

# DETERMINISTIC DESIGN OF MULTIBEND HOA LATTICES

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Stop fishing  
in the dark

### 3<sup>rd</sup> generation light sources Bessy II

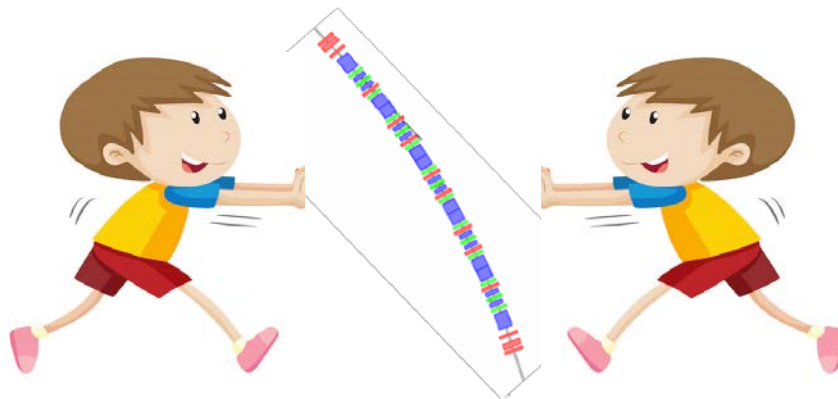


2 dipoles  
9 quadrupoles  
7 sextupoles  
19 drifts  
Consider symmetry  
=> 20+ parameters to optimize

### 4<sup>th</sup> generation light sources Bessy III



6 dipoles  
10 reverse bends  
24 quadrupoles  
19 sextupoles, octupoles?  
~ 50 drifts  
=> **too many parameters to handle**



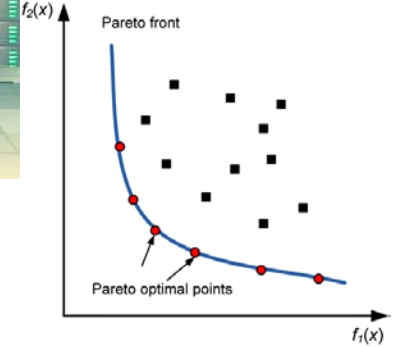
**OPTION A:** Take existing lattice and push  
towards own needs and demands

## OPTION B: Use Multi-Objective Genetic Algorithms



**Both methods show good results, but:**

You don't know how close you are to the optimum.  
There always *might* be an even better solution.



## HZB approach: Deterministic Lattice Design

- 'LEGO'-approach: Optimize smaller, generic subsections of the lattice individually  
*Cuts down on the number of parameters*
- Understand basic functionalities of elements in subsections  
*Why a reverse bend? Combined function or separate function magnets? How to order the magnets?*
- Deviation from strict 'LEGO' approach, asymmetries, injection straights, super bends ...  
*– all regarded as perturbations from the generic baseline lattice that do not alter the basic design choices*

## The layout of the talk

- Premises, goals, and limitations
- Optimization of the unit cell
- Optimization of the dispersion suppression cell
- Focusing towards the straight section
- First look at non-linearities
- Summary

## Premises:

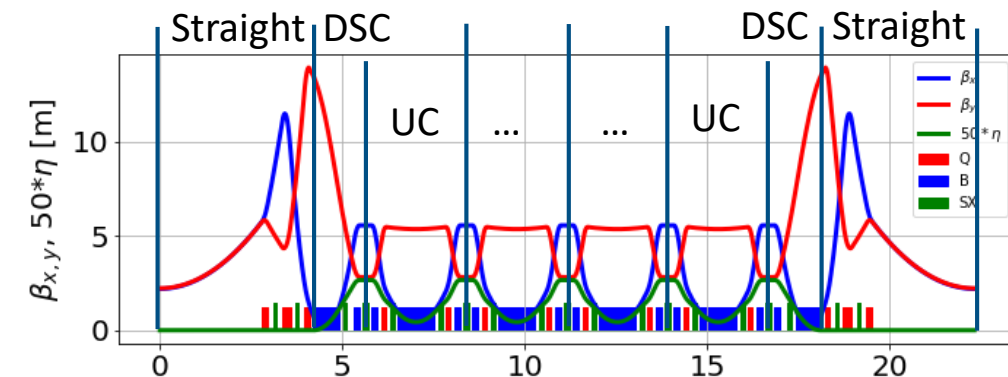
- **Multi-bend achromatic structure**  
*repetitive unit cell,*  
*dispersion suppression cell,*  
*straight section*
- **RB - Reverse bend**  
*reduces emittance*  
*decouples dispersion from betas*  
*reduces length*
- **HOA – higher-order achromat**  
*Obey certain phase advance rules*  
*to cancel out 2<sup>nd</sup> and 3<sup>rd</sup> order*  
*driving terms*

## Goals:

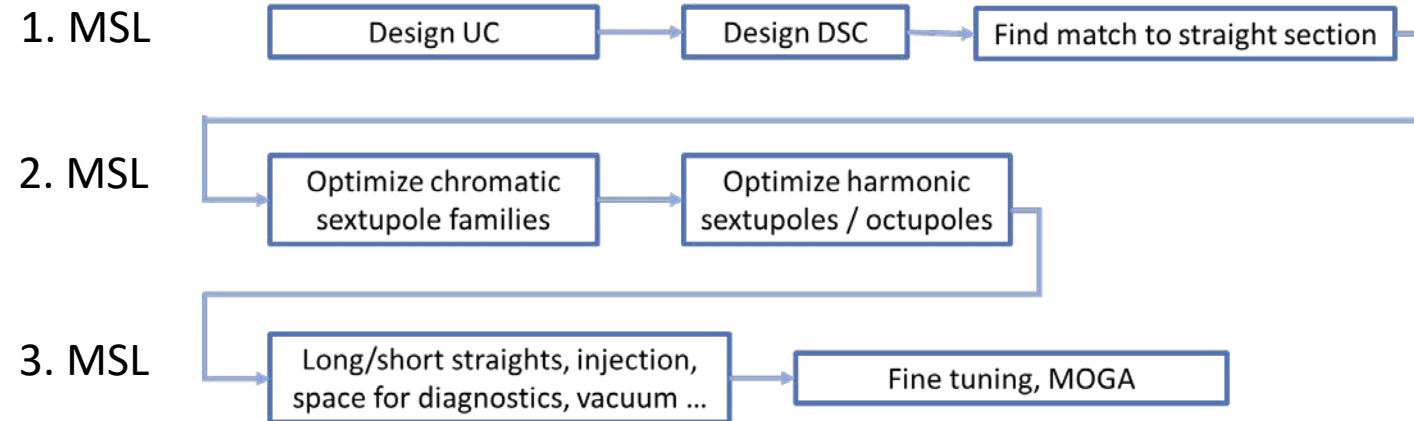
- 2.5 GeV
- Diff. limited at 1keV
- Low emittance 100pm
- Moderate, positive  $\alpha \sim 1e-4$
- At least 16 straights
- 5.6m long straight sections
- Equal betas in straight, <3m

