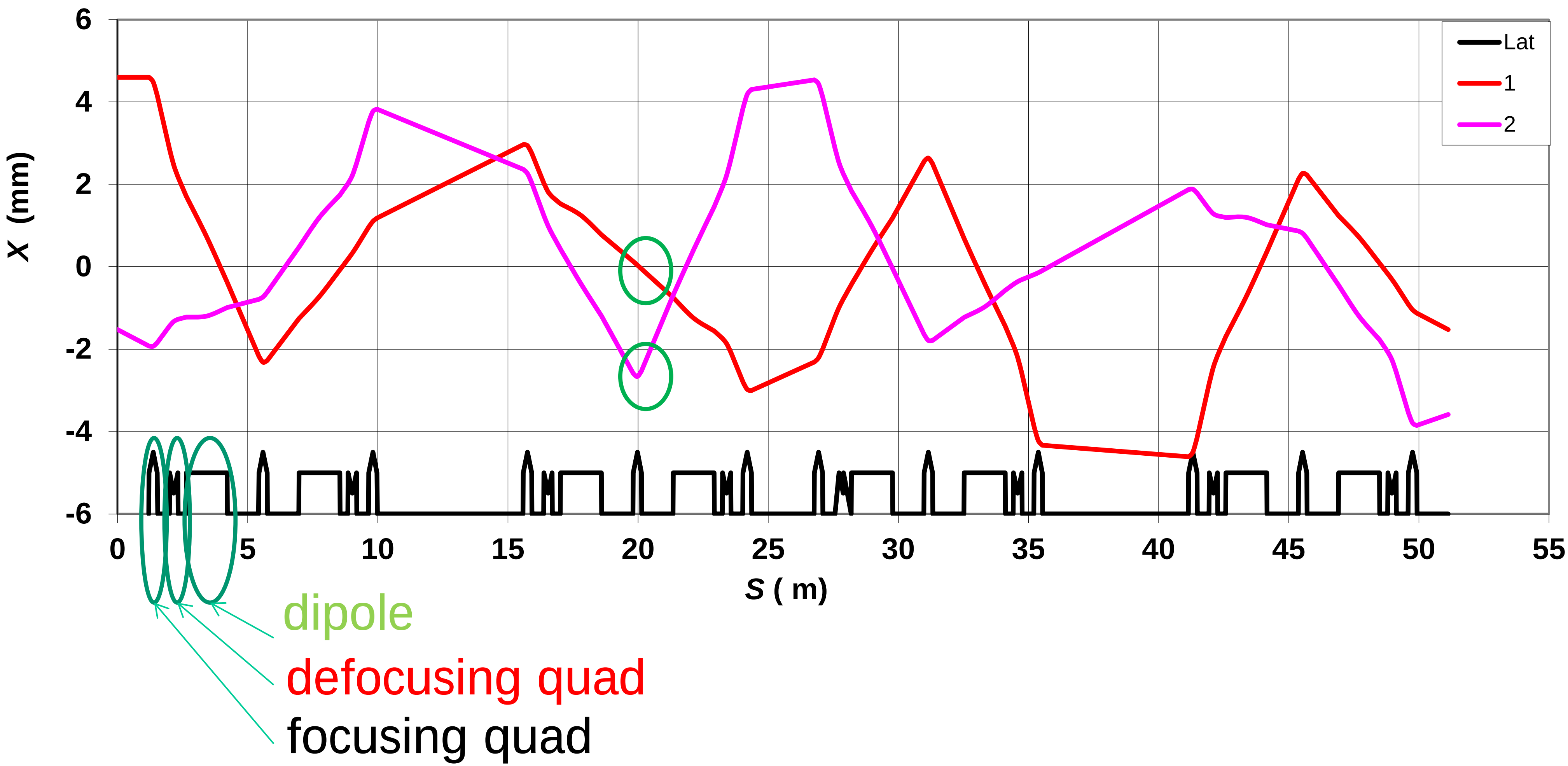
The solution are the **betatron** oscillations:

$$\boxed{x(s) = \sqrt{\boldsymbol{J}_x \beta_x(s)}\cos(\mu_x(s) - \boldsymbol{\mu_0})}$$

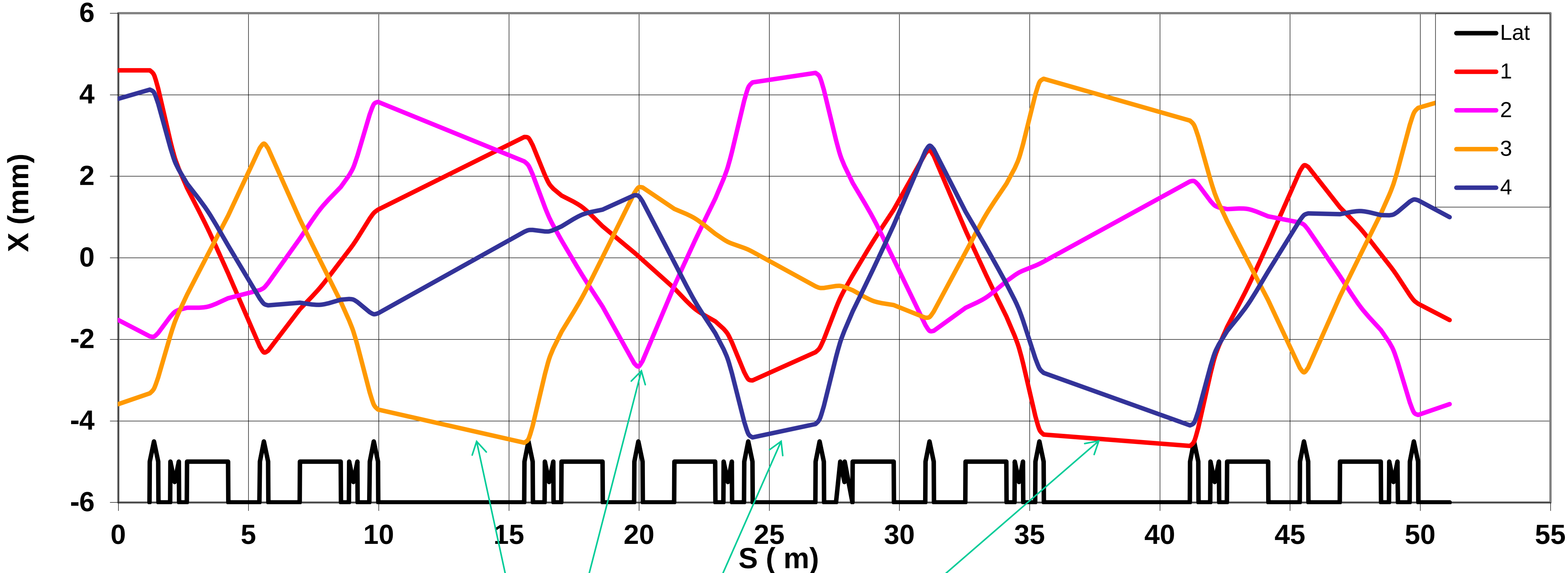
- J: it is a **constant…**
- $\beta(s)$: **beta function**: varies along the ring and periodic $\beta(s+L) = \beta(s)$
- μ_0: initial phase: depends on initial conditions
- μ: betatron phase: grows not linearly:

$$\mu(s) = \int_0^s \frac{ds}{\beta(s)}$$

Betatron oscillations

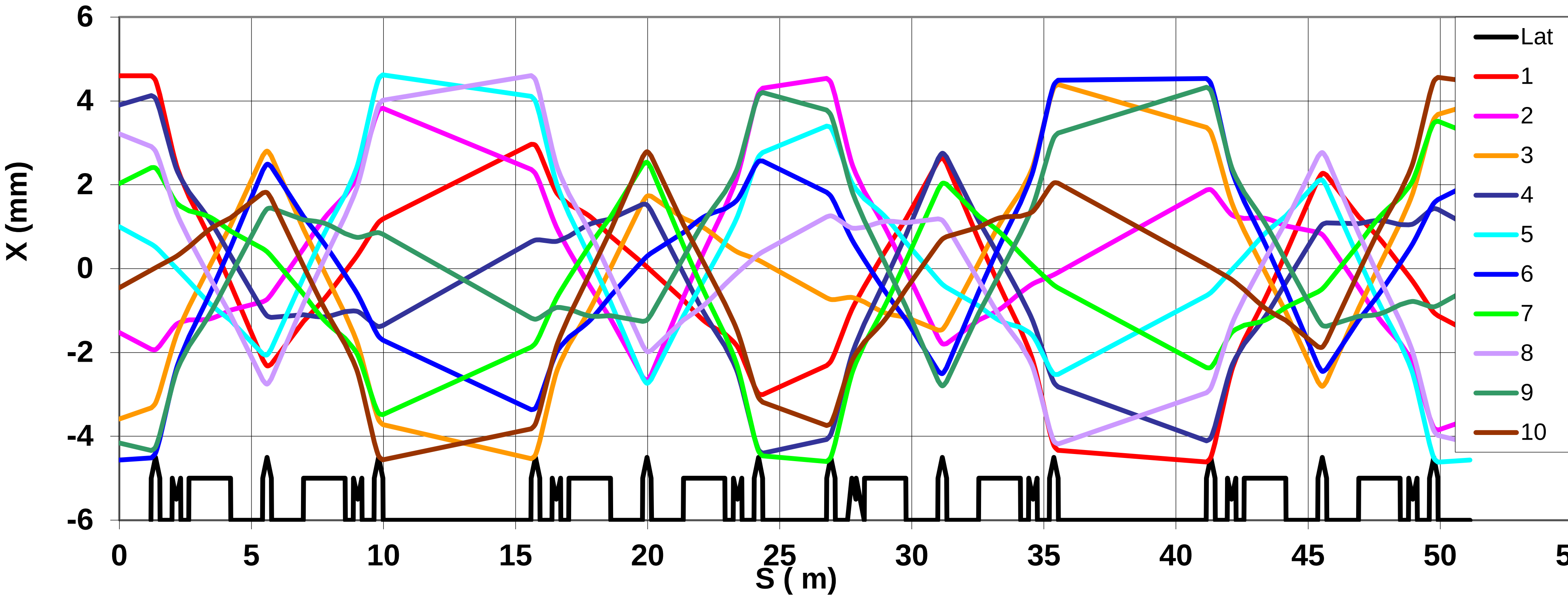


Betatron oscillations



$$x(s) = \sqrt{J\beta(s)}cos(\mu(s) - \boldsymbol{\mu_0})$$

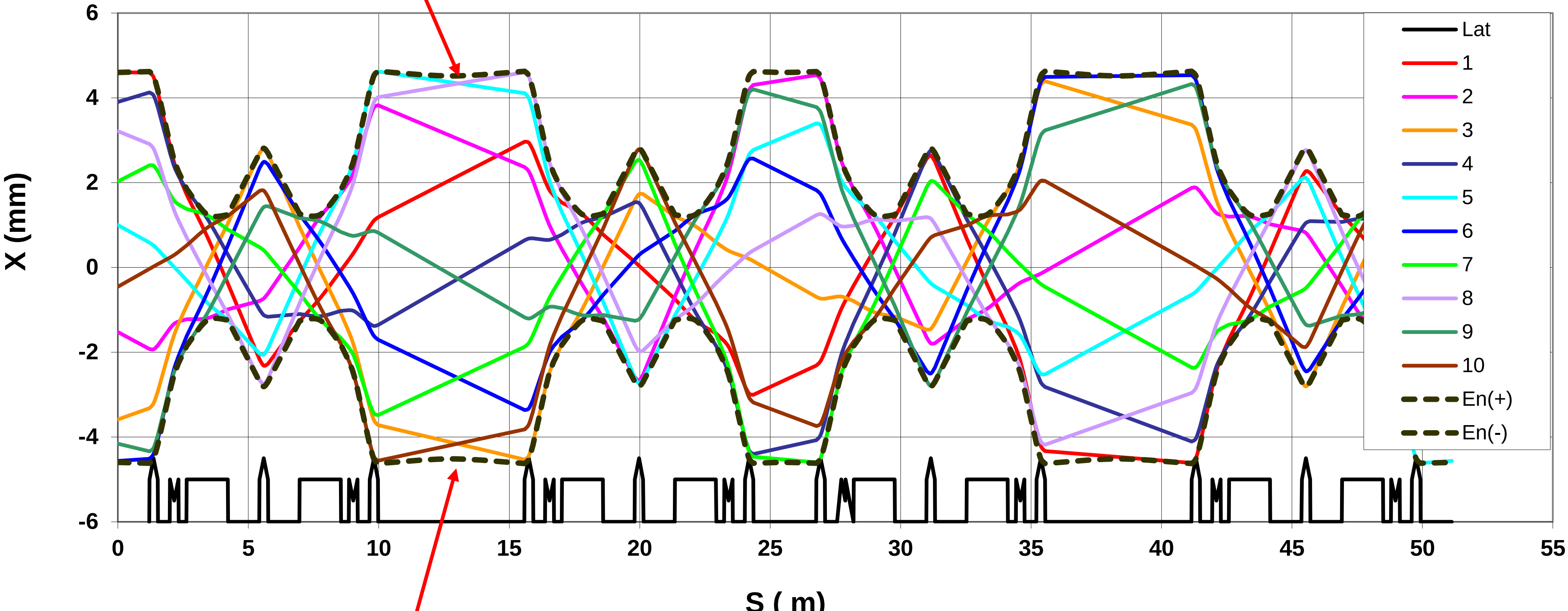
Betatron oscillations



The tracks correspond to different turns of the same particle, but can be regarded also as a single turn of a bunch of particles starting with different initial phases...