## Limitations:

- Short circumference  $\sim 350m$
- ‘Off-the-shelf’ technology  
*(included from the beginning):*  
Bends: 1.3T  
Gradient: 80T/m  
Sextupoles: 4000T/m<sup>2</sup>  
Drifts: 0.10m
- Homogeneous bend radiation for PTB (few bends for main stack holder)



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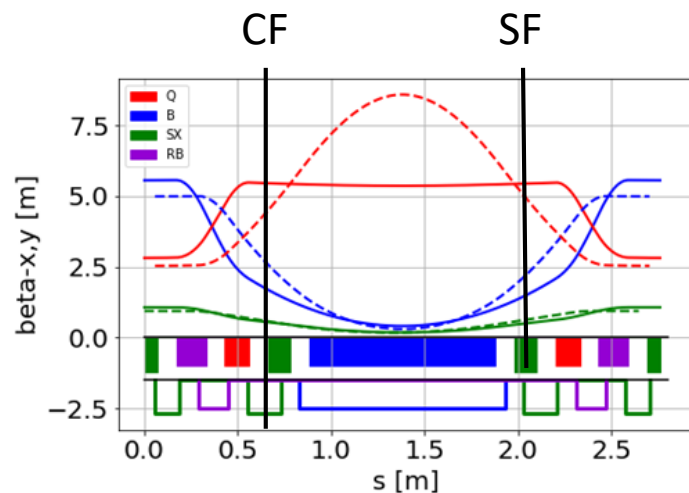
## The uniqueness of Unit Cell:

- Fix  $\beta_x$ ,  $\eta$  at center,  $\alpha_{x,y} = 0 - 2$  gradients => **unique solution**
- 2 phase advances (HOA) – 2 gradients => **unique solution**
- Freedom in dispersion by RB

## Magnetic set up of the Unit Cell:

### 6 magnet permutations

- include QD into the main bend or not (CF/SF)
- place the RB or SF at the outside
- (SF-UC) place QD or SD next to the central dipole



Explanation:  
Better separation of  
beta-functions

	Bend	UC type	$\epsilon$	$\xi_x$	$\xi_y$	SF [1/m <sup>2</sup> ]	SD [1/m <sup>2</sup> ]
1	CF	SF last	96	-0.69	-0.37	-25.1	19.6
2	CF	RB last	96	-0.81	-0.27	-25.5	20.0
3	SF	SF last	94	-0.75	-0.28	-17.8	10.2
		SD central					
4	SF	RB last	94	-0.85	-0.22	-27.5	16.4
		SD central					
5	SF	SF last	96	-0.75	-0.29	-18.6	15.4
		QD central					
6	SF	RB last	97	-0.82	-0.25	-26.0	22.3
		QD central					

For similar emittance and horizontal chromaticity, the SX-strength to compensate chromaticity can vary by a factor of >2!  
SF-UC has 40-50% lower SX-strength than CF-UC.

### Higher-Order Achromat condition for Unit Cell:

- Condition on phase advance in UC:  
 $\varphi_{x,y} / n = N$      $n$ : number of UC,  $N$ : lower integer
- Prerequisite for large mom. acceptance and large aperture
- Natural phase advance  $\varphi_x \sim 0.4$ ,  $\varphi_y \sim 0.1$
- $\varphi_x \sim 0.5$  not achievable
- $\varphi_x \sim 0.33$  longer cell/higher emittance  
 $\Rightarrow$  Need to build 6-bend
- 18, 20 super periods exceed circumference  
 $\Rightarrow$  Need to build 16 SP

**But: How to reach 100pm? Need to optimize dipoles!**

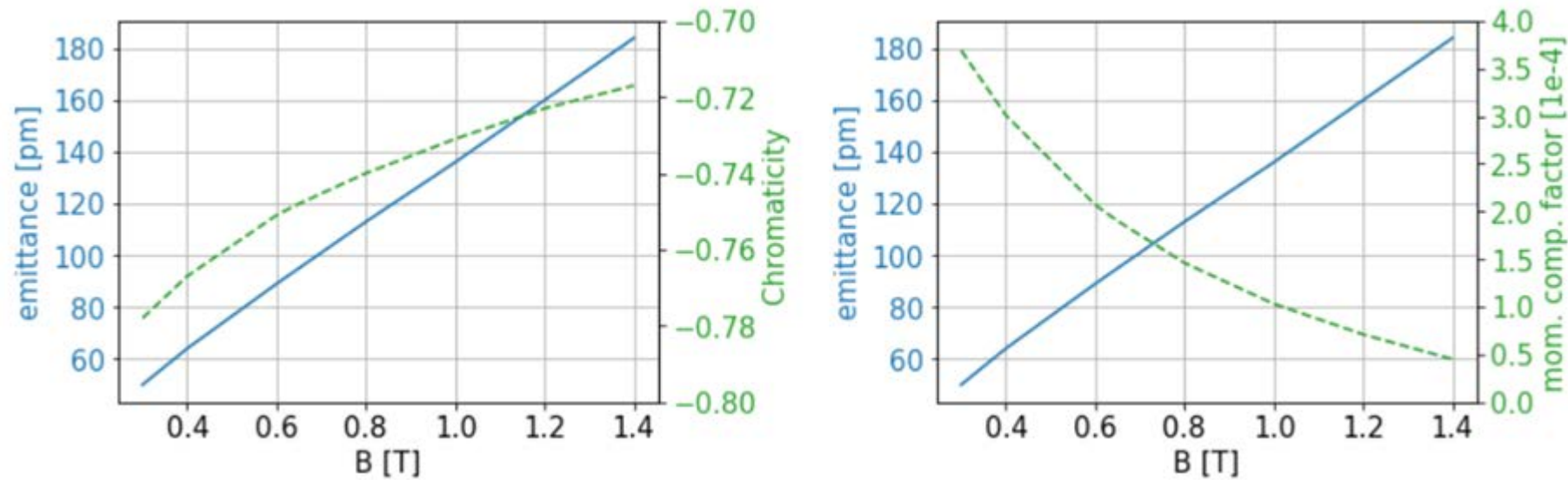
Table 2: Summary of Lattice Options.

SP	bends per SP	$\theta$ [°]	$\phi_x/2\pi$ HOA	$\beta_0$ [m]	$\varepsilon[\text{pm}]$ @ $\eta = 0.004$
16	7	3.75	0.33	0.66	271
	6	4.5	0.40	0.40	246
	5	5.625	0.50	-	-
18	7	3.67	0.33	0.64	215
	6	4.0	0.40	0.39	196
	5	5.0	0.50	-	-
20	7	3.0	0.33	0.63	176
	6	3.6	0.40	0.39	162
	5	4.5	0.50	-	-

*The conditions on the phase advance per unit cell is the strongest restriction in the design of the UC. Only a 16-period 6-MBA is feasible for BESSY III. (more bends show higher emittance due to necessarily larger  $\beta$ ,  $\eta$  in dipole)*



**Field strength of main dipole:** Change field/length of the dipole, fit HOA-condition (QD, QF), plot  $\varepsilon$ ,  $\xi$ ,  $\alpha$



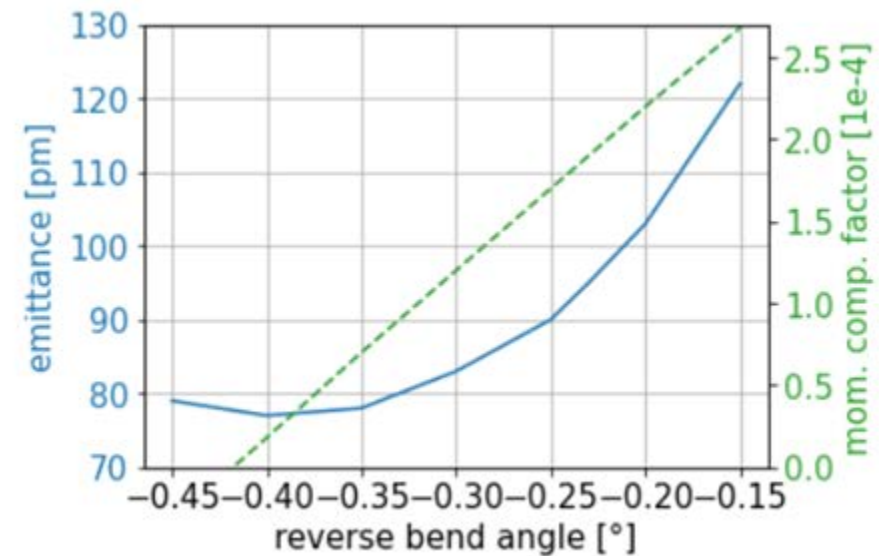
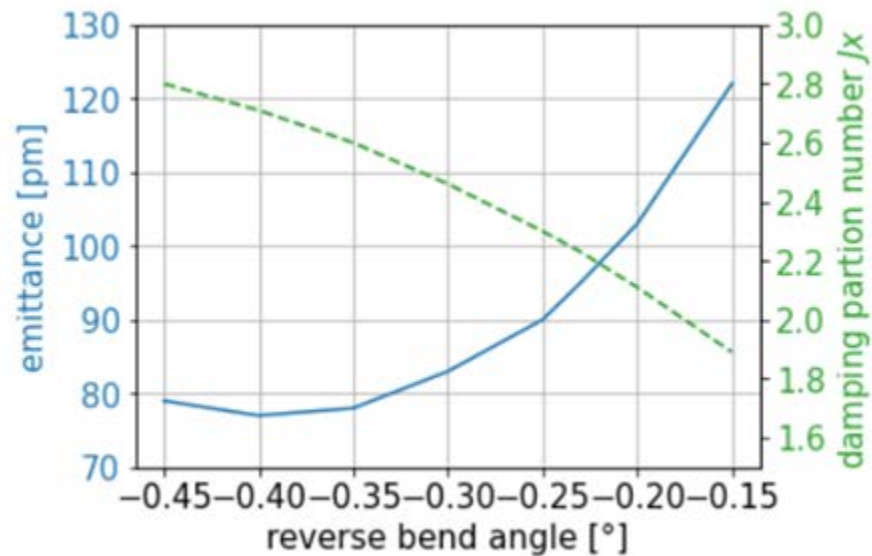
The emittance decreases by >70% for  $\Delta\xi \sim 8\%$ !  $\alpha: 0.5 \cdot 10^{-4} \rightarrow 3.7 \cdot 10^{-4}$

Need  $\alpha$  in UC  $\sim 2e-4 \Rightarrow B \sim 0.6T, L = 1.0m$

Plus: Long dipoles relax the focusing - TME conditions:  $\beta_{TME} = \frac{L}{\sqrt{15}}$   $\eta_{TME} = \theta \frac{L}{6}$   $\varepsilon_{TME} \propto \theta^3 \frac{2}{3\sqrt{15}}$



**Field strength of reverse bend:** Change displacement of RB/QF, fit HOA-condition (QD, QF), plot  $\varepsilon$ ,  $\alpha$ ,  $J_x$