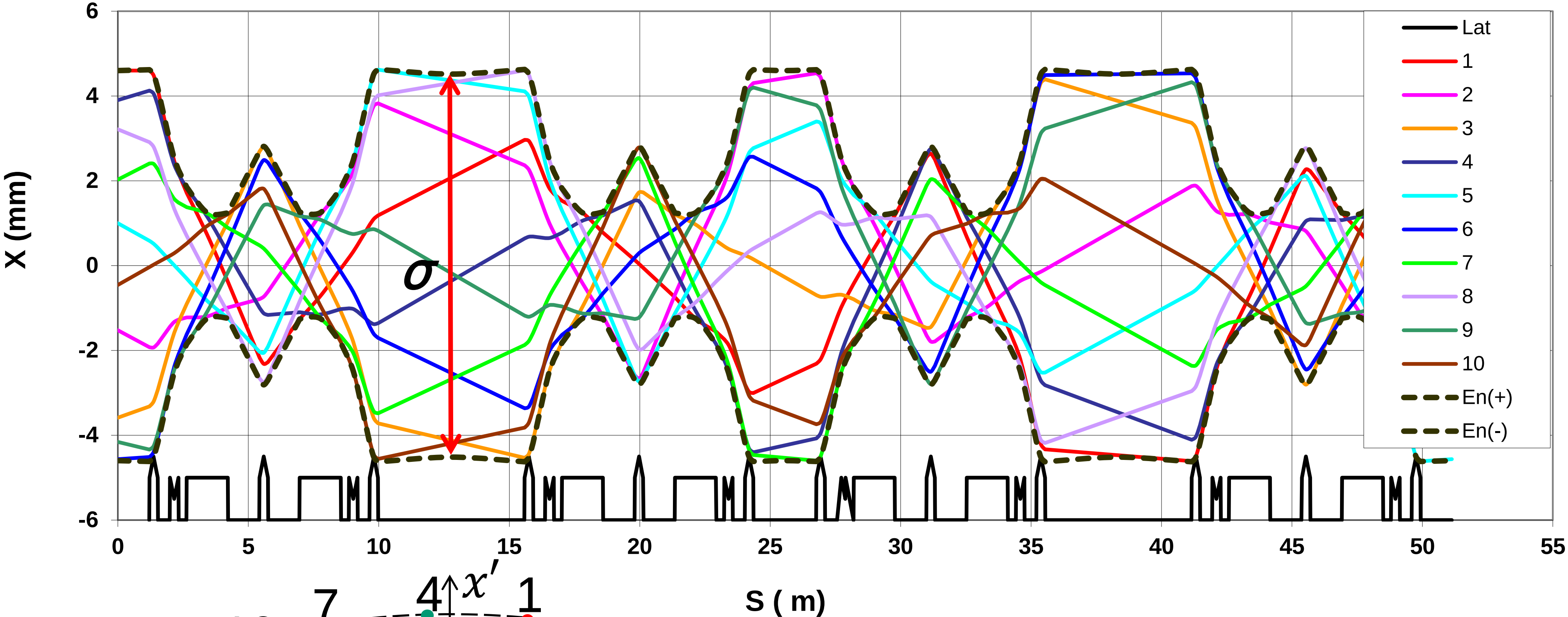
Betatron oscillations

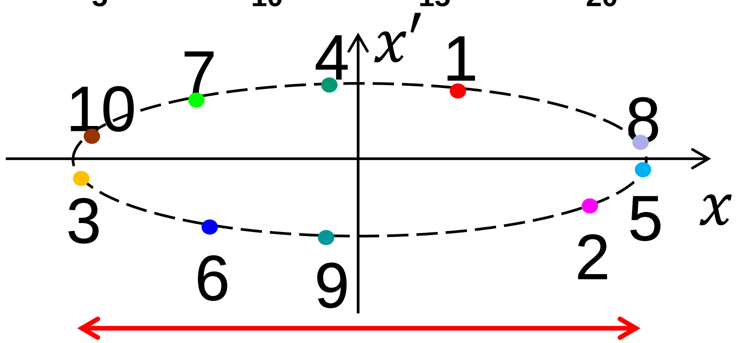
$+\sqrt{J\beta(s)}$



$-\sqrt{J\beta(s)}$

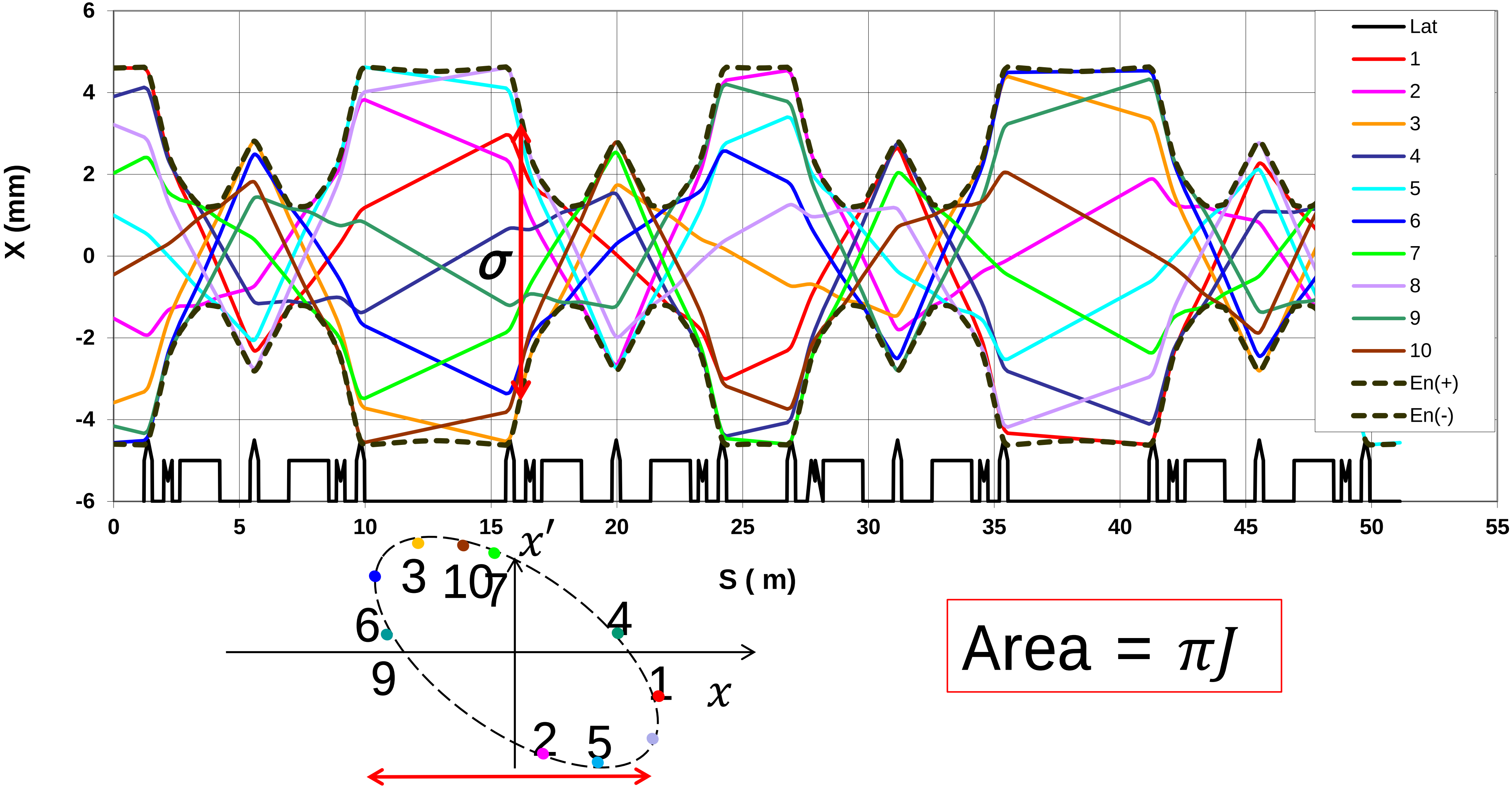
Betatron oscillations





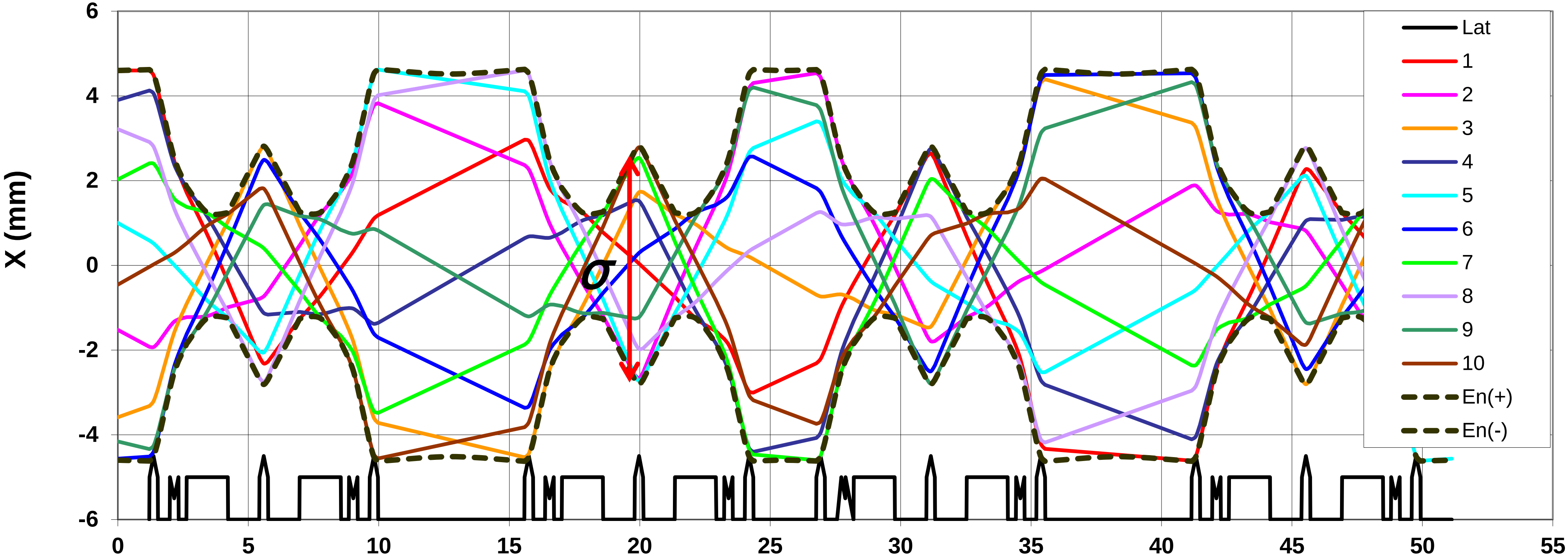
$$\boxed{\text{Area} = \pi J}$$

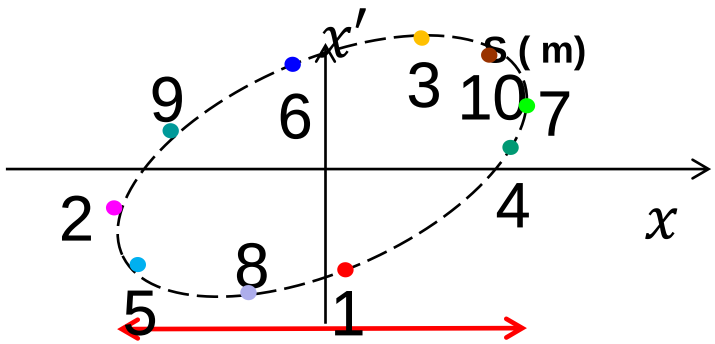
Betatron oscillations



$\boxed{\text{Area} = \pi J}$

Betatron oscillations

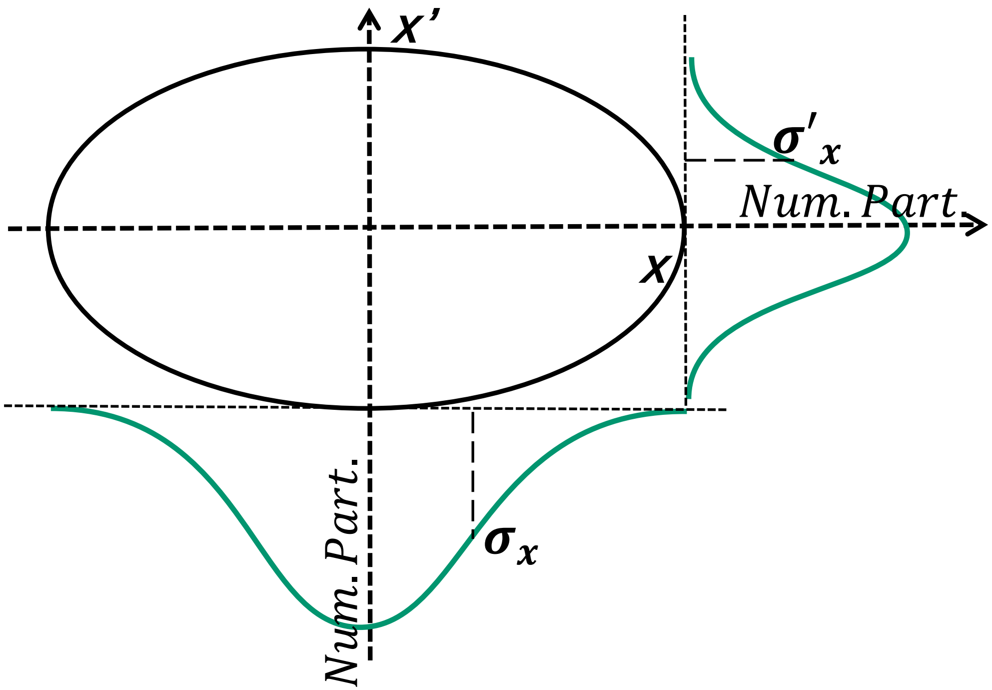




$$\boxed{\text{Area} = \pi J}$$