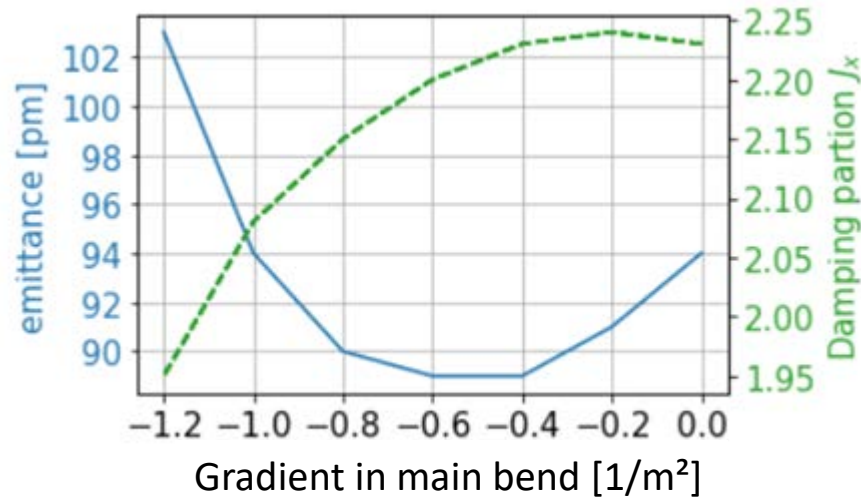


The emittance decreases by >60%, but:  $\alpha : 2.7 \cdot 10^{-4} \rightarrow 0.4 \cdot 10^{-4}$

$J_x$  increases because  $J_x(\text{main bend})$  decreases

Keeping  $\alpha \sim 2e-4$  limits the RB displacement (as well as technical limits)  $\Rightarrow RB \sim 0.225T$

**Combined function magnet - revisited:** Increase gradient in main bend, fit HOA-condition (QD, QF), plot  $J_x$   $\varepsilon$



Damping partition number:

$$J_x = 1 - \int_0^c \frac{\eta}{\rho} + 2k\eta\rho \, ds$$

*The damping partition number of the UC decreases (!) with raising gradient in the main bend.*

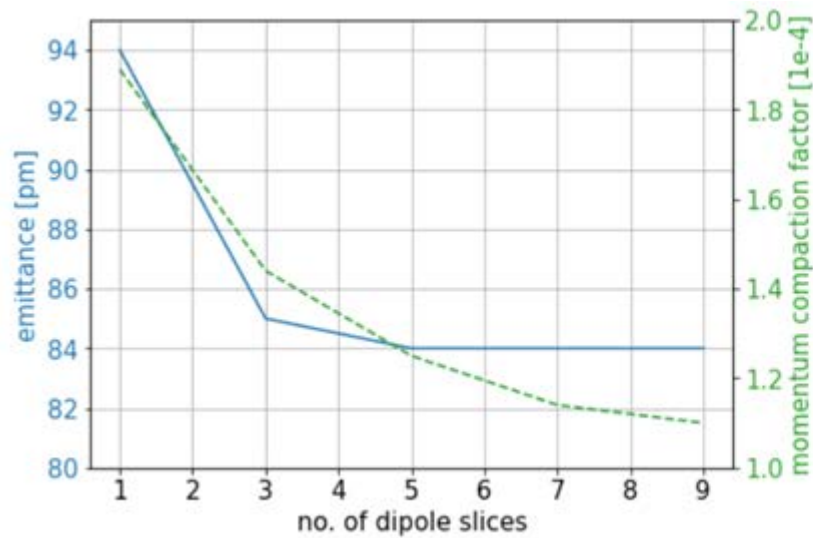
*Why? The contribution to  $J_x$  of the gradient is small compared to that of RB. (~0.3 per dipole, ~160 per RB)*

*The smaller field of QD leads to a smaller  $\eta$  and  $k$  of RB.*

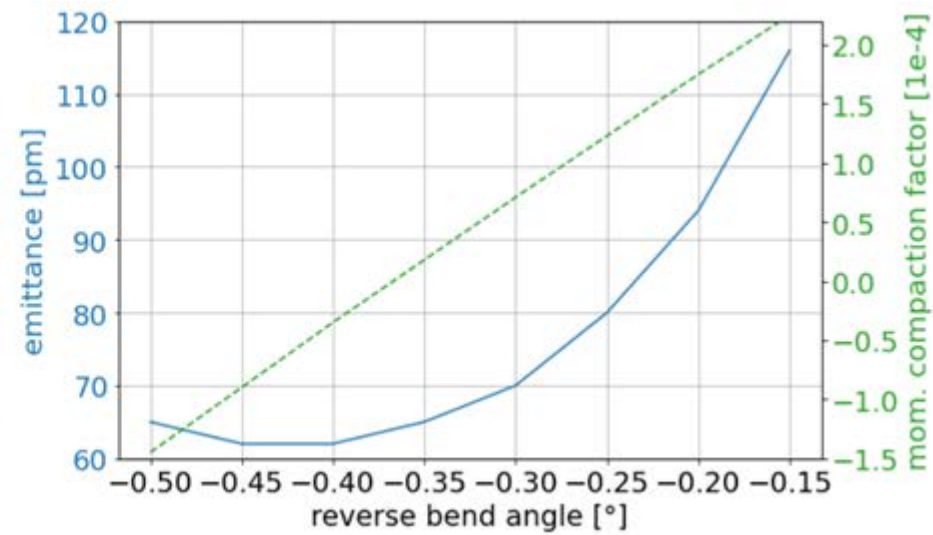
***The benefit of a gradient dipole is therefore disputable when an RB is used.***