Beam size and emittance

The beam contains particles with a distribution of positions ***x*** and divergences ***x'***:



The transverse **beam size** $\boldsymbol{\sigma_x}$ and **divergence** $\boldsymbol{\sigma'_x}$ vary along the ring as function of $\beta(s)$:

$$\boldsymbol{\sigma_x}(\boldsymbol{s}) = < x > = \sqrt{\varepsilon_x \beta(s)}$$

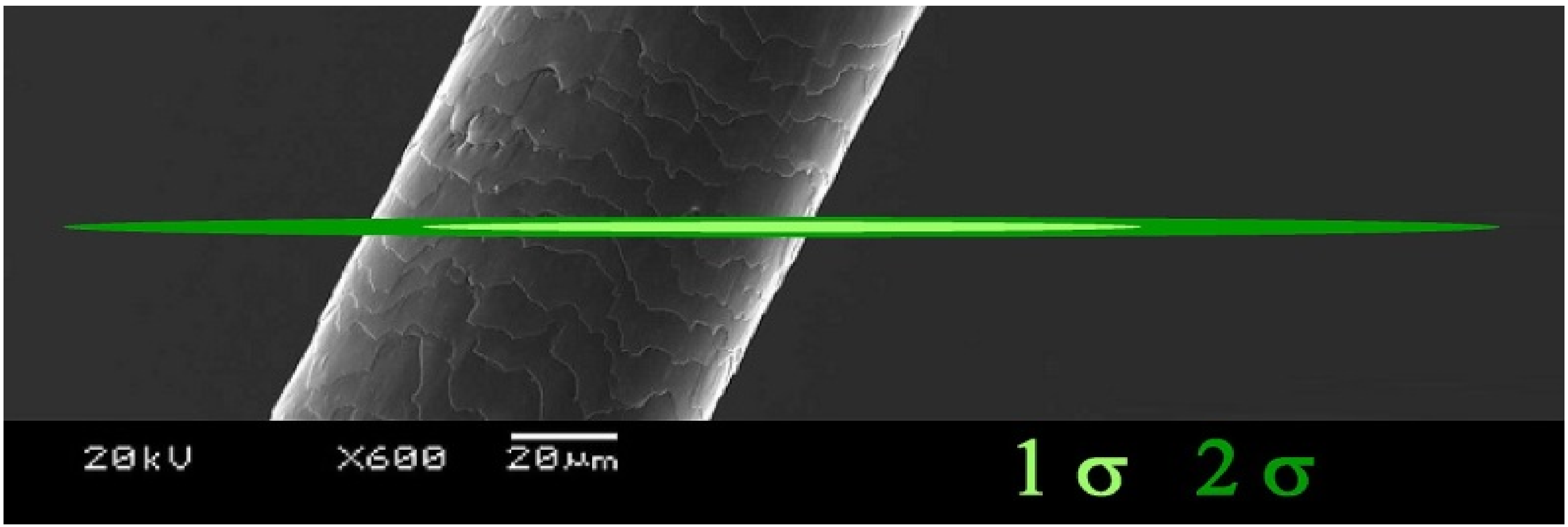
$$\boldsymbol{\sigma'_x}(s) = < x' > = \sqrt{\frac{\varepsilon_x}{\beta(s)}}$$

BUT their product is constant and is called **emittance**:

$$\boldsymbol{\varepsilon_x = \sigma_x \sigma'_x}$$

Beam size and emittance

In modern light sourced (picture from SLS), beam sizes are of the order of a hair thickness:



Beam size and emittance

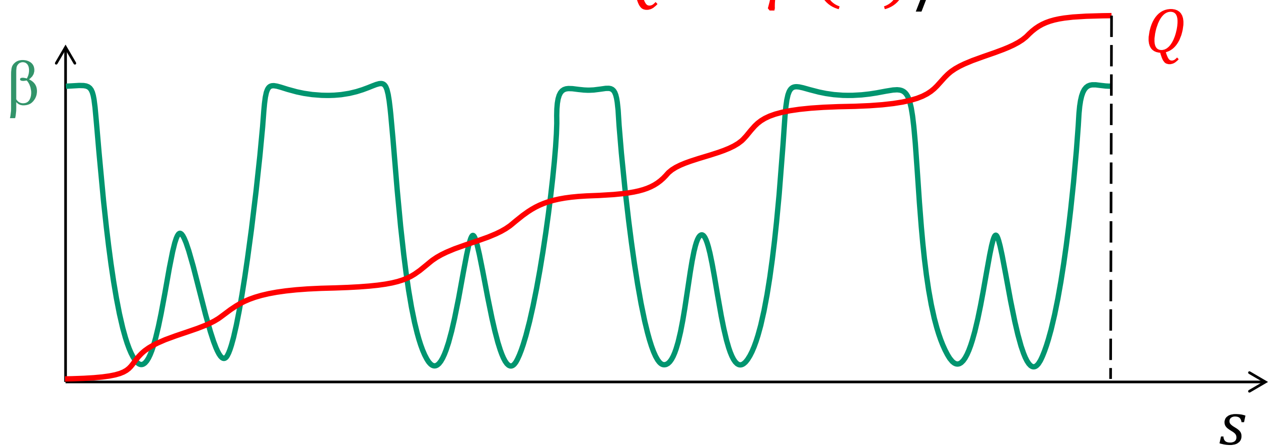
After this “master class” on the emittance, may you answer...

... which beam has smaller emittance?

The betatron tune

The number of betatron oscillations in one turn is called betatron tune:

$$Q = \mu(L)/2\pi$$



$$\mu(s) = \oint \frac{ds}{\beta(s)}$$

The two values of the horizontal and vertical tunes are called **"working point"** of the storage ring:

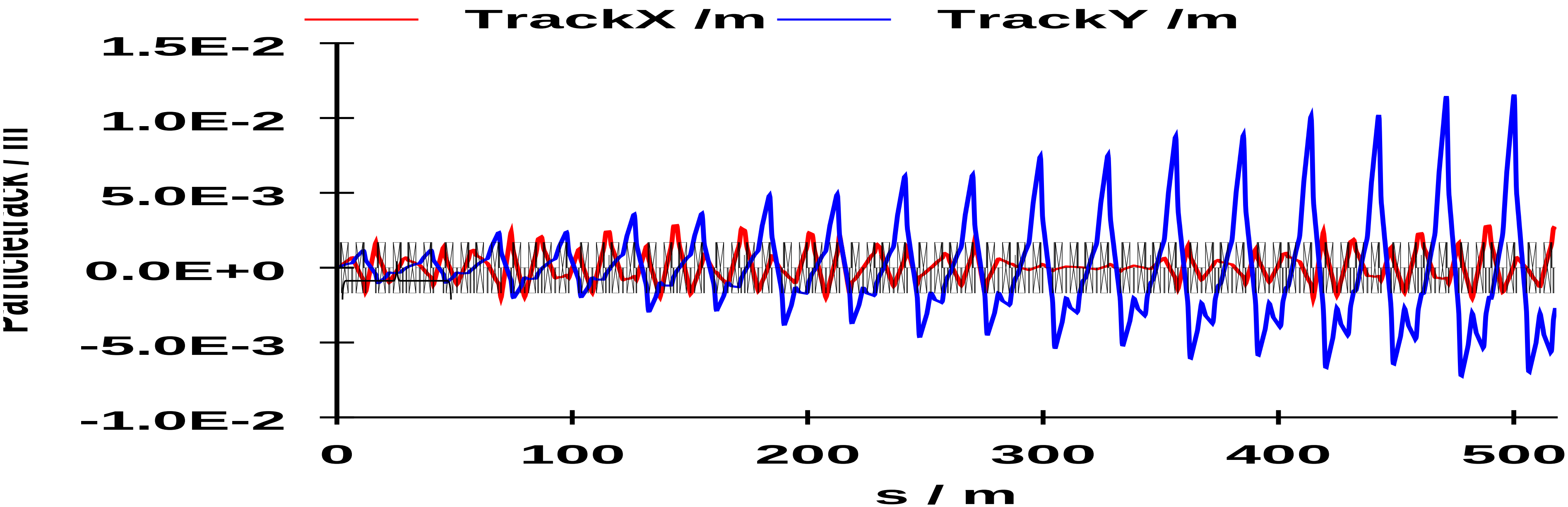
$$W.P. = (Q_x, Q_y)$$

Watch out for resonances!

Simulation of trajectory for many turns.

W.P. is $(Q_x, Q_y) = (4.16, 2.00)$

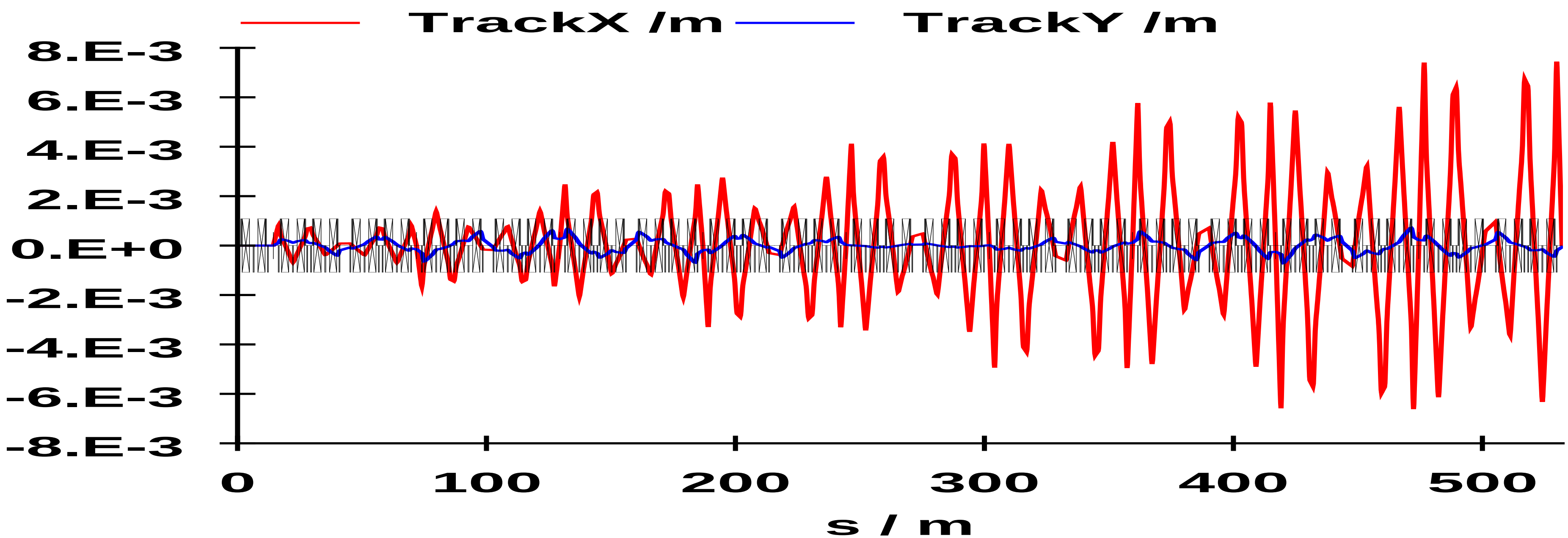
And a small dipole error introduced in the lattice:



Amplitude in the vertical direction is growing and the beam will be lost: a resonance condition is happing.