## Longitudinal gradient bend:

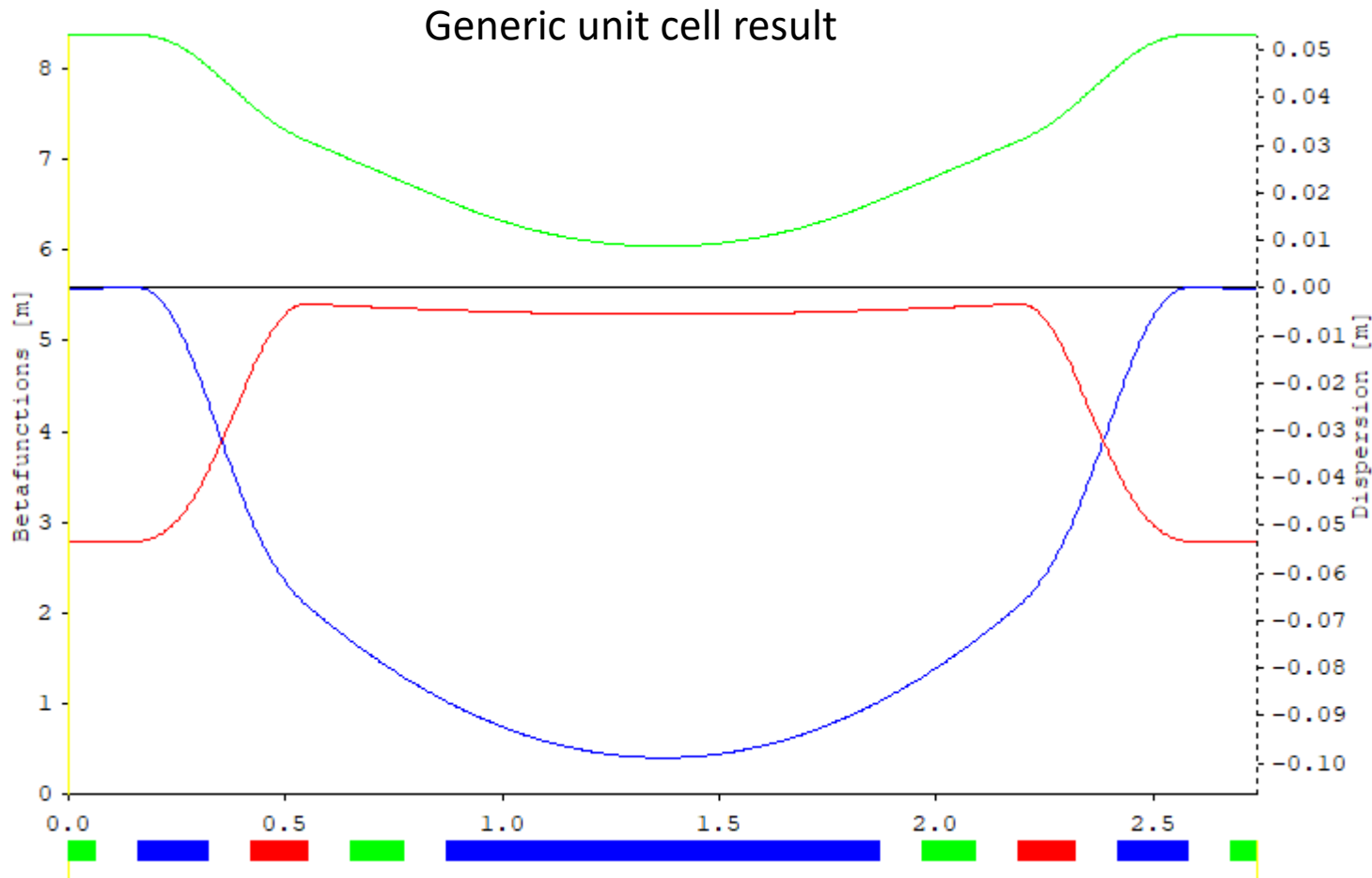
Split main bend into slices, optimize field distribution with OPA, fit HOA-condition (QD, QF), plot  $\varepsilon$ ,  $\alpha$ . For 3 slices, vary RB field.



$\beta_0$  is fixed by the phase advance  
 $\eta_0$  is fixed by the RB  
 More slices for fixed RB field don't help



50% emittance gain for increased RB field.  
 Limited by decreasing  $\alpha$ .



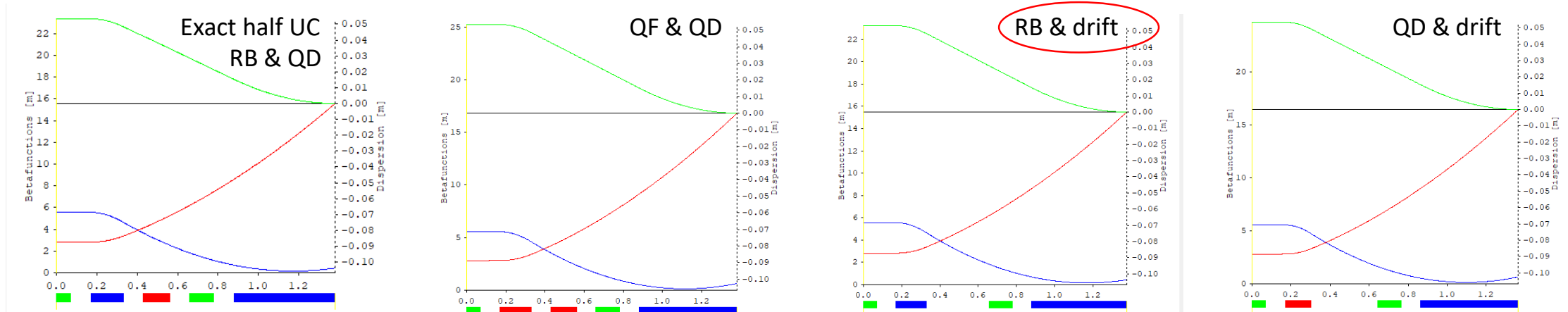
L [m]	2.76
$\theta$ [°]	4.0
$\theta_{\text{abs}}$ [°]	4.92
$\varepsilon$ [pm]	95
$\varepsilon_{\text{TME}}$ [pm] (main bend only)	93
$\alpha$	1.90e-4
$\xi_x, \xi_y$	-0.75, -0.28
$J_x, J_y$	2.23, 1.0
$\beta_{x0}, \beta_{y0}$	0.4, 5.4
$\eta_0$	0.0088

## Dispersion Suppression Cell:

- Guideline: As close as possible to half unit cell – to keep phase between SX
- Boundary conditions unit cell &  $\eta, \eta' = 0 \Rightarrow$  2 parameters for fitting  $\Rightarrow$  unique solution

	emittance [pm]	$\alpha [10^{-4}]$	RB/QF [ $1/m^2$ ]	QD [ $1/m^2$ ]	drift [m]
RB and QD	172	0.72	6.77	0.13	0.1
QF and QD	272	2.23	7.26	0.14	0.1
RB and drift	172	0.72	6.85	-	0.33
QF and drift	273	2.26	8.98	-	0.34

*$\beta$ -functions are similar, emittance lower with RB, QD has a negligible (positive!) gradient*



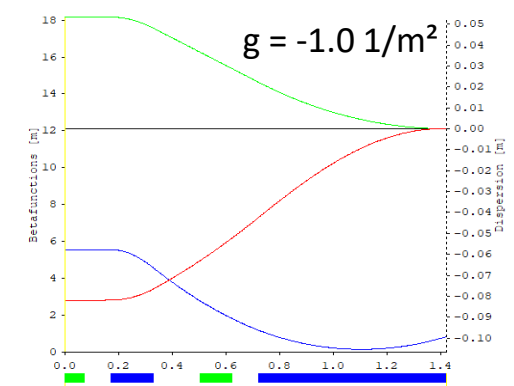
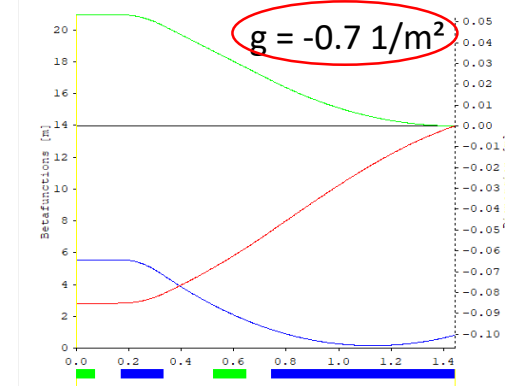
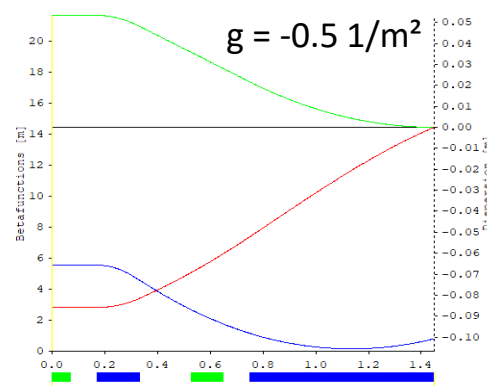
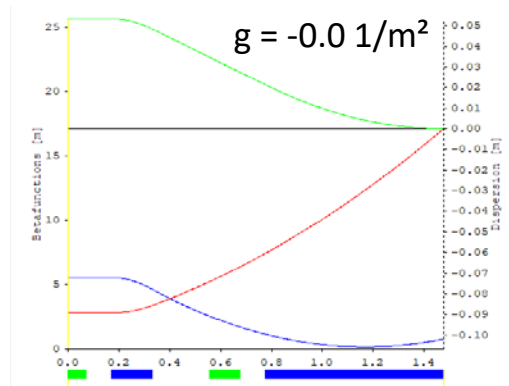
## Effect of a gradient in DSC-dipole:

- Homogeneous bend for PTB in UC
- Increase length to 70cm (technical feasibility)

gradient [ $1/m^2$ ]	emittance [pm]	$\alpha$ [ $10^{-4}$ ]	$\tilde{RB}$ [ $1/m^2$ ]	ML1B/UL1 [m]	$\xi_x$	$\xi_y$
0.0	127	0.41	6.86	0.23/0.00	-1.41	-1.08
-0.5	121	0.42	7.17	0.20/0.04	-1.45	-0.88
-1.0	116	0.43	7.49	0.18/0.11	-1.51	-0.72
-1.5	111	0.43	7.82	0.15/0.23	-1.55	-0.61

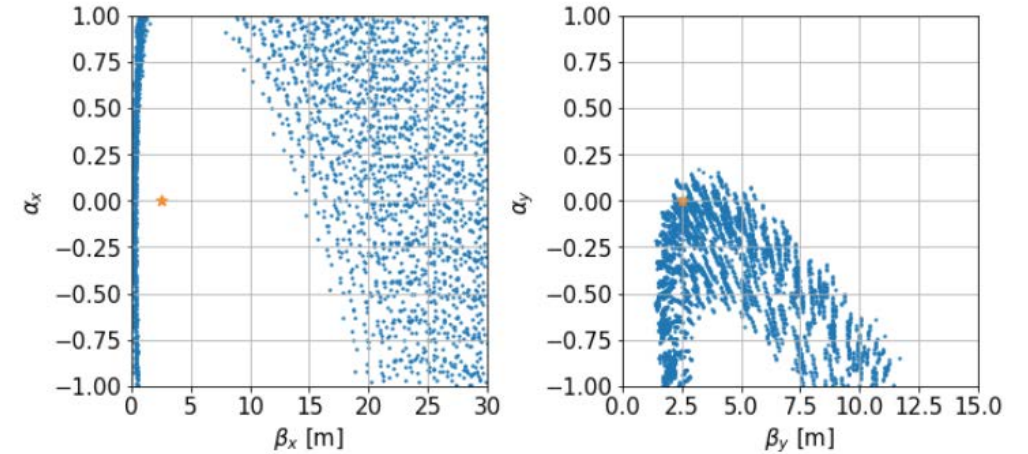
*The gradient lowers the emittance (no conditions on phase advance) and  $\beta_y$ ,  $\alpha_y$  towards straight => lower chromaticity  
(For the calculation of  $\alpha$  and  $\xi$ , the contribution of a generic triplet is taken into account.)*