Watch out for resonances!

Simulation of trajectory for many turns.
W.P. is (Q_x ,Q_y) = (4.50, 1.69)
with a small quadrupole error introduced in the lattice:



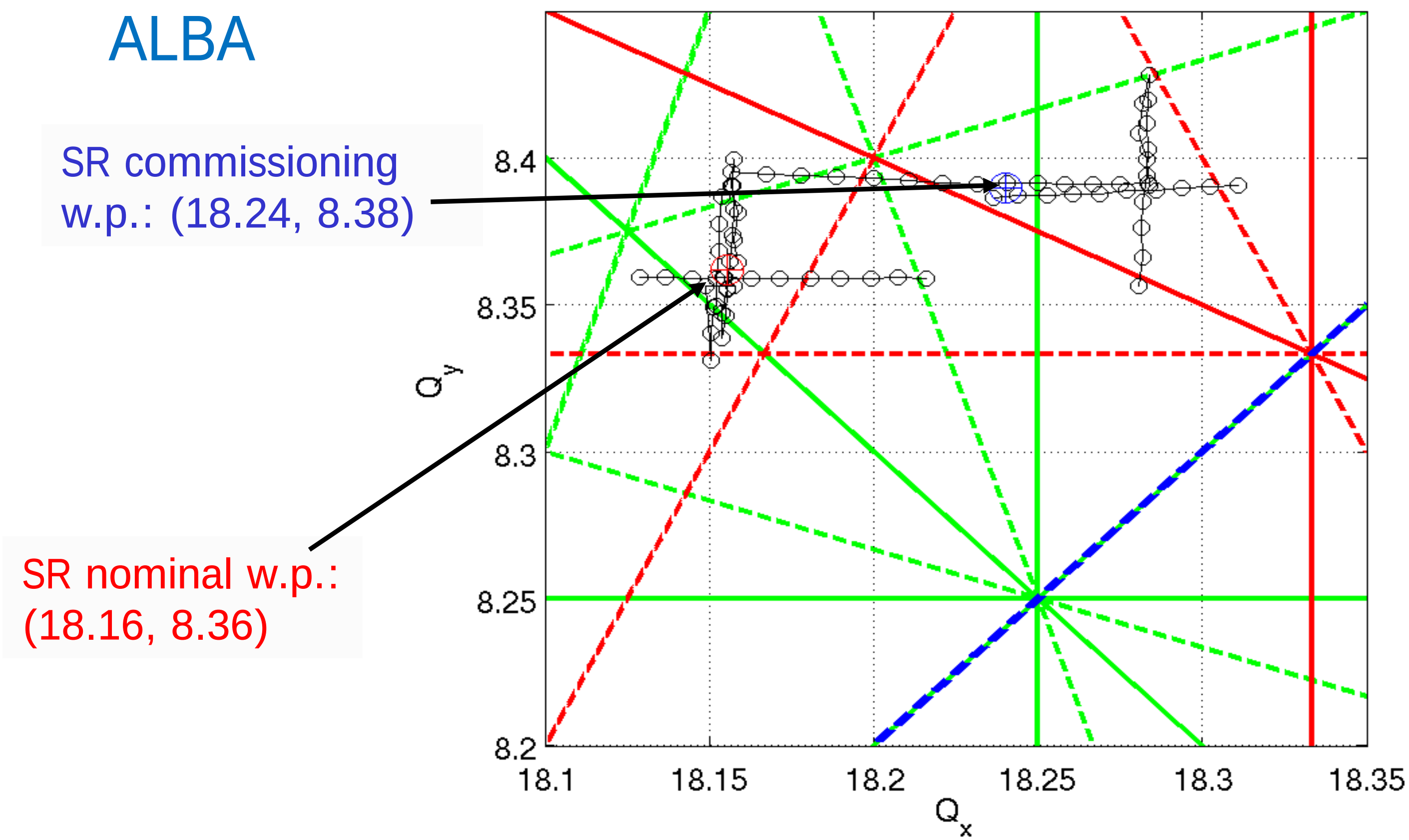
Amplitude in the horizontal direction is growing and the beam will be lost: again a resonance condition happens.

Watch out for resonances!

More in general, circular accelerators must avoid **resonant lines**:

$$nQ_x + mQ_y = q$$

ALBA



Octupole
$4Q_y = 33$
$Q_x - 3Q_y = -7$
$Q_x + 3Q_y = 43$
$2Q_x - 2Q_y = 20$
$2Q_x + 2Q_y = 53$
$3Q_x - Q_y = 46$
$3Q_x + Q_y = 63$
$4Q_x = 73$
Sextupole
$3Q_y = 25$
$1Q_x + 2Q_y = 35$
$2Q_x - Q_y = 28$
$2Q_x + Q_y = 45$
$3Q_x = 55$
Quadrupole
$Q_x - Qy = 10$

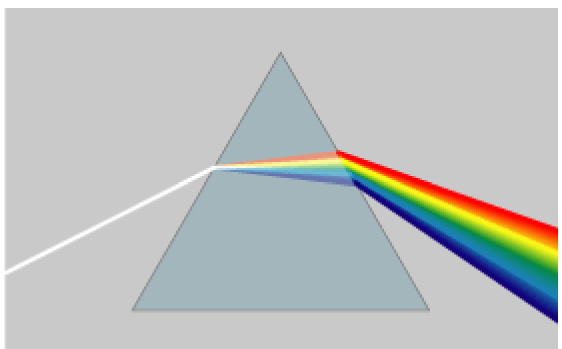
Tune scan measurements performed during in 2012 to adjust the ALBA W.P.

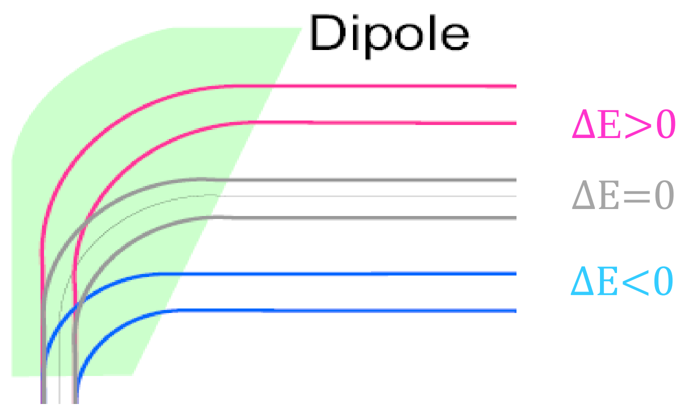
Dispersion and Chromaticity

The beam has a natural energy spread, i.e. particles have a distribution of energies around the nominal one.