$L_B = 0.7m$

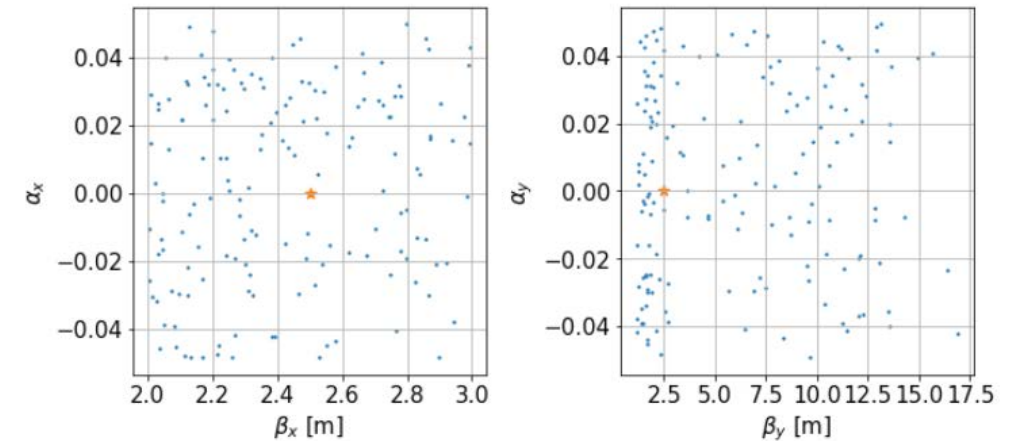


### Use numerical scan:

- Boundary condition of DSC, and straight section
- Scan over drift lengths and gradients
- Analyze results graphically
- Select solutions close to  $\beta_{x,y} = 2.5\text{m}$ ,  $\alpha_{x,y} = 0$
- Some re-fitting with optics program
- Try doublet: 2 gradients, 2 drifts
- Try triplet: 3 gradients, 3 drifts



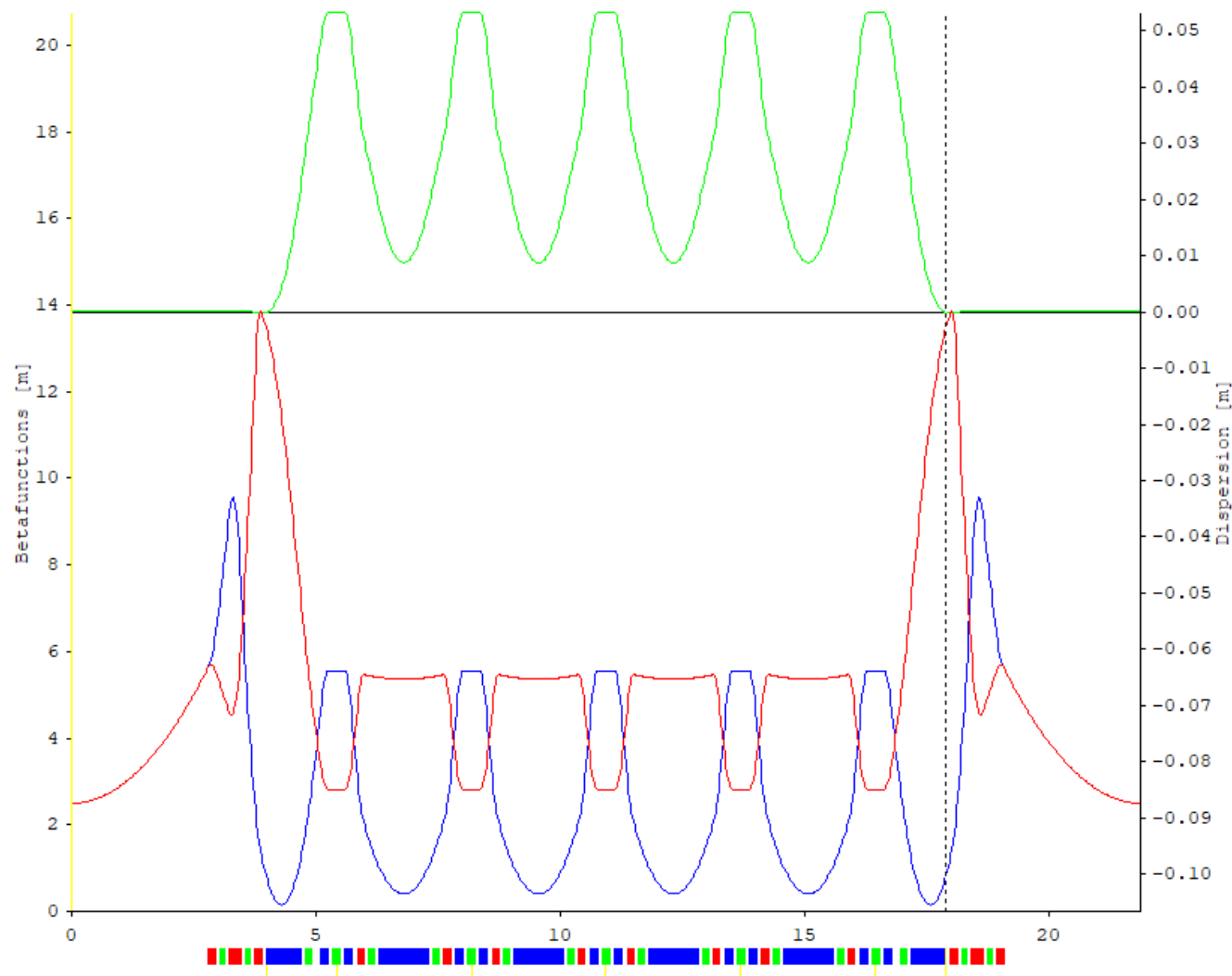
*Doublet: No solution for  $\beta_x$*



*Triplet: Chose the appropriate solution, f.e lowest chromaticity*



The composition of LEGO Blocks now yields a baseline lattice, that fulfills all demands  
=> First milestone lattice



L [m]	350.06
$Q_x / Q_y$	44/13
$\varepsilon$ [pm]	101
$\alpha$	$1.17e-4$
$\xi_x / \xi_y$	-95, -44
$J_x / J_y$	2.06, 1.0

**Chromaticity compensation – 2 chromatic sextupole families:**

- No harmonic sextupoles, no octupoles

CrX lin	0.00	-0.03
CrY lin	0.00	0.01
Qx	H21000	5.45
3Qx	H30000	2.00
Qx	H10110	4.84
Qx-2Qy	H10020	14.86
Qx+2Qy	H10200	3.37
2Qx	H20001	2.48
2Qy	H00201	0.64
Qx	H10002	0.03
CrX sqr	0.00	191.40
CrY sqr	0.00	46.13
dQxx	0	-147508.1
dQxy, yx	0	-62118.4
dQyy	0	-33487.2

Driving term by OPA

$$SD = -29.9 \text{ 1/m}^2$$

$$SF = 17.0 \text{ 1/m}^2$$

**Chromaticity compensation – 4 sextupole families:**

- No harmonic sextupoles, no octupoles

CrX lin	0.00	-0.03
CrY lin	0.00	0.01
Qx	H21000	21.18
3Qx	H30000	7.42
Qx	H10110	6.02
Qx-2Qy	H10020	11.19
Qx+2Qy	H10200	9.46
2Qx	H20001	0.84
2Qy	H00201	0.77
Qx	H10002	0.03
CrX sqr	0.00	64.69
CrY sqr	0.00	76.98
dQxx	0.00	3085.77
dQxy, yx	0.00	8272.42
dQyy	0.00	-33651.98

4 SX-families suppress the TS with momentum as well as the TS with amplitude

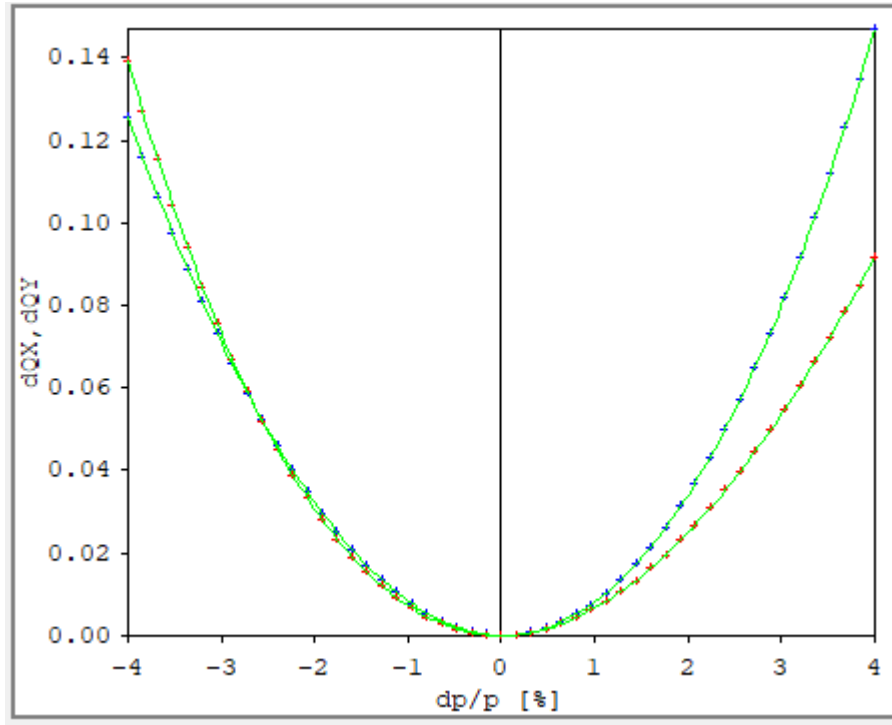
$$SD1 = -26.7 \text{ 1/m}^2$$

$$SF1 = 12.6 \text{ 1/m}^2$$

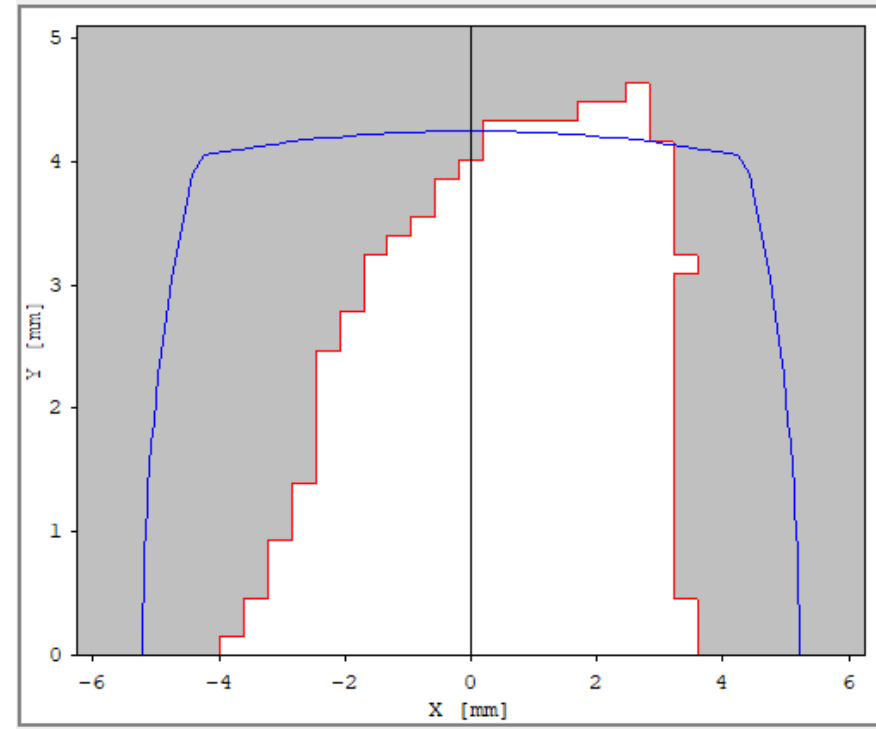
$$SD2 = -39.8 \text{ 1/m}^2$$

$$SF2 = 23.9 \text{ 1/m}^2$$

- No harmonic sextupoles, no octupoles



*Momentum acceptance 3-4%*



*Initial dynamic aperture ( $\beta_{x,y} = 2.5\text{m}$ )*

## Summary and conclusion

- Careful analysis of the substructures of MBA lattices leads to a better understanding of design options
- Promising baseline lattices can be deterministically constructed
- Technical feasibility should be included from the start
- Reducing the sextupole strength in the linear optimization yields promising non-linear properties
- Splitting of chromatic sextupole families is mandatory (  $\geq 4$  )

We are not finished:

- Octupoles and/or harmonic sextupoles
- Tune scan
- All drifts are 0.1m – can we improve by variation?
- Need for injection straight? Super-bend? .....
- ...
- MOGA for fine tuning

**I'm really sorry that I couldn't join this workshop and I hope that  
you are having a good time, fruitful discussions, and a great dinner!  
I'm looking forward to Paul's report!  
Bettina**