For the ALBA storage ring: $\frac{\sigma_E}{E_0} = 0.1\%$

prism

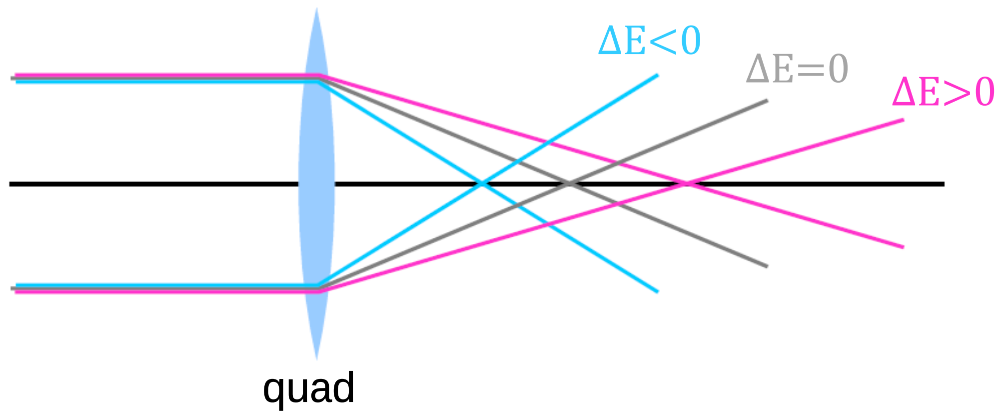




Like a prism with light beams, dipole magnets generate **dispersion** of particle trajectories with displaced energies.

Dispersion and Chromaticity

The chromaticity effect can be visualized as a different focal length depending on energy deviation of the particles:



Chromaticity means a **lack of focusing** for particles with $\Delta E > 0$.

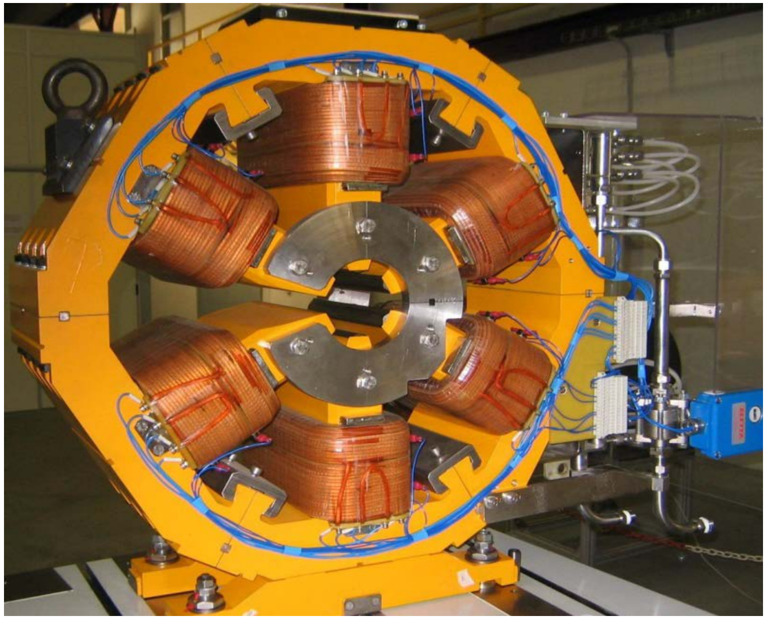
The chromaticity generated by quadrupoles is **always negative** in both planes x and y.

Dispersion and Chromaticity

Chromaticity must be corrected to zero or positive values in any accelerator to avoid instabilities which arise at high stored currents.

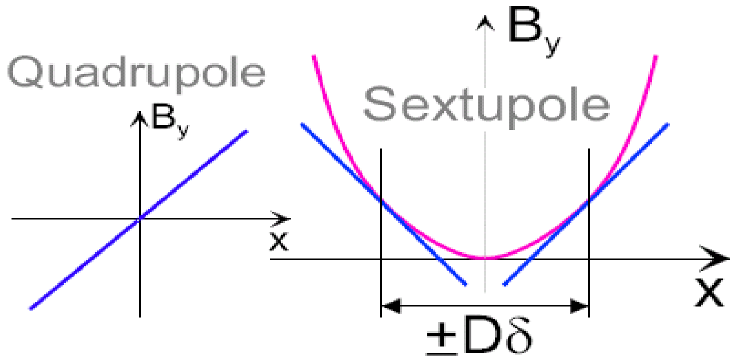
Sextupole magnets are used for this.

They are similar to quadrupoles, but have 6 poles instead:

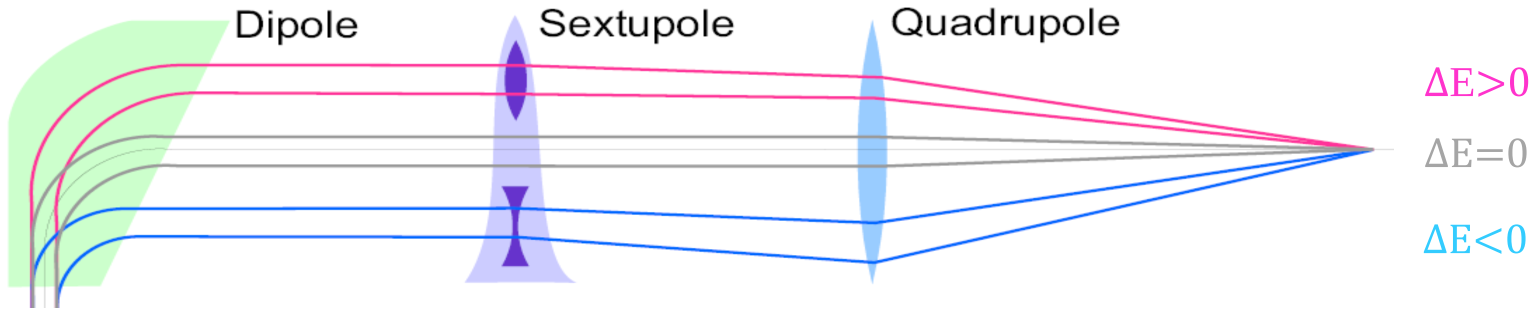


Dispersion and Chromaticity

Sextupole have parabolic magnetic field:

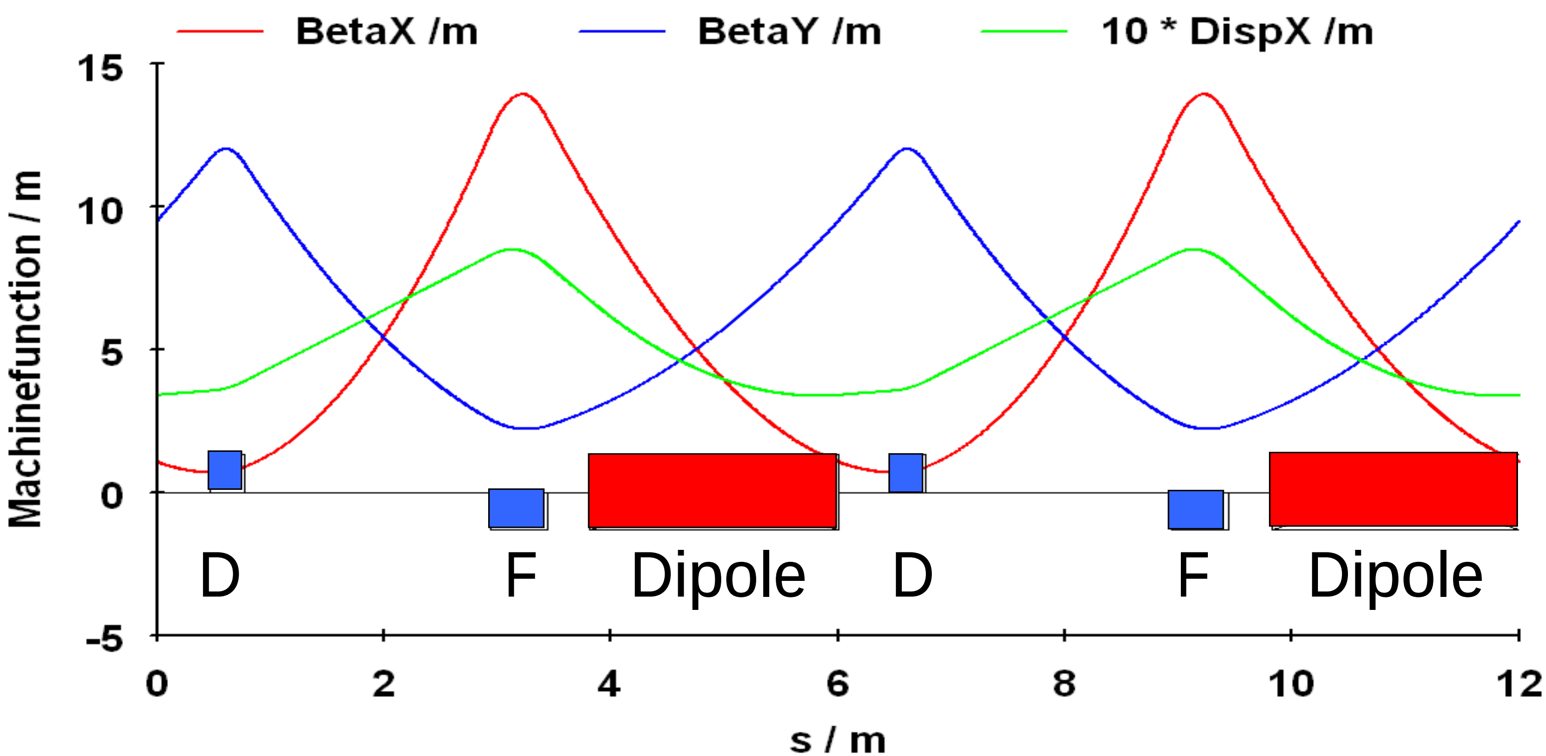


Placing sextupoles at locations with dispersion $D(s)$ allows to correct the chromaticity:



Putting all together... : FODO cell

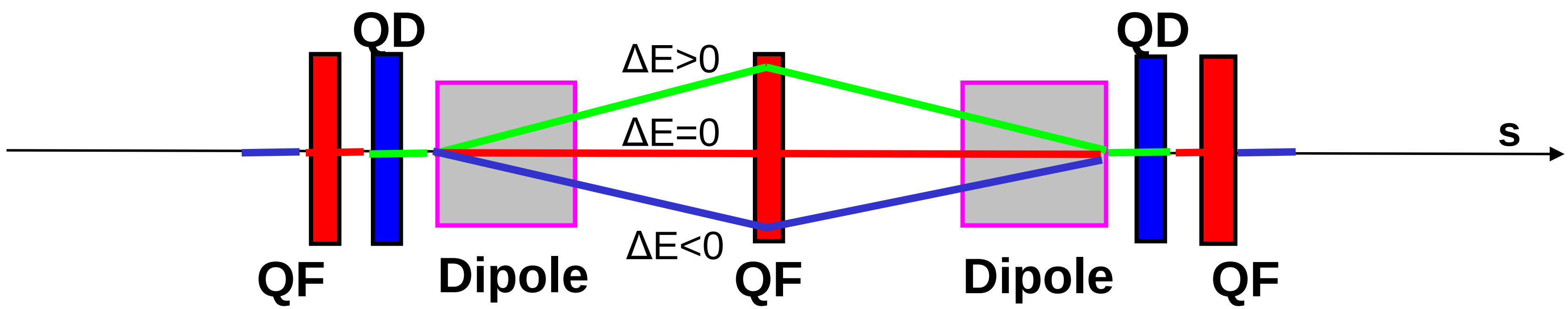
The simplest cell (the unit block to build a ring) consists of alternating FOcusing and DefOcusing quadrupoles (FODO):



This unit cell is typically used at booster synchrotrons, but light source storage rings need more complex lattices.

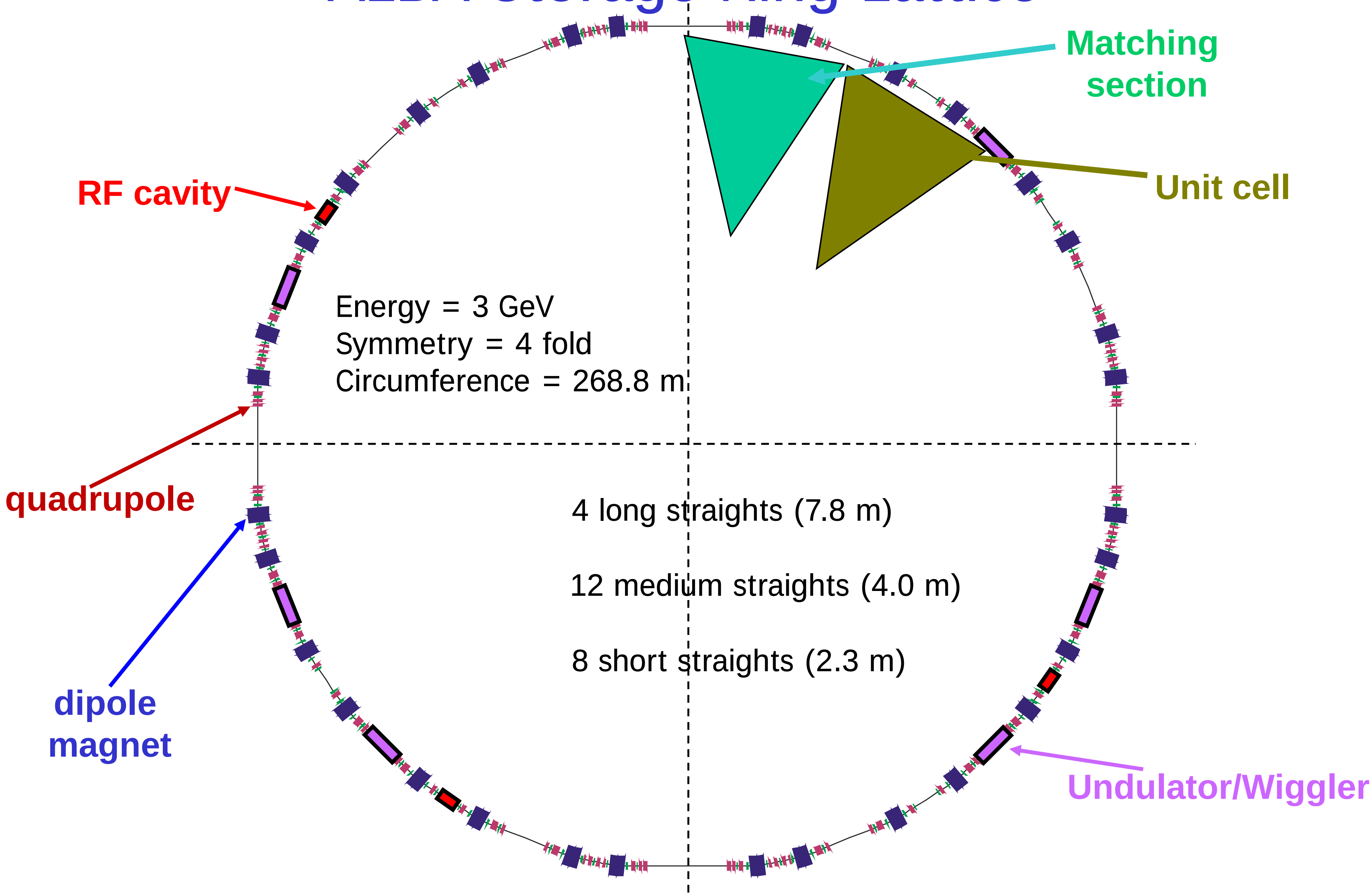
Putting all together... : DBA cell

Bending magnets and quads can be arranged in such a way that the dispersion vanish at the beginning and at the end of the cell:



This cells is called **Double Bend Achromat** and it's the typical building block of many synchrotrons, which need many long **straight sections with zero (or low) dispersion where to install Insertion Devices**.

ALBA Storage Ring Lattice



Many thanks to:

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Thanks!

Questions